Investigating a New Approach to Black Hole Quasinormal Modes: Physics-Informed Neural Networks

A. M. Ncube, G. E. Harmsen and A. S. Cornell
Department of Physics, University of Johannesburg

Contact Information:
C1 Lab 125, Department of Physics
Auckland Park Kingsway Campus
University of Johannesburg
South Africa

Phone: +27 61 982 6658
Email: ncubeanele@gmail.com

Introduction

In response to small perturbations, a black hole (BH) produces damped waves called quasinormal modes (QNMs) as it returns to its equilibrium state [1]. The QNM frequencies (QNFs), denoted by ω, are complex-valued to account for this damping . Computing them requires solving second-order differential equations (DEs). The present research investigates a potential new approximation method for computing QNFs in the form of *Physics-informed neural networks* (PINNs). As a first step, PINNs were applied to solve an inverse problem (i.e. where the unknown parameter, ω, within the DEs is computed using data for the wave-functions satisfying the DEs). We focused on the perturbations of near extremal Schwarzschild-de Sitter (SdS) and Reissner-Nordström-de Sitter (RNdS) black holes whose effective potential is given exactly by an inverted Pöschl-Teller potential.

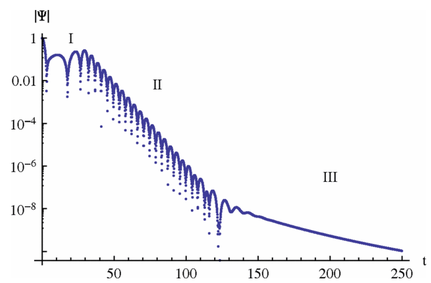

Figure 1: A time-domain profile for gravitational perturbations of a Schwarzschild black hole. At early times (II) the QNM signal dominates the signal [2, 3].

Methodology

PINNs are neural networks that are trained to solve DEs by imposing physics-constraints such as boundary and/or initial conditions [4]. To build the PINNs, we are using the DeepXDE library in Python [5].

Two components of PINNs:

1. The physics-uninformed *feed-forward neural network* (FNN) represents the surrogate of the solution to the DEs (left of figure 3).
2. The *loss-function* in which the physics-constraints are embedded that make FNN approximations physics-informed (right of figure 3).

The PINN Algorithm

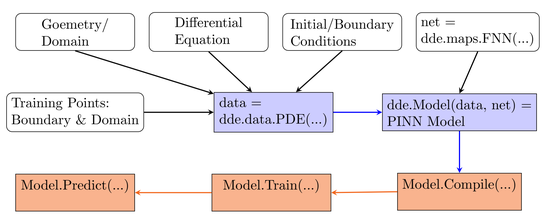

Figure 2: The stages for setting up and training PINNs. In the training phase, weights and biases within the FNN are iteratively updated via minimization of the physics-informed loss function to steer the FNN towards an accurate approximate solution.

Perturbations of spherically symmetric BHs

Spherically symmetric black holes have space-time metrics generally given by [6]:

$$ds^2 = -f dt^2 + f^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2),$$

where r is the radial distance from the center of the BH. The metric function (f) for SdS & RNdS BHs is [6, 7]:

$$f = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{\Lambda r^3}{3},$$

where M and Q are the mass and electric charge of the BH in geometrical units and Λ is the cosmological constant ($Q = 0$ for SdS BHs).

Perturbation equations of BHs are Schrödinger-like differential equations that encode the dynamics of BH perturbations. In general, we have [6]:

$$\frac{d^2\psi}{dx^2} + (\omega^2 - V(r))\psi = 0,$$

where x is the "tortoise" coordinate related to r: $dr/dx = f$ [2]. Additionally the effective potential for massless scalar perturbations of SdS and RNdS BHs is [7, 8]:

$$V(r) = f\left[\frac{l(l+1)}{r^2} + \left(\frac{2M}{r^3} - \frac{2Q^2}{r^4} - \frac{2\Lambda}{3}\right)\right],$$

where l is the multipole quantum number and $Q = 0$ for SdS BHs.

QNM wave-functions are solutions to the Schrödinger-like perturbation equations that satisfy the boundary conditions:

$$\psi(x) \sim \exp(\pm i\omega x), \quad x \to \pm\infty.$$

Physically, this implies plane waves going down the BH horizon and going out at spatial infinity [9].

Effective potential in the near extremal limit

Near extremal SdS & RNdS black holes refer to a cases where the cosmological horizon is very close (in the r coordinate) to the BH horizon. In this limit, the effective potential of the SdS and RNdS are given exactly by an *inverted Pöschl-Teller potential* [6, 7]:

$$V(x) = \frac{V_0}{\cosh^2(\kappa_b x)},$$

where $V_0 = \kappa_b^2 l(l+1)$ for massless scalar perturbations and κ_b is the surface gravity associated with the BH horizon. For massless scalar fields, the exact QNM wave-functions and QNFs are [6, 7, 10], respectively:

$$\psi(x) = [\xi(\xi-1)]^{i\omega/2\kappa_b} \cdot {}_2F_1\left(1+\beta+i\frac{\omega}{\kappa_b}, -\beta+i\frac{\omega}{\kappa_b}; 1+i\frac{\omega}{\kappa_b}; \xi\right),$$

$$\frac{\omega}{\kappa_b} = \sqrt{\left(l(l+1) - \frac{1}{4}\right)} - i\left(n + \frac{1}{2}\right), \quad n = 0, 1, 2, \ldots$$

Here $\xi^{-1} = 1 + \exp(-2\kappa_b x)$ and $\beta = -1/2 + (1/4 - V_0/\kappa_b^2)^{1/2}$.

