

The Equation of State (EoS) of hadronic matter from the microscopic Ultra-relativistic Quantum Molecular Dynamics (UrQMD) model

T.E Nemakhavhani¹, M Younus², A Muronga³

¹Physics Department, University of Johannesburg, P.O.Box 524. Auckland Park 2006, Johannesburg, RSA

² Department Physics , Nelson Mandela University, Summerstrand, Port Elizabeth, 6031, South Africa

³ Faculty of Science, Nelson Mandela University, Summerstrand, Port Elizabeth, 6031, South Africa

E-mail: tenemakhavhani@gmail.com

Abstract. The Equation of State (EoS) of a hot and dense hadron matter is studied using a microscopic transport model which can support the Large Hadron Collider energies of up to $\sqrt{s_{nn}} = 14$ TeV, namely the Ultra-relativistic Quantum Molecular Dynamics (UrQMD 3.3). The molecular dynamics simulation is performed for a system of light meson species (π ; ρ and K) in a box with periodic boundary conditions. The equilibrium state is investigated by studying the chemical equilibrium and the thermal equilibrium of the system. The particle multiplicity equilibrates with time, and the energy spectra of different light meson species have the same slopes and common temperatures when thermal equilibrium is reached. The solution of the EoS allows for better understanding of the final state of interactions, which is dominated by hadrons produced during ultra-relativistic heavy ion collisions.

1. Introduction

A large number of studies in heavy ion physics and high energy physics have been done using the results from the Relativistic Heavy Ion Collider (RHIC). Now with the restart of the Large Hadron Collider (LHC) physics programme, the field of high energy nuclear physics, and especially heavy ion physics, has gone into a new era. It is now possible to explore the properties of Quantum-Chromo-Dynamics (QCD) at unprecedented particle densities and temperatures, and at much higher energies than that produced at RHIC, from $\sqrt{s} = 200$ GeV to $\sqrt{s} = 14$ TeV at the LHC [1].

High energy heavy ion reactions are studied experimentally and theoretically to obtain information about the properties of nuclear matter under extreme conditions at high densities and temperatures, as well as about the phase transition to a new state of matter, the quark-gluon plasma (QGP) [2, 3, 4, 5]. This work reports on the EoS of hadronic matter consist of lighter mesons only extracted using the UrQMD model. The knowledge of the EoS is important for better understanding of the final state of interactions which is dominated by hadrons produced during ultra-relativistic heavy ion collisions. The EoS of nuclear matter is one of the key points to gain further understanding since EoS directly provides the relationship between the pressure and the energy at a given net-baryon density [6]. The thermodynamic properties, transport

coefficients and EoS for hadron gas are a major source of uncertainties in dissipative fluid dynamics [4, 7, 8].

In order to study the EoS we look at the system in equilibrium state. The equilibrium state of the system which is made up of by a light meson species is studied by evaluating particle number densities from chemical equilibrium, energy spectra as well as the temperatures from thermal equilibrium of different light meson species in a cubic box, which imposes periodic boundary conditions. The infinite hadronic matter is modelled by initializing the system with light meson species namely, the pion (π), the rho (ρ) and the kaon (K). The system of light mesons species is used in order to find the transition temperature of hadron matter to QGP which is believed to be around $T < 70$ MeV in which the results will be used in future to study the Thermal Conductivity of a pion gas.

We focus on the hadronic scale temperature ($100 \text{ MeV} < T < 200 \text{ MeV}$) and net zero baryon number density, which are expected to be realized in the central high energy nuclear collisions [4]. We then change energy density from $\varepsilon = 0.1 - 2.0 \text{ GeV}/\text{fm}^3$ and for each energy density we run the system with n-number of events while keeping the volume and baryon number density constant until the equilibrium state is reached.

The rest of the paper is organized as follows: In section 2 we study the description of the UrQMD model. In section 3 we study equilibration properties of the system. In section 4 we study the equation of state (EoS) of the hadron matter.

