Unifying Dark Matter and Dark Energy in Chaplygin Gas Cosmology: Confronting the Model with Observational Data

Anna-Mia Swart[1], Renier Hough[2], Shambel Sahlu[1,3,4], Heba Sami[1], Thato Tsabone[1], Rubby Aworka[5], Maye Elmardi[2] and Amare Abebe[1]

[1] Center for Space Research, North-West University, Mahikeng 2735, South Africa
[2] Center for Space Research, North-West University, Potchefstroom, 2520, South Africa
[3] Astronomy and Astrophysics Department, Entoto Observatory and Research Center, Ethiopian Space Science and Technology Institute, Addis Ababa, Ethiopia
[4] Department of Physics, College of Natural and Computational Science, Wolkite University, Wolkite, Ethiopia
[5] African Institute for Mathematical Science (AIMS), Ghana

E-mail: `annamiaswart@gmail.com`

Abstract. In this paper we study the accelerating expansion of the Universe by unifying dark matter and dark energy with an exotic fluid - the so-called Chaplygin gas. We consider the cosmological background expansion of a universe model filled with a radiation-baryonic matter-Chaplygin gas fluid system and show that such a model can solve the dark matter and dark energy problems, at least at the level of the background expansion. We present the numerical results of the deceleration parameter and luminosity distance, and show that they correlate well with observational data.

1. Introduction

One of the most active areas of research in cosmology today involves trying to understand the nature of dark matter and dark energy. The Universe is made up of different components, amongst them matter, radiation and dark matter. Recently, the Chaplygin gas (CG) model has been proposed. This model mimics the effects of dark energy and dark matter, and can be a possible substitution for our current standard model of cosmology. The negative pressure associated with the CG model is related to a positive energy density by a characteristic equation of state given as

$$p = -\frac{A}{\rho^{\alpha}} \,, \tag{1}$$

where p is the pressure and ρ is the energy density, both in a comoving reference frame with $\rho > 0$. A and α are positive constants. The values for α are given by the generalized Chaplygin gas (GCG) model ($0 < \alpha \leq 1$), and the original Chaplygin gas (OCG) ($\alpha = 1$) [1]. From the above equation of state, the energy density of the exotic fluid reads

$$\rho_{ch}(a) = \left[A + \frac{B}{a^{3(1+\alpha)}}\right]^{\frac{1}{1+\alpha}} \,, \tag{2}$$

where $B = e^{C(1+\alpha)}$ with C a constant of integration, which means that B is a positive constant.

We can now look at equation (2) and determine how the energy density of the CG evolves during different epochs. In the limiting case where $\frac{B}{A} \gg a^{3(1+\alpha)}$ for the early universe, the energy density in OCG can be given by

$$\rho_{ch}(a) \approx \sqrt{\frac{B}{a^6}} = \frac{\sqrt{B}}{a^3} \; . \tag{3}$$

From this we can conclude that the CG corresponds to dust-like matter (or dark matter) in the early Universe as $\rho_{ch} \sim a^{-3}$.

On the other hand, $\frac{B}{A} \ll a^{3(1+\alpha)}$ at late times, and for this case, the energy density becomes

$$\rho_{ch}(a) \approx \pm\sqrt{A} \; . \tag{4}$$

This tells us that $\rho_{ch} \sim$ constant, and thus the CG corresponds to a cosmological constant [2]. In this manuscript, we study different cosmological scenarios and present the numerical solutions describing the evolution of the Universe filled with a CG-radiation-barionic matter system. In this preceding paper we assume a flat universe ($\Omega_k = 0$, where Ω_k is the density due to the curvature of spacetime).

The layout of the manuscript is as follows: in the following section we review the basic Friedmann equation by unifying the exotic fluid with other components like radiation and baryonic matter fluids. In Section 3, we present the numerical analysis of some key cosmological constraints to examine the general evolutionary features of the Universe. We then devote Section 4 to discussions of our results and the conclusions.

2. A Universe consisting of CG, radiation and baryonic matter fluids

We have now concluded that the CG complies to the behaviour of dark matter and dark energy. It thus now becomes necessary to include radiation and baryonic matter so that we can get a comprehensive view of this model to describe the expansion of the Universe. To do this, we will consider a universe in which we assume the energy density to consist of CG, baryons and radiation. The fluid equations for radiation (r), baryonic matter (b) and CG (ch) hold independently as

$$\dot{\rho}_i + 3\frac{\dot{a}}{a}(\rho_i + p_i) = 0 \; , \quad i = \{r, b, ch\}. \tag{5}$$

Here $\rho_b = (\rho_0)_b a^{-3}$ is the energy density of baryonic matter with constant initial value $(\rho_0)_b$ and pressure $p_b = 0$ (the equation of state parameter for baryonic matter is $\omega = 0$). The energy density of radiation is found to be $\rho_r = (\rho_0)_r a^{-4}$, where $(\rho_0)_r$ is the constant initial value for the energy density of radiation, with pressure $p_r = \frac{1}{3}\rho_r$. The values for the pressure and energy density of the CG are given by equations (1) and (2), respectively.

