On the Effects of Dissipation Range Turbulence on the Perpendicular Diffusion Coefficients of Cosmic Ray Electrons

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OUTLINE

INTRODUCTION

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These dissipation range quantities were not included in the calculation of perpendicular diffusion coefficients.

The aim is to investigate the effects of the omission of the dissipation range quantites on the perpendicular diffusion coefficients.

Turbulence Background

Fluctuations are made up of eddies of different sizes.
Each eddy tends to break up into smaller eddies, due to instabilities within the system.
When the eddies reach a minimum size, their energy dissipates to the surrounding environment in the form of thermal energy.

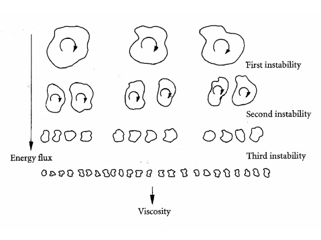

Figure: [Davidson, 2004]

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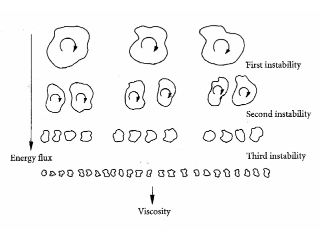

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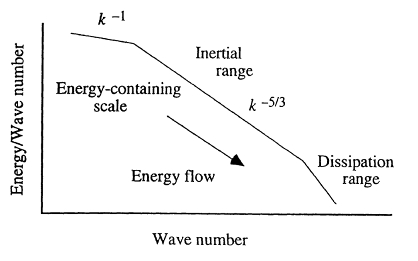
This process is called the energy cascade.

The amount of energy contained in the turbulent structures of the energy cascade is given by the turbulence power spectrum.
This spectrum can be divided into ranges as follows:



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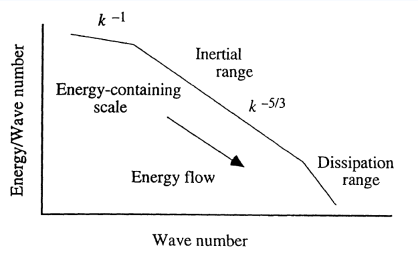

Figure: [Nel, 2015]

Due to physical considerations an inner range can also be included for the lowest wavenumbers.

THE TURBULENT HELIOSPHERIC MAGNETIC FIELD

We can express the heliospheric magnetic field as

$$\mathbf{B}(x, y, z) = B_0\hat{\mathbf{z}} + \mathbf{b}(x, y, z)$$

where B_0 is the uniform background component and **b** is the fluctuating component.

Different models can be used for **b**, namely

- slab model-assumes fluctuations are propagating in the z-direction
- 2D model- assumes fluctuations are propagating in the (x,y)-plane

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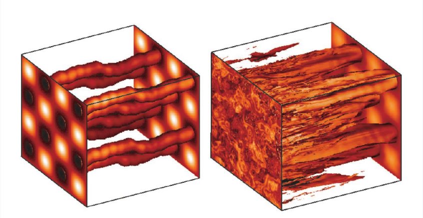

Figure: **Left:**slab turbulence **Right:** composite turbulence [Matthaeus et al., 2003]

A composite turbulence model consisting of 20% slab and 80% 2D turbulence can be used to approximate the heliospheric magnetic field at Earth [Bieber et al., 1996].

Calculation of the Diffusion Coefficients

The Scattering Theory

The Random Ballistic Decorrelation interpretation of Non-linear Guiding Centre Theory [Ruffolo et al., 2012] presents a linear equation for $\kappa_\perp$ that can be solved analytically.

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The Perpendicular Diffusion Coefficient:

$$\kappa_\perp = \frac{a^2 v^2}{3B_0^2}\sqrt{\frac{\pi}{2}} \int \frac{\varepsilon_{2D}(k_\perp)}{\sqrt{\sum_i k_i^2 \langle v_i \rangle^2}} \times erfc\left[\frac{\frac{v^2}{3\kappa_\parallel} + \gamma(k_\perp)}{2\sqrt{\sum_i k_i^2 \langle v_i \rangle^2}}\right] dk_\perp$$

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- Axisymmetry
- Transverse fluctuations

Furthermore, we assume magnetostatic turbulence s.t. $\gamma(k_\perp) = 0$

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Note that: $\kappa = \frac{v\lambda}{3}$

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The Omnidirectional Power Spectrum

$$\varepsilon_{2D}(k_\perp) = \frac{C_0 \lambda_{2D} \delta B_{2D}^2}{2\pi k_\perp} \begin{cases} (\frac{\lambda_{2D}}{\lambda_{out}})^{-1} (\lambda_{out} k_\perp)^{-p} & , |k_\perp| < \frac{1}{\lambda_{out}} \\ (\lambda_{2D} k_\perp)^{-1} & , \frac{1}{\lambda_{out}} \leq |k_\perp| < \frac{1}{\lambda_{2D}} \\ (\lambda_{2D} k_\perp)^{-s} & , |k_\perp| \geq \frac{1}{\lambda_{2D}} \end{cases}$$

where $p = 3$ and $s = \frac{5}{3}$. δB_{2D}^2 is the total magnetic variance.

The normalisation constant C_0 can be determined by setting:

$$\int_0^\infty 2\pi k_\perp \varepsilon_{2D}(k_\perp) dk_\perp = \delta B_{2D}^2$$

This yields:

$$C_0 = \frac{(p+1)(s-1)}{p + s + (p+1)(s-1)\log(\frac{\lambda_{out}}{\lambda_{2D}})}$$

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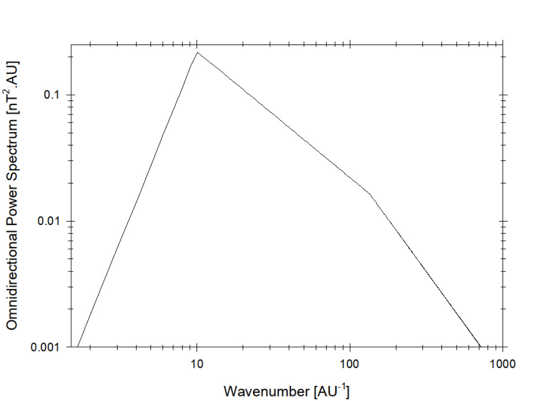

The New Omnidirectional Power Spectrum

$$\varepsilon_{2D}(k_\perp) = \frac{C_0 \lambda_{2D} \delta B_{2D}^2}{2\pi k_\perp} \begin{cases} (\frac{\lambda_2 D}{\lambda_{out}})^{-1} (\lambda_{out} k_\perp)^{-p} & , |k_\perp| < \frac{1}{\lambda_{out}} \\ (\lambda_{2D} k_\perp)^{-1} & , \frac{1}{\lambda_{out}} \leq |k_\perp| < \frac{1}{\lambda_{2D}} \\ (\lambda_{2D} k_\perp)^{-s} & , \frac{1}{\lambda_{2D}} \leq |k_\perp| < \frac{1}{\lambda_D} \\ (\frac{\lambda_D}{\lambda_{2D}})^{s} (\lambda_D k_\perp)^{-q} & , |k_\perp| \geq \frac{1}{\lambda_D} \end{cases}$$

where $p = 3$, $s = \frac{5}{3}$ and $q = 3$.

Normalisation yields:

$$C_0 = \left[\frac{\lambda_{2D}^2}{(p+1)\lambda_{out}^2} + \frac{\lambda_{2D}}{(q-1)\lambda_D} + \frac{\lambda_D - \lambda_{2D}\left(\frac{\lambda_D}{\lambda_{2D}}\right)^s}{(s-1)\lambda_D} + \log\left(\frac{\lambda_{out}}{\lambda_{2D}}\right) \right]^{-1}$$

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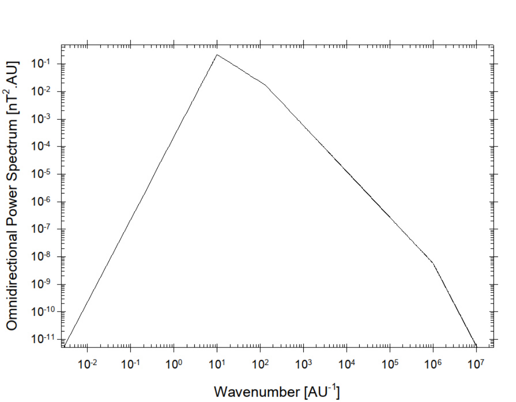


The equaitons for the parallel mean free path were derived from two different models, namely:

The Damping Model

$$\lambda_{||} = \frac{3s}{\sqrt{\pi}(s-1)}\frac{R^2}{k_{min}}\left(\frac{B_0}{\delta B_{slab}}\right)^2\left[\frac{1}{4\sqrt{\pi}} +_2 F_1(1, \frac{1}{p-1}, \frac{p}{p-1}; -\frac{\pi a}{f_1}Q^{p-2})\frac{\sqrt{\pi a}}{f_1 R^s Q^{p-s}} + \frac{2}{\sqrt{\pi}(2-s)(4-s)}\frac{1}{R^s}\right]$$

and

The Random Sweeping Model

$$\lambda_{||} = \frac{3s}{\sqrt{\pi}(s-1)}\frac{R^2}{k_{min}}\left(\frac{B_0}{\delta B_{slab}}\right)^2\left[\frac{1}{4\sqrt{\pi}} + \left(\frac{1}{\Gamma(q/2)} + \frac{1}{\sqrt{\pi}(q-2)}\right)\frac{b^{q-2}}{Q^{q-s}R^s} + \frac{2}{\sqrt{\pi}(2-s)(4-s)}\frac{1}{R^s}\right]$$

where

$f_1 = \frac{2}{p-2} + \frac{2}{2-s}$,

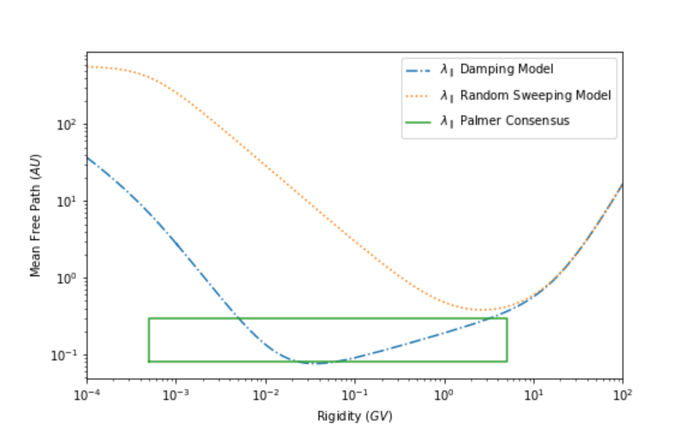
$R = \frac{P}{B_0}{}_{min}$,

$Q = \frac{P}{B_0}k_D$ and

$b = \frac{v}{2\alpha_d v_A}, \quad \alpha_d \epsilon [0, 1]$

The Parallel Mean Free Path

Evaluation at 1 AU (using solar minimum turbulence quantities) yields:



Results

The Perpendicular Diffusion Coefficient

$$\kappa_\perp = \frac{\sqrt{3} v a C_0 B_0 \delta B^2}{3 B_0 \sqrt{\delta B_{tot}}} \times$$

$$\Bigg[\frac{\sqrt{\pi}\lambda_{2T}}{p} \left(Erfc[\sqrt{3} X \lambda_{2T}] - \frac{B_0 \sqrt{\frac{3}{\pi}} \lambda_{2T} E_{\frac{1+p}{2}}[3X^2\lambda_{2T}^2]}{a\sqrt{\delta B_{tot}}\lambda_{\|}} \right)$$

$$+\sqrt{\pi} \left(\frac{a\sqrt{\delta B_{tot}}\lambda_{\|}(e^{-3X^2\lambda_{2D}^2} - e^{-3X^2\lambda_{2T}^2})}{B_0\sqrt{3\pi}} - \lambda_{2D} Erfc[\sqrt{3}X\lambda_{2D}] + \lambda_{2T} Erfc[\sqrt{3}X\lambda_{2T}] \right)$$

$$+\frac{3^{\frac{-s}{2}}\lambda_{2D}}{s}(X\lambda_{2D})^{-s} \left[\left(3^{s/2}\sqrt{\pi} \left((X\lambda_{2D})^s Erfc[\sqrt{3}X\lambda_{2D}] - (X\lambda_D)^s Erfc[\sqrt{3}X\lambda_D] \right) - \Gamma\left[\frac{1+s}{2}, 3X^2\lambda_{2D}^2\right] + \Gamma\left[\frac{1+s}{2}, 3X^2\lambda_D^2\right] \right) \right]$$

$$+\frac{3^{-q/2}\lambda_{2D}}{q} \left(\frac{\lambda_D}{\lambda_{2D}}\right)^s (X\lambda_D)^{-q} \left(3^{q/2}\sqrt{\pi}(X\lambda_D)^q Erfc[\sqrt{3}X\lambda_D] + \Gamma\left[\frac{1+q}{2}\right] - \Gamma\left[\frac{1+q}{2}, 3X\lambda_D^2\right] \right) \Bigg]$$