2. Brief Description of the UrQMD Model

The UrQMD is a microscopic model based on a phase space description of nuclear reactions. We use version 3.3 of the UrQMD model for this study. The UrQMD 3.3 hybrid approach extends previous ansatzes to combine the hydrodynamic and the transport models for the relativistic energies. The combination of these approaches into one single framework, it is done for a consistent description of the dynamics

The UrQMD model describes the phenomenology of hadronic interactions at low and intermediate energies from a few hundreds MeV up to the new LHC energy of $\sqrt{s} = 14$ TeV per nucleon in the centre of mass system [9, 10]. The UrQMD collision term contains 55 different baryon species and 32 meson species, which are supplemented by their corresponding anti-particles and all the isospin-projected states [9, 11, 12]. The properties of the baryons and the baryon-resonances which can be populated in UrQMD can be found in [11, 12], together with their respective mesons and the meson-resonances. A collision between two hadrons will occur if

$$d_{trans} \leq \sqrt{\frac{\sigma_{tot}}{\pi}}, \quad \sigma_{tot} = \sigma(\sqrt{s}, type), \quad (1)$$

where d_{trans} and σ_{tot} are the impact parameter and the total cross-section of the two hadrons respectively [9, 12]. In the UrQMD model, the total cross-section σ_{tot} depends on the isospins of colliding particles, their flavour and the centre-of-mass (c.m) energy \sqrt{s} . More details about the UrQMD model is presented in [9, 10, 12].

3. Equilibration of Hadronic Matter

To investigate the equilibration of a system, the UrQMD model is used to simulate the ultra-relativistic heavy ion collisions. A multi-particle production plays an important role in the equilibration of the hadronic gas [4]. The cubic box used for this study is initialised according to the following numbers of baryons and mesons: zero protons, 80 pions, 80 rhos and 80 kaons. For this study a cubic box with volume V and a baryon number density $n_B = 0.00 \text{ fm}^{-3}$ is considered. The energy density ε , volume V and the baryon number density n_B in the box are fixed as input parameters and are conserved throughout the simulation. The energy density is defined as $\varepsilon = \frac{E}{V}$, where E is the energy of N particles and the three-momenta p_i of the particles

in the initial state are randomly distributed in the centre of mass system of the particles as shown in the equations below.

$$E = \sum_{i=1}^N \sqrt{m_i^2 + p_i^2}, \quad \sum_{i=1}^N p_i = 0. \quad (2)$$

3.1. Chemical Equilibrium

In this subsection the chemical equilibrium is studied from the particle number densities of different light meson species in a box with $V = 1000 \text{ fm}^3$, $n_B = 0.0 \text{ fm}^{-3}$ at different energy densities using UrQMD box calculations. Figure 1 (a) and (b) represents the time evolutions of the various meson number densities at $\varepsilon = 0.3$ and 0.4 GeV/fm^3 energy densities.

In figures 1 (a) and (b), the meson species indicate that the system does indeed reach chemical equilibrium. It is observed that the pions have large particle number densities and the reason could be the decay in the heavier mesons and other particles produced in the system to form pions. The saturation of the particle number densities indicate the realization of a local equilibrium. In conclusion, the chemical equilibrium of the system has been reached, as in both figures the saturation times are the same for all three mesons, regardless of the shape of each meson. In figure 1 (a) where $\varepsilon = 0.3 \text{ GeV/fm}^3$, the equilibrium time for all meson species is around $t = 22 \text{ fm/c}$ and for figure 1 (b) at a higher energy density of $\varepsilon = 0.4 \text{ GeV/fm}^3$, the equilibrium time is observed to have increased to $t = 32 \text{ fm/c}$. These results show that an increase in energy density influences the particle multiplicity inside the periodic box, which affect the equilibration time.

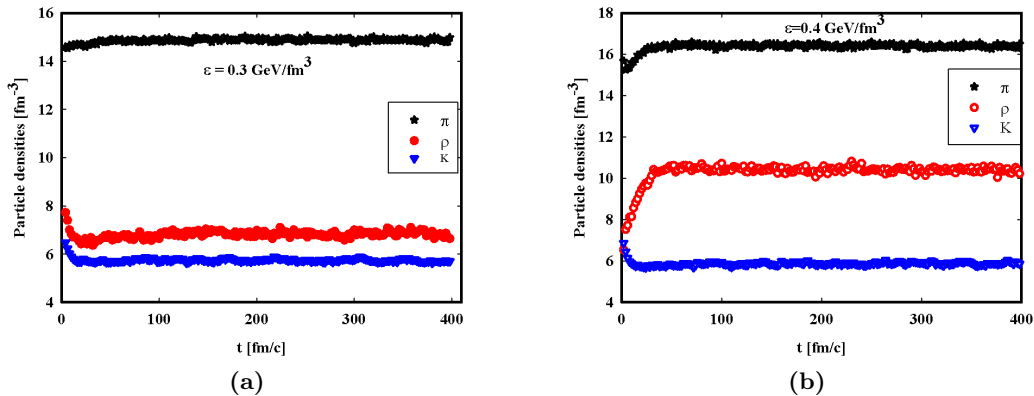


Figure 1: The time evolution of particle number densities of light meson species (π , ρ and K) at (a) $\varepsilon = 0.3 \text{ GeV/fm}^3$ and (b) $\varepsilon = 0.4 \text{ GeV/fm}^3$.

3.2. Thermal Equilibrium and Temperature

In this subsection the thermal equilibrium and the temperature from the energy distributions of different light meson species are studied. The possibility of the thermal equilibrium of the hadronic matter is studied by examining the energy distribution of the system in a box with periodic boundary conditions using the UrQMD model. The particle spectra are given by the momentum distribution as [13]

$$\frac{dN_i}{d^3p} = \frac{dN}{4\pi E p dE} \propto C e^{(-\beta E_i)}. \quad (3)$$

Figure 2 (a) and (b) represent the time evolutions of energy spectra of different meson species. The linear lines are fitted with the logarithm of the Boltzmann distribution, which is approximated by $C \exp(-\beta E_i)$ from Eq. 3, where $\beta = 1/T$ is the slope parameter of the distribution and E_i is the energy of particle i . The results are plotted as a function of kinetic energy $K = E - m$, so that the horizontal axes for all the particle species coincide [14]. In figure 2 (a) and (b) it is observed that the slopes of the energy distribution converge to common values of temperatures at different times above $t = 180$ fm/c for $\varepsilon = 0.2$ GeV/fm³ and above $t = 250$ fm/c for $\varepsilon = 0.3$ GeV/fm³. In thermal equilibrium the system is characterized by unique temperature T [14]. The thermal temperatures were extracted from the equilibrium state using the Boltzmann distribution such that $T = 118.3$ MeV for $\varepsilon = 0.2$ GeV/fm³ and $T = 150.1$ MeV for $\varepsilon = 0.3$ GeV/fm³.

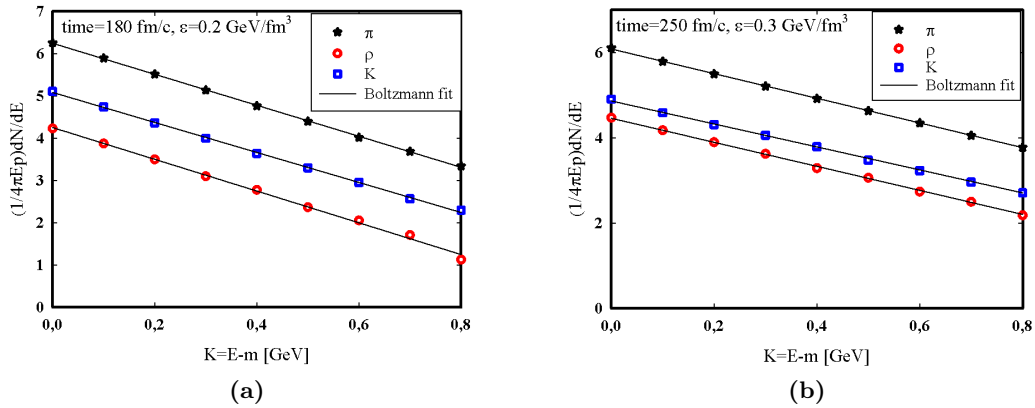


Figure 2: The energy distributions of light meson species (π , ρ and K) at (a) $\varepsilon = 0.2$ GeV/fm³ and $t = 180$ fm/c and (b) $\varepsilon = 0.3$ GeV/fm³ and $t = 250$ fm/c. The lines are the Boltzmann fit which gives the extracted temperatures of $T = 118.3$ MeV for (a) and $T = 150.1$ MeV for (b). [4, 15, 16]

4. Hadronic gas model (Equation of State)

The hadron abundances and the ratios have been suggested as the possible signatures for the exotic states and the phase transitions of the nuclear matter [17]. The equation of state of hot and dense hadron matter provides the valuable information regarding the nature of the hadron matter. These signatures have been applied to the study of chemical equilibration in the relativistic heavy ion reactions. The properties like the temperatures, the entropies and chemical potentials of the hadronic matter have been extracted assuming thermal and chemical equilibrium [9].