We can write the energy density as the sum of the different energy density components:

$$\rho_{tot}(a) = \left[A + \frac{B}{a^{3(1+\alpha)}}\right]^{\frac{1}{1+\alpha}} + (\rho_0)_r\, a^{-4} + (\rho_0)_b\, a^{-3} \; . \tag{6}$$

The total pressure can also be given in a similar fashion:

$$p_{tot}(a) = -\frac{A}{[\rho_{ch}(a)]^\alpha} + \frac{1}{3}\rho_r(a) \; . \tag{7}$$

The fluid equation for the total energy density then becomes

$$\dot{\rho}_{tot} + 3\frac{\dot{a}}{a}(\rho_{tot} + p_{tot}) = 0 \; . \tag{8}$$

We can also make a change of variables and use redshift, z, instead of the scale factor, a, as this is a physically measurable quantity, and the scale factor is not. We do this by substituting $1+z=\frac{a_0}{a}$, where a_0 is the scale factor at the present time which we normalize to 1. The basic Friedmann equation consequently reads as

$$3H^2(z)=\left\{\left[A+B(1+z)^{3(1+\alpha)}\right]^{\frac{1}{1+\alpha}}+(\rho_0)_r(1+z)^4+(\rho_0)_b(1+z)^3\right\}\ , \tag{9}$$

where $H=\dot{a}/a$ is the Hubble parameter. Dividing equation (9) by $3H_0$ (H_0 being the value of the Hubble parameter today), we obtain

$$h^2=\left[D+E(1+z)^{3(1+\alpha)}\right]^{\frac{1}{1+\alpha}}+(\Omega_0)_r(1+z)^4+(\Omega_0)_b(1+z)^3\ , \tag{10}$$

where $h\equiv\frac{H}{H_0}$ is the normalized Hubble parameter, and $D=\frac{A}{9H_0^4}$, $E=\frac{B}{9H_0^4}$, $(\Omega_0)_r\equiv\frac{(\rho_0)_r}{3H_0^2}$ and $(\Omega_0)_b=\frac{(\Omega_0)_b}{3H_0^2}$ are dimensionless density parameters. The density parameters for radiation, baryonic matter and CG can be expressed as

$$\Omega_r(z)=(\Omega_0)_r(1+z)^4\ ,\quad \Omega_b(z)=(\Omega_0)_b(1+z)^3\ ,\quad \Omega_{ch}(z)=\left[D+E(1+z)^{3(1+\alpha)}\right]^{\frac{1}{1+\alpha}}\ , \tag{11}$$

respectively. The total density parameter for a universe where CG, baryonic matter and radiation dominate can be written as

$$1=\Omega_r(z)+\Omega_b(z)+\Omega_{ch}(z)\ . \tag{12}$$

3. Numerical analysis

In this section, we analyse different cosmological constraints, such as the deceleration parameter and luminosity distance, by considering the CG in addition to the other components of the Universe.

3.1. Deceleration parameter

From the Friedmann equation, we obtain the acceleration equation of the total fluid as

$$\frac{\ddot{a}}{a}=-\frac{1}{6}\left(\rho_{tot}+3p_{tot}\right)\ . \tag{13}$$

With the use of equations (6) and (7), we get the acceleration equation into the form

$$\begin{aligned}\frac{\ddot{a}}{a}=-\frac{1}{6}\bigg\{&\left[A+B(1+z)^{3(1+\alpha)}\right]^{\frac{1}{1+\alpha}}-3A\left[A+B(1+z)^{3(1+\alpha)}\right]^{-\frac{\alpha}{1+\alpha}}\\&+2(\rho_0)_r(1+z)^4+(\rho_0)_b(1+z)^3\bigg\}\ .\end{aligned} \tag{14}$$

There exists a deceleration parameter, q, which gives us an understanding of the rate of expansion of the Universe:

$$q\equiv-\frac{\ddot{a}}{a}\frac{1}{H^2}\ . \tag{15}$$

By substituting equations (9) and (14), the deceleration parameter can be written as

$$q(z)=\frac{\left[A+B(1+z)^{3(1+\alpha)}\right]^{\frac{1}{1+\alpha}}-3A\left[A+B(1+z)^{3(1+\alpha)}\right]^{-\frac{\alpha}{1+\alpha}}+2(\rho_0)_r(1+z)^4+(\rho_0)_b(1+z)^3}{2\left[A+B(1+z)^{3(1+\alpha)}\right]^{\frac{1}{1+\alpha}}+2(\rho_0)_r(1+z)^4+2(\rho_0)_b(1+z)^3}\ . \tag{16}$$