where $X = \frac{B_0}{a\sqrt{\delta B_{tot}}\lambda_{\|}}$

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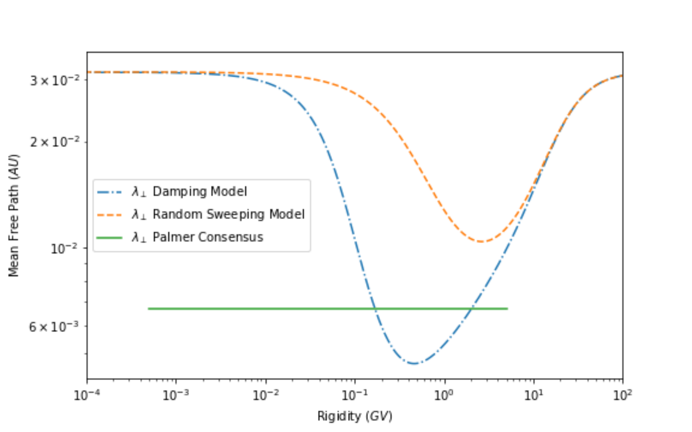
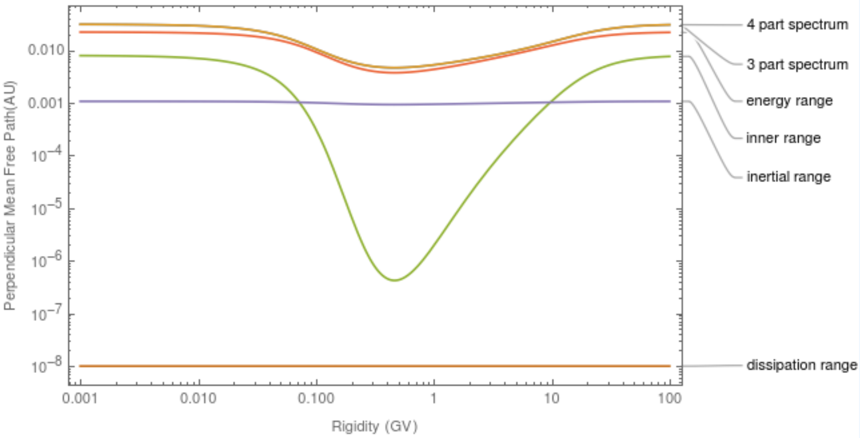
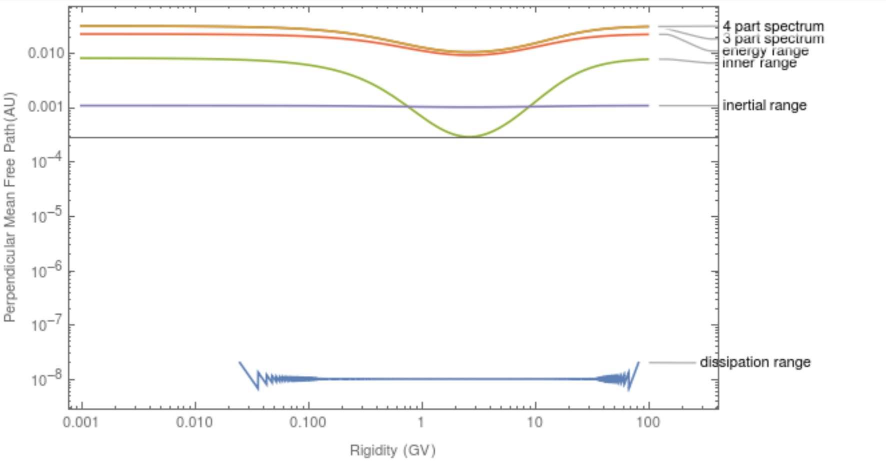


Damping Model:



Random Sweeping Model:



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By approximating the power spectrum as only the energy containing range term, the calculations are greatly simplified.

In the future:

- Solve $\kappa_\perp$ for dynamical turbulence, i.e. $\gamma(k_\perp) \neq 0$.
- Investigate the effects of these results on cosmic ray transport

Turbulence Quantities at 1 AU

Outer scale:
$\lambda_{2D} = 0.0074 AU$ [Weygand et al., 2011]

Turnover scale:
$\lambda_{2T} = 0.1 AU$ [Engelbrecht, 2019]

Dissipation scale:
$\lambda_{D} = 10^{-5} AU$ [Leamon et al., 2000]

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