**Relativistic oscillator under the influence of a magnetic field in Noncommutative space**

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**Abstract**

It is known that the relativistic harmonic oscillator is one of the models of fundamental physics that contains bound states with non-zero residual energy, which is explained by the quantum confinement effect. In this thesis we investigated the thermal properties of the oscillator by KleinGordon (KGO) under the action of a uniform magnetic field in a non-commutative space, in which the exact energy eigenvalues ​​and normalized wave functions are obtained analytically. In this study we found that the spatial geometric deformation of (KGO) is equivalent to the behavior of the KleinGordon equation in a commutative space, which describes the oscillating motion of a particle without spikes exposed to the action of a constant magnetic field

**Keywords:** KG Oscillator, Non-Commutative space, Thermodynamic Properties

1. **Introduction**

Non-commutative space theories have been widely studied over the past few years (for a review see Ref. [1]).M-theory compactification, [2],string theory in nontrivial contexts,[3] and quantum Hall effect are related to non-commutative field theories.[4] Inclusion of non-commutativity in quantum field theory can be accomplished in one of two separate ways through Moyal \* product on the space of ordinary functions, or by defining the field theory of an intrinsically non-commutative coordinate operator space.[1,5] The equivalence between the two approaches has been nicely defined in Ref.[6]. The study of the one particle market, which sparked an interest in the study of non-commutative quantum mechanics, may provide a simple insight into the role of non-commutativity in field theory (NCQM)[7-8].

2- **Theoretical study**

The noncommutative spaces can be realized as spaces where the coordinate operator satisfies the commutation relations:

|  |  |
| --- | --- |
|  | (1) |

 Where, is an antisymmetric tensor of space dimension (length)2 In the context of this deformation the product of any two functions is equivalent to the star Moyal product defined by

|  |  |
| --- | --- |
|  | (2) |

Where *f* and are two arbitrary functions and assumed to be infinitely differentiable. The non-commuting coordinate operator can be expressed in terms of commuting coordinate operators and their momentum operators in the form, [8-9]

|  |  |
| --- | --- |
| ,  | (3) |

With the parameter antisymmetric tensor *θ* chosen

|  |  |
| --- | --- |
|  | (4) |

The Klein--Gordon oscillator in a two-dimensional ((2+1)-dimensional space-time) commutative space and in a magnetic field has the following form:

|  |  |
| --- | --- |
|  | (5) |

Where

 (6)

With (7)

Following a straightforward calculation, we get exactly the following equation:

 (8)

Which can be written as:

 (9)

 With

,

 (10)

To get the exact solution of (9), we use the polar coordinates in momentum space *(r, ϕ),* and we used a separate form containing the azimuthal quantum number *l*

 (11)

Therefore, the expression of the equation will be such:

 (12)

With:

 (13)

By using the following transformation then, we obtain

 (14)

Therefore, we use Nikiforov Uvarov method in order to obtain the energy eigenvalues and the wave function; we get the spectrum as:

 (15)

Where we have used the definition and is the cyclotron frequency.

Returning to the wave function Ψ(r,ϕ) written as

 (16)

Here *C* noted as the normalization constant

**3. THERMODYNAMIC PROPERTIES:**

In this section, we examine the effect of non-commutative geometry on the thermal properties of KG oscillator. Returning to the spectrum (15) which can be written as:

 (17)

With:

 (18)

 (19)

Because all the thermodynamic quantities can be obtained from the Z partition function, we first calculate this function of the system, which is defined according to the temperature T as

 (20)

With, is the Boltzmann constant, and is ground state energy*. (l=0)*

In addition, for the states of negative energy, the function of the Z-partition of the oscillator of K-G at finite temperature T has the form:

 (21)

Now, to evaluate this function, we use the formula Euler-Maclaurin. For carrying out our analysis on the thermodynamics of the K-G oscillator, we will limit to the stationary states of positive energy

By replacing (17) in (21), we obtain the partition function of the K-G oscillator in deformed space as:

 (22)

Then we employ Euler-Maclaurin formula; we obtain the partition function:

 (23)

Where

The thermodynamic properties of the physical system, such as free energy F, mean energy U specific heat C, and entropy S, can be calculated from the following expressions:

 (24)

Such:

 (25)

Following the same manner to obtain the other properties. In the following, all the profiles of the thermodynamic quantities as a function of the temperature variable were presented for different values of the deformation parameter 𝜃



 

**Fig1**: the free energy F as the function of T for **Fig2:** the mean energy U as the function T deformation values. for deformation values.



 

**Fig3**: the capacity heat C as the function of T for **Fig4:** the entropy S as the function T

 deformation values. for deformation values.

Where we have used the Hartree atomic units

We observe that the thermodynamic properties have influenced by the parameter .

**4. CONCLUSION**

In this framework, we have exposed an explicit calculation of the Klein Gordon oscillator equation under the influence of a uniform magnetic field in a noncommutative space relatively to a particle of spin 0.type. We have obtained the analytical expressions for bound state energies and wave functions by using the NU.method. Finally, we have plotted the thermal properties in the regime of high temperature, T, which have been, affected by NC algebra.

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