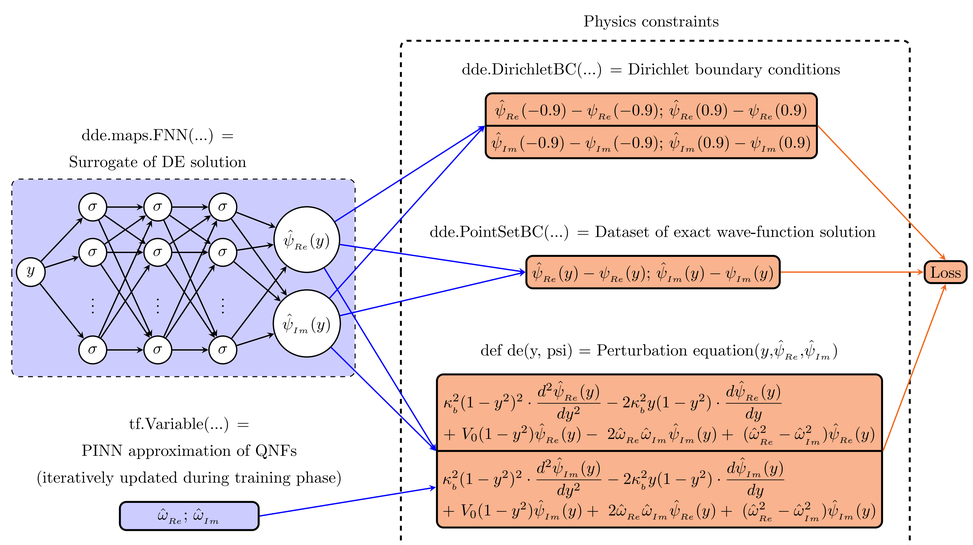

Figure 3: The set-up for the PINN used to solve our perturbation equation with the effective potential given by an inverted Pöschl-Teller potential. The equation is in terms of $y = \tanh(x)$, which gives a finite domain (-1,1), and it is split into real and imaginary parts for easier implementation in the DeepXDE library. The training hyperparameters used for all our computations: FNN with 3 hidden layers, 20 nodes per layer; *tanh* as a non-linear activation function; learning rate of 0.001; "adam" as an optimiser; training data consisting of 100 domain points and a dataset with 50 uniformly distributed true values of the wave-funtion $\psi(y)$ in the spatial domain [-0.9,0.9].

Results

Table 1: PINN approximations of the QNFs (in geometrical units) for massless scalar perturbations of SdS & RNdS BHs.

		PINN		Exact [6, 7]		L_2
n	l	ω_{Re}/κ_b	ω_{Im}/κ_b	ω_{Re}/κ_b	ω_{Im}/κ_b	relative error
0	1	1.32287	-0.49997	1.32288	-0.50000	0.00214%
	2	2.39790	-0.50002	2.39792	-0.50000	0.00106%
	3	3.42761	-0.50032	3.42783	-0.50000	0.01105%
	4	4.44096	-4.98165	4.44410	-0.50000	0.08128%
	5	5.43513	-0.50716	5.45436	-0.50000	0.37463%

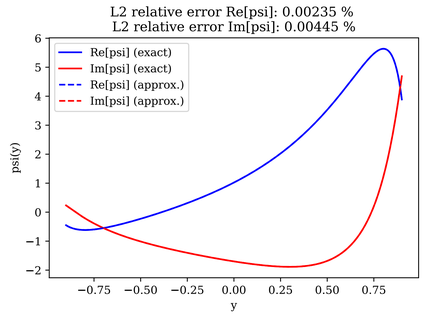

Figure 4: Plot of the PINN approximation of QNM wave-function for $n = 0$, $l = 1$ from table 1.

Conclusion

Our results indicate that PINNs have the capacity to accurately approximate QNFs for massless scalar perturbations of SdS and RNdS BHs which solved as inverse problems. Future considerations include *going beyond inverse-problems* by applying PINNs on general black hole perturbation equations without data for QNM wave-functions (e.g. asymptotically flat Schwarzschild black hole).

Acknowledgements

I gratefully acknowledge the support of my supervisors and the funding granted to me by the Faculty of Science, University of Johannesburg.

References

[1] S. Dey and S. Chakrabarti. Eur. Phys. J. C, 79(6):504, 2019.

[2] R. A. Konoplya and A. Zhidenko. Rev. Mod. Phys., 83:793–836, 2011.

[3] K. D. Kokkotas and B. Schmidt. G.Living Rev. Rel., 2:2, 1999.

[4] G. Karniadakis, Y. Kevrekidis, L. Lu, P. Perdikaris, S. Wang, and L. Yang. Nature Reviews Physics, 3(6):1–19, 05 2021.

[5] L. Lu, X. Meng, Z. Mao, and G. E. Karniadakis. SIAM Review, 63(1):208–228, 2021.

[6] V. Cardoso and J. P. S. Lemos. Phys. Rev. D, 67:084020, 2003.

[7] C. Molina. Phys. Rev. D, 68:064007, 2003.

[8] H. T. Cho, A. S. Cornell, J. Doukas, T. R. Huang, and W. Naylor. Adv. Math. Phys., 2012:281705, 2012.

[9] L. C. Stein. Journal of Open Source Software, 4(42):1683, 2019.

[10] V. Ferrari and B. Mashhoon. Phys. Rev. D, 30:295–304, 1984.