A Quantum Look at Difference-Frequency Generation

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Introduction

Nonlinear optical processes can be used in new quantum communication schemes. Here we look at difference-frequency generation (DFG).

- DFG has applications in quantum teleportation schemes, measurement-free error correction, etc.,
- DFG is the **stimulated** form of spontaneous parametric down-conversion (SPDC),
- application requires a quantum optical description of DFG.

Difference-Frequency Generation

DFG is a second order nonlinear optical process. The nonlinear polarization (P_3) of the medium produces the difference-frequency beam (E_3):

$$E_3 \;\propto\; P_3 = 2\,\varepsilon_0\,\chi^{(2)} E_1\,E_2^*. \qquad (1)$$

- **Pump** (ω_1) and **signal** (ω_2) beams enter the nonlinear crystal and produce an **idler** beam (ω_3),
- $\omega_3 = \omega_1 - \omega_2$ (difference-frequency),
- in classical optics, DFG is used for parametric amplification or phase conjugation.

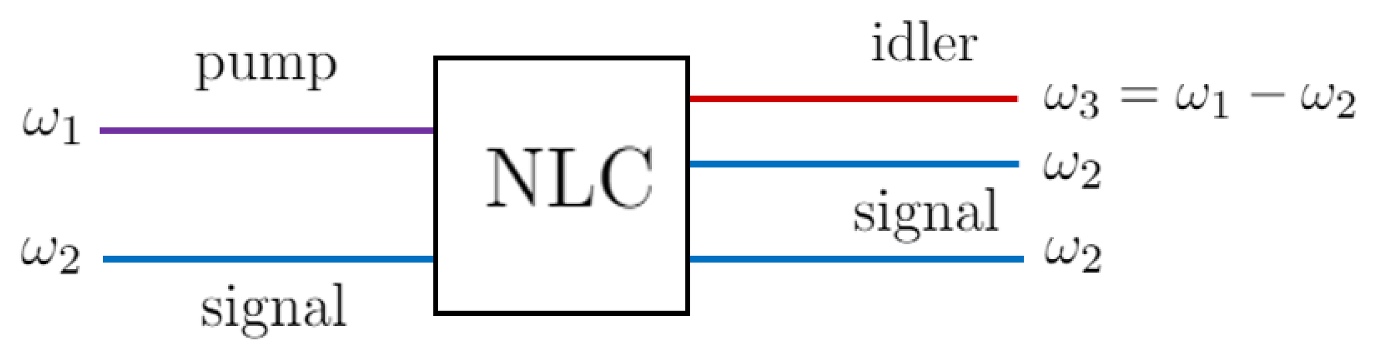


Figure 1:Single photon level of DFG process

Classical Optics

Coupled wave equations for the signal (A_s) and idler (A_i) amplitudes are obtained from the nonlinear Maxwell equations, and solved, giving:

$$A_s(z) = A_s(0)\,\cosh\kappa z, \qquad (2)$$

$$A_i(z) = i\left(\frac{n_2\,\omega_3}{n_3\,\omega_2}\right)^{1/2}\frac{A_p}{|A_p|}A_s^*(0)\,\sinh\kappa z. \qquad (3)$$

Here z is the propagation distance, κ is a generalised wavenumber and the respective refractive indices of the beams are n_2 and n_3.

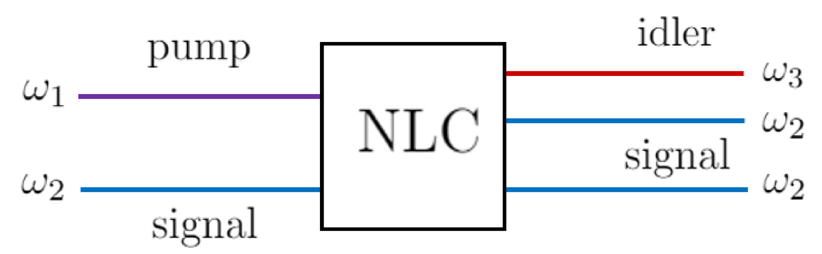
Quantum Formulation

The nonlinear process is described by the interaction term of the quantized Hamiltonian, with the right combination of **annihilation and creation operators** that will destroy a photon in the pump mode and create photons in the signal and idler modes,