In this section the EoS for a hadron matter is studied from the UrQMD simulation. The equation of state can be studied using a statistical model which is described by the grand canonical ensemble of non-interacting hadrons in an equilibrium state at temperature T . A large number of studies have been done to study EoS of hadron matter as presented in [4, 16, 12, 18, 19]. For this study the focus is only on the EoS of hadron matter made out of the π , the ρ and the K calculated from the UrQMD model. This is done through studying the evolution of pressure and energy density with temperature. The thermodynamic quantities used to calculate EoS are the energy density given by

$$\varepsilon = \frac{1}{V} \sum_{i=1}^{all-particles} E_i, \quad (4)$$

and the pressure which is given by

$$P = \frac{1}{3V} \sum_{i=1}^{all-particles} \frac{p_i^2}{E_i}. \quad (5)$$

Figure 3 represent the EoS of hadronic matter namely (a) the pressure and, (b) the energy density as function of temperature. The energy density used here Eq. (4) is the same energy density used as an input parameter during the initialization of the simulation. The pressure is then calculated from the collision results using Eq. (5). The thermal temperatures used here are extracted from the thermal equilibrium (subsection 3.2) using the Boltzmann distribution function given as $C \exp(-\frac{E}{T})$ [20].

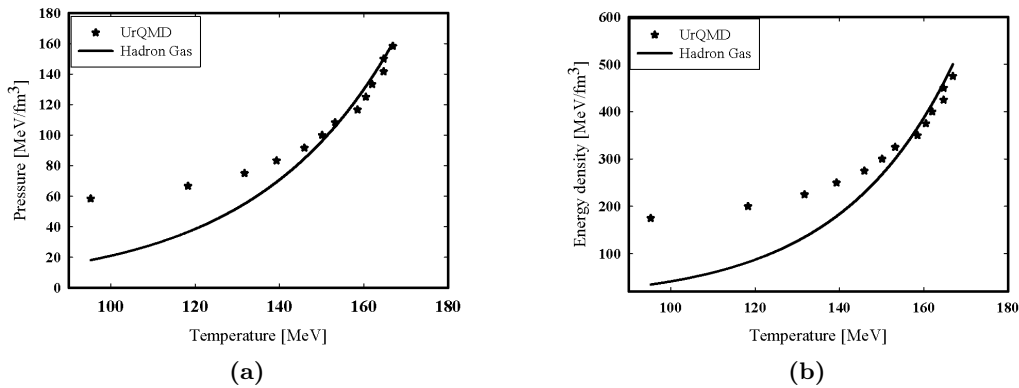


Figure 3: The equation of state of a mixed hadron gas of the π , the ρ and the K at finite temperature ($100 \text{ MeV} < T < 200$) (a) the pressure of hadronic matter is plotted as a function of temperature (b) the energy density of hadronic matter is plotted as a function of temperature.

From the above figure 3 (a) and (b), both pressure and energy density increase with an increase in the temperature. The results are in good agreement with those obtained in [4, 12]. The fitted line in figure 3 (a) and (b) represents the power law fit. The focus is on the hadronic scale temperature of ($100 \text{ MeV} < T < 200 \text{ MeV}$) and the zero net baryon number density which is expected to be realized in the central high energy nuclear collisions [4]. The pressure and energy density grow with temperature and start to saturate just after $T = 150 \text{ MeV}$ which might indicate that there is a change in phase transition of the hadron gas. In figure 3 (a) and (b) at low temperatures between $T = 90 \text{ MeV}$ to $T = 150 \text{ MeV}$ the power law T^2 of hadron gas behaves differently than the power law T^4 at high temperature between $T = 155 \text{ MeV}$ to $T = 170 \text{ MeV}$. The power law of T^4 can be perfectly compared to that of the Stefan–Boltzmann law at high temperatures [21]. At lower temperature the results does not fit well together because the URQMD model results only start at higher energy density which has to correspond with the initial energy density included in the URQMD model input file during the simulation which was $\varepsilon = 0.175 \text{ GeV/fm}^3$ which yield the temperature of $T = 95.23 \text{ MeV}$. The reason for such initial energy density was because we had already chosen the temperature range, we expected the hadron gas to exist [4].

5. Conclusion

From the presented results it has been shown that both chemical and thermal equilibrium has been reached. The temperature extracted from thermal equilibrium was used to study the EoS of hadron matter. Both the pressure and the energy density increase exponentially with the temperature until around $T = 170$ MeV which corresponds to the expected temperature range of $100 < T < 200$ MeV in which hadron gas is expected to exist. At higher temperatures the URQMD results are in good comparison with that of Stefan-Boltzmann law of T^4 . The results of the EoS are in good comparison with those reported by other authors. In future these results will be used to study the transport coefficient of hadron matter using the Green-Kubo formula.

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