It is now possible to plot the deceleration parameter versus the redshift as shown in Figure 1. This plot shows an accelerated expansion in negative z until $z \sim 0.6$, and a decelerated expansion after this, which means that the Universe's expansion was decelerating early in time, and later (including the present and the future) the expansion is accelerating. This result is consistent with what one would expect the expansion history of the Universe to look like: in the current concordance model, there should be a decelerating matter-dominated phase followed by a dark-energy-driven late time acceleration phase.

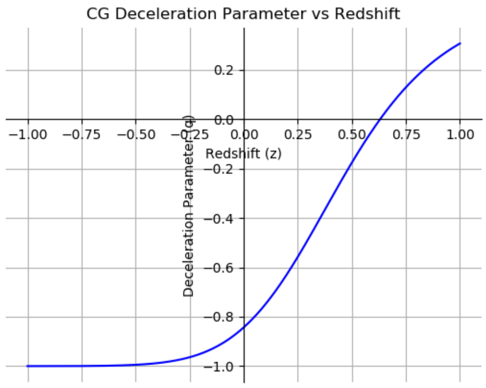

Figure 1. A plot of the deceleration parameter, q, versus the redshift, z.

3.2. Luminosity distance D_L and distance modulus μ

To test our model, equation (10) is useful. We can use the distance modulus, μ, for Supernovae Type 1A data and calculate the corresponding distance modulus for the CG model at different redshifts. The luminosity distance relates two bolometric quantities, namely the luminosity, L, and the flux, f, of a distant object such as a supernova [3]. From the luminosity distance, the distance modulus can be derived as a function of redshift. Since we need the luminosity distance, D_L, in terms of redshift, we have to relate it to the transverse comoving distance, D_M, to obtain

$$D_L = (1+z)D_M, \tag{17}$$

Furthermore, we have D_M as a function of Ω_k [4], given by

$$D_M = \begin{cases} D_H \frac{1}{\Omega_k} \sinh\left(\sqrt{\Omega_k}\frac{D_c}{D_H}\right) & \text{for } \Omega_k > 0, \\ D_c & \text{for } \Omega_k = 0, \text{ and} \\ D_H \frac{1}{|\Omega_k|} \sin\left(\sqrt{|\Omega_k|}\frac{D_c}{D_H}\right) & \text{for } \Omega_k < 0\ , \end{cases} \tag{18}$$

where Ω_k is the density due to the curvature of spacetime and D_H is the Hubble Distance. Since $\Omega_k = 0$ we have $D_M = D_c$, where D_c is called the line-of-sight comoving distance (LSCD) and is defined as

$$D_c = \int \frac{cdt}{a} = D_H \int_0^z \frac{dz'}{h(z')}\ , \tag{19}$$

where z' is the redshift of the supernova you are observing. From the LSCD definition, in conjunction with equation (10), we can calculate the distance modulus. Using the above given distance definitions, the resulting distance modulus (in Mpc), is given as

$$\mu = m - M = 25 - 5 \times \log_{10} \left[3000\bar{h}^{-1}(1+z) \int_0^z \frac{dz'}{h(z')} \right] , \tag{20}$$

where m is the apparent magnitude and M is the absolute magnitude of the object [5]. We use the Hubble distance as $D_H = 3000\bar{h} \ \frac{km}{s \cdot Mpc}$, where $\bar{h}$ is the Hubble uncertainty parameter.

Now that we have a way of relating the Chaplygin gas model to data, we need to discuss the dataset that we will use. We obtained the Supernovae Type 1A dataset from the SDSS-ll/SNLS3 Joint Light-curve Analysis (JLA). It contains 123 low-redshift $(0.01 < z < 0.1)$ supernovae and 236 intermediate redshift $(0.01 < z < 1.1)$ supernovae. We used the B-filter magnitudes, and found the absolute magnitudes of these particular supernovae in the research papers [6], [7], and [8]. The reason we use these types of supernovae is due to the fact that their luminosity is relatively similar to one another, since all White Dwarfs (WD) are known to have a relative size and composition. This particular type of supernovae is caused by a WD accreting a low-mass companion main sequence star. Since their luminosity is relatively the same, the difference in the flux received, from one WD to another, is a direct consequence of the distance the light had to travel. We can use redshift to approximate the distance.

To fit the distance modulus to the data, we will use a Markov-Chain-Monte-Carlo (MCMC) simulation[1]. The MCMC simulation is able to search for the most probable free parameter value, given certain physical constrains. The *EMCEE Hammer Python* package was used to execute the MCMC simulation. Furthermore, the MCMC calculates the most probable best fit by calculating the likelihood function for the given free parameters. We assumed that the data has a Gaussian distribution. The best fit calculated free parameters for the CG model on the supernovae data is shown in Figure 2.

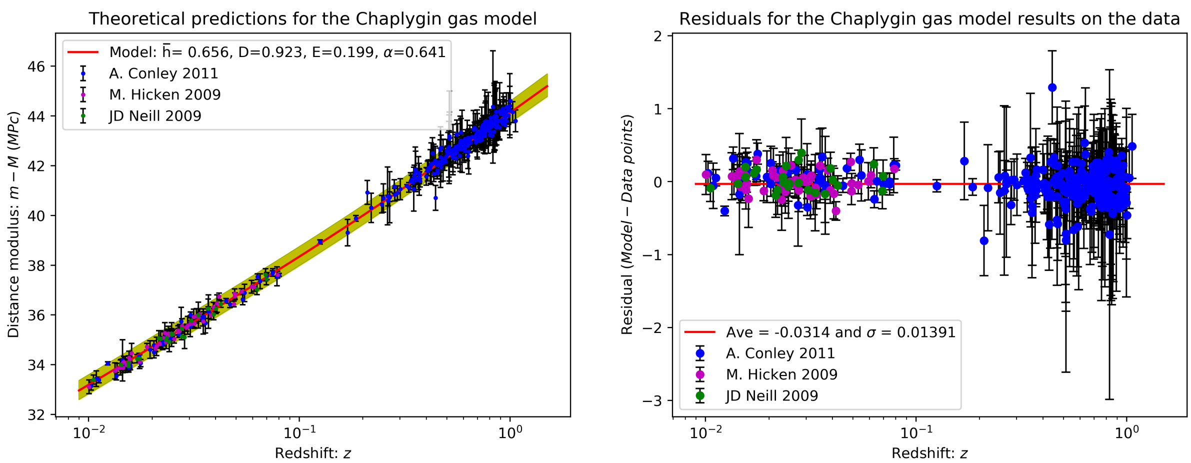

Figure 2. The Chaplygin gas model's (10) best fit free parameters to the Supernovae Type 1A data with cosmological parameter values $\bar{h} = 0.6561^{+0.074}_{-0.041}$, $D = 0.9226^{+0.251}_{-0.263}$, $E = 0.1986^{+0.059}_{-0.064}$, and $\alpha = 0.6411^{+0.255}_{-0.362}$ as calculated by the MCMC-simulation (L.H.S. panel). The cosmological parameter values were fixed to $\Omega_b = 0.049$ and $\Omega_r(0.674)^2 = 2.47 \times 10^{-5}$. The R.H.S panel shows the residuals between the predicted model values and the data points.

Figure 2 clearly shows in the R.H.S panel that we have obtained the best fit model for equation (10), since the average residual deviation is $\bar{x}_{res} = -0.0314$ with a standard deviation

[1] This entire section, including the MCMC simulation code, is similar to work done in the conference proceedings paper [9]. We used the code developed in that paper to test the model.

of $\sigma_{res} = 0.0139$. Furthermore, the residuals show no over- or under-estimation on either low-redshift or intermediate redshift. This suggests that the CG model can accurately predict the late-time accelerated expansion. However, it is worth noting that the best fit Hubble constant parameter for the CG model includes within the error approximation, the measured Hubble constant found by [10]. This is an interesting result, since it is known that the predicted Hubble constant values between Cosmic Microwave Backgroud (CMB) related results, such as Planck2018 [10], and supernovae results have a discrepancy between them with the latter's results for the ΛCDM model predicting a higher value for the Hubble constant [11].

Thus the CG model is able to give, not only realistic values for the Hubble constant, but it also lowers the discrepancy between cosmic microwave background (CMB) radiation and supernovae results. From this we can conclude that the CG model is a viable alternative model to explain the late-time acceleration.

4. Conclusion

In this paper, we examined the accelerating expansion of the Universe using a theoretical exotic fluid which mimics dark energy and dark matter. After we reviewed the background of the Universe, we presented the numerical results of some important cosmological parameters like the deceleration parameter and luminosity distance by unifying this exotic fluid with the usual cosmological fluids (baryonic matter and radiation). From the numerical results, we concluded that the CG is an alternative candidate to explain the current accelerating universe and that this model is a possible way to solve the dark matter and dark energy problems. It would be possible to get a more accurate model by adding more constraints on the A, B and α terms using observational data, and this is a task we plan to undertake in the near future.

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