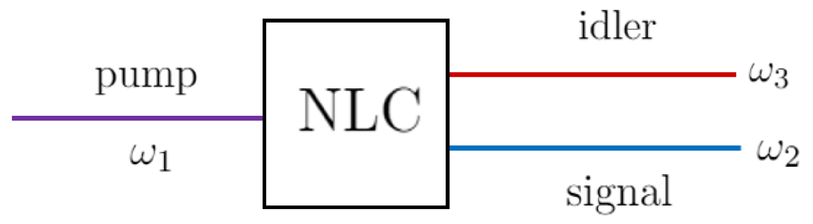
$$\hat{H}_I = \frac{-i\chi_{eff}^{(2)}\hbar^{3/2}}{\sqrt{\varepsilon_0}(2\pi)^3}\int_V \mathrm{d}^3\mathbf{r}\int \mathrm{d}^3\mathbf{k}_1\mathrm{d}^3\mathbf{k}_2\mathrm{d}^3\mathbf{k}_3 e^{-i(\mathbf{k}_3+\mathbf{k}_2-\mathbf{k}_1)\cdot\mathbf{r}}\; e^{i(\omega_3+\omega_2-\omega_1)t}\;\hat{a}(\mathbf{k}_1)\hat{a}^\dagger(\mathbf{k}_2)\hat{a}^\dagger(\mathbf{k}_3) + h.c.\;. \qquad (4)$$

Initial state: $|\psi_i\rangle = |1_{U_p}\rangle_p\,|1_{S_l}\rangle_s\,|0\rangle_i$.
Output state after crystal (here $\mathbf{q}$ denotes transverse wave vectors):

$$|\psi_{out}\rangle = \int \mathrm{d}^2\mathbf{q}_2\mathrm{d}^2\mathbf{q}_3\;\Phi(\mathbf{q}_2,\mathbf{q}_3)\;\hat{a}^\dagger(\mathbf{q}_2)\;\hat{a}^\dagger(\mathbf{q}_3)\,|1_{S_l}\rangle_s\,|0\rangle_i\,. \qquad (5)$$



DFG



SPDC

Main Results

In order to calculate $|\psi_{out}\rangle$, we need an appropriate expression for the mode function Φ. We found that the **decomposition of** Φ is written in the product basis:

$$\Phi(\mathbf{q}_2,\mathbf{q}_3) = \sum_{k,m} N^p_{k,m}\,\mathcal{I}_k(\mathbf{q}_3)\,\mathcal{S}_m(\mathbf{q}_2), \qquad (6)$$

which gives,

$$|\psi_{out}\rangle = \sum_k N^p_{k,l}\sqrt{2}\,|2_{S_l}\rangle\,|1_{I_k}\rangle + \sum_{m\neq l}\sum_k N^p_{k,m}\,|1_{S_l},1_{S_m}\rangle\,|1_{I_k}\rangle\,. \qquad (7)$$

Output state of DFG is a product state:

$$|\psi_d\rangle \equiv \left(\sum_k N^p_{k,l}\,|1_{I_k}\rangle\right)\sqrt{2}\,|2_{S_l}\rangle\,, \qquad (8)$$

$$\text{where}\quad N^p_{k,l} = \frac{1}{(2\pi)^2}\int \mathrm{d}^2\boldsymbol{\rho}\;\mathrm{I}_k^*(\boldsymbol{\rho})\;\mathrm{S}_l^*(\boldsymbol{\rho})\;\mathrm{U}_p(\boldsymbol{\rho}). \qquad (9)$$

The coefficients $N^p_{k,l}$ can be thought of as the overlap, $N^p_{k,l} = \langle 1_{I_k}|1_{I_d}\rangle$, of an idler photon with a spatial mode given by $\mathrm{I}_k(\boldsymbol{\rho})$ and the idler mode of the photon created by DFG which we have called $\mathrm{I}_d(\boldsymbol{\rho})$. Here $\boldsymbol{\rho}$ is the transverse position coordinates.
The **mode of the idler** is therefore given by $\mathrm{I}_d(\boldsymbol{\rho}) = \mathrm{S}_l^*(\boldsymbol{\rho})\;\mathrm{U}_p(\boldsymbol{\rho})$, a product of the complex conjugate signal mode and of the pump mode.

Conclusion

- General mode decomposition is used instead of Schmidt decomposition,
- we derive a quantum optical description of DFG photon modes,
- the derived result agrees with the classical result, shows phase conjugation, amplification and that the output state is not entangled.