Constraining *f(R)*-gravity models with recent cosmological data

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Abstract. In this work, we look at the cosmological constraints of some *f(R)*-modified gravity models such as $f(R) = \beta R^n$ (a toy model) and more realistic ones like the Starobinsky and Hu-Sawicki models. We use 236 intermediate-redshift and 123 low-redshift Type 1A Supernovae data obtained from the SDSS-II/SNLS3 Joint Light-curve Analysis (JLA), with absolute magnitudes, for the B-filter, found on the NASA Extragalactic Database (NED). We then develop a Markov Chain Monte-Carlo (MCMC) simulation to find the best fit (firstly to the ΛCDM model), to obtain the cosmological parameters (Ω_m and $\bar{h}$). We then use the concordance model results to constrain the priors for the *f(R)*-gravity models on the MCMC simulation. We assume a flat universe $\Omega_k = 0$ and a radiation density Ω_r that is negligible in both the ΛCDM model and *f(R)*-gravity models. Thus, the only difference between the ΛCDM model and *f(R)*-gravity models will be dark energy and the arbitrary free parameters. This will tell us if there exist viable *f(R)*-gravity models when we compare them to the results of the ΛCDM model and thus constrain the generic *f(R)*-gravity models with cosmological data.

1. Introduction

Since the theory of General Relativity (GR) was proposed by Einstein in 1915, it has developed into the accepted theory to explain gravity. GR is a generalization of Newtonian gravity in the presence of extreme gravitational fields. The reason behind the acceptance, among others, was due to the discovery by Hubble in 1929 that the Universe is expanding. GR was able to explain this discovery, and this led to the Hot Big Bang theory model, which uses GR as the physical basis. With the observational discovery in more recent times that the expansion of the Universe is accelerating, which is not in line with GR predictions, the Hot Big Bang model had to be improved. An unknown pressure force acting out against gravity, dubbed "dark energy" was added to explain why gravity on cosmological scales is not able to slow down the expansion.

The cosmological field equations in standard cosmology are derived by the using variational principle on the Einstein-Hilbert action

$$A = \frac{c^4}{16\pi G}\int d^4x\sqrt{-g}\big[R + 2(L_m - \Lambda)\big] \ , \tag{1}$$

where Λ is the cosmological constant representing the "dark energy" pressure force, and L_m is the standard matter Lagrangian [1]. These field equations are given by

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu} \ , \tag{2}$$

where $R_{\mu\nu}$ and R are the Ricci tensor and Ricci scalar respectively, $g_{\mu\nu}$ is the metric tensor and $T_{\mu\nu}$ represents the energy-momentum tensor. The two most important cosmological equations in Eq. (2) are the Friedmann equations (we assume that in the geometric unit system $c = 1 = 8\pi G$), which in the Friedmann-Lemaître-Robertson-Walker (FLRW) spacetime metric read

$$\left[\frac{\dot{a}(t)}{a(t)}\right]^2 = \frac{\rho(t)}{3} - \frac{\kappa}{a^2(t)} + \frac{\Lambda}{3} \,, \tag{3}$$

$$\frac{\ddot{a}(t)}{a(t)} = -\frac{1}{6}\Big(\rho(t) + 3P(t)\Big) + \frac{\Lambda}{3} \,, \tag{4}$$

where $a(t)$ is the scale factor (describing the relative size of the Universe at a certain time), $\rho(t)$ is the energy density, $P(t)$ is the isotropic pressure, and κ is the 3D (spatial) curvature. To close the above system of expansion equations, we relate ρ and P through the equation of state

$$P(t) = \omega\rho(t) \,, \tag{5}$$

where we assume a perfect-fluid system with a constant equation of state parameter ω.

1.1. Problems faced by GR and proposed solutions ∼ f(R)-gravity

The Friedmann equations are used to mathematically describe the Big Bang theory and the ongoing expansion (with the inclusion of dark energy to explain the late-time acceleration) of the Universe. The inclusion of dark energy provides one of the problems faced by the ΛCDM model, since dark energy is an unknown pressure force acting out against gravity, but have been shown to make up $\sim 68\%$ of the Universe, [2]. Furthermore, an early-time accelerated expansion, called the inflation period, added other problems to the ΛCDM model such as the horizon problem and the coincidence problem. Other arising problems faced by the ΛCDM model also include the Magnetic monopole problem (none has been found) and the Universe's matter/anti-matter ratio, which is expected to be equal to 1, but is close to zero [3].

Due to the problems faced by the ΛCDM model, there exist proposed solutions in the form of modified gravity models. In some of these modified theories, you may add extra fields or go to higher dimensions. We will be looking at a higher-order derivative theory, called *f(R)*-gravity model. For these models, the modification occurs when changing the Ricci scalar in the Einstein-Hilbert action (1) to a function of the Ricci scalar, namely *f(R)*. Re-deriving the Einstein field equations, we obtain

$$f'(R)R_{\mu\nu} + g_{\mu\nu}\Box f'(R) - \nabla_\mu\nabla_\nu f'(R) - \frac{1}{2}g_{\mu\nu}f(R) = T_{\mu\nu}, \tag{6}$$

where $\Box = \nabla_\sigma\nabla^\sigma$ is the covariant d'Alembert operator. As you will notice in equation (6), we do not have a dependency on the cosmological constant, since this modified theory tries to explain the accelerated expansion without the inclusion of dark energy. We can then re-derive the Friedmann equations for *f(R)*-gravity, and obtain

$$\begin{aligned} \left[\frac{\dot{a}(t)}{a(t)}\right]^2 &= \frac{\rho(t)}{3f'(R)} - \frac{\kappa}{a^2(t)} + \frac{1}{6}\Big(R - \frac{f(R)}{f'(R)}\Big) - H\dot{R}\frac{f''(R)}{f'(R)}, \\ \frac{\ddot{a}(t)}{a(t)} &= -\frac{\rho(t)}{3f'(R)} + \frac{f(R)}{6f'(R)} + H\dot{R}\frac{f''(R)}{f'(R)}. \end{aligned} \tag{7}$$

2. Supernovae Type 1A data and MCMC simulations

2.1. Distance modulus

To test the *f(R)* Friedmann equations (7), we use Supernovae Type 1A data. The reason for this is due to the fact that Type 1A Supernovae (White Dwarf (WD) accreting a low mass companion star) are regarded as standard candles, since their luminosities can be calibrated, due to light-curve correlations between width and amplitude, to be similar to one another [4].

This would mean that the measured flux is only dependent on the distance to supernovae and not the composition or mass of the WD. We will use redshift to approximated the distance. Thus, an expanding universe, where the distance to the supernovae is changing, can be used to find the best fitting distance modulus for our Friedmann equation. For simplicity, we will assume a flat universe $\Omega_k = 0$, with a negilible radiation density $\Omega_r \approx 0$ [2].

We will be using data obtained from SDSS-II/SNLS3 Joint Light-curve Analysis (JLA). From that particular dataset, we will use 123 are low-redshift supernovae with a redshift between $0.01 < z \leq 0.1$ and 236 supernovae with an intermediate redshift between $0.1 < z \leq 1.1$. The reason we will using low-redshift data, is due to the fact that we will be testing the model for a late-time acceleration, thus a redshift below ~ 0.5. Furthermore, we will be using the calculated absolute magnitudes of these supernovae for the B-filter, that can be found in [5, 6, 7].

The distance modulus can be derived from the *luminosity distance* D_L, which relates two bolometric quantities, namely the luminosity L and the flux f of the distant supernovae. We can then relate D_L to the *transverse comoving distance*, by using redshift and obtaining

$$D_L = (1 + z)D_M. \tag{8}$$

By using the conditions for the *transverse comoving distance* as a function of the curvature of spacetime density (Ω_k) found in [8], we can determine that $D_M = D_c$, where D_c is the *line-of-sight comoving distance*. We also have to define the *Hubble distance* to be $D_H = 3000\bar{h}\frac{km}{s.Mpc}$ and $h(z)$ to be the normalized Hubble parameter in terms of redshift. By using the definition of the *distance modulus* (in *Mpc*), and the aforementioned different distance definitions, we obtain

$$\mu = m - M = 25 - 5 \times \log_{10}\left(3000\bar{h}^{-1}(1 + z)\int_0^z \frac{dz'}{h(z')}\right), \tag{9}$$

where m is the apparent magnitude and M is the absolute magnitude of the measured Supernovae [6]. This method is called supernova cosmology [9].

2.2. Markov Chain Monte Carlo (MCMC) simulations

To fit the data to the distance modulus, we will use MCMC simulations. The MCMC simulation is able to search for the most probable free parameter value, given certain physical constrains. It starts searching at some initial given value, by calculating the likelihood of the distance modulus. It then takes a random step for each parameter in the parameter space away from the initial values. Then it calculates the likelihood for all possible combinations between the initial parameter values and the new parameter values to find the combination with the largest likelihood of occurring. The simulation then finds an acceptance ratio between the initial parameter values and the new largest likelihood combination parameter value. Using the new information on the best combination, it repeats the steps until it converges to the most probable best fit parameter values. We will use the *EMCEE Hammer Python* package to execute the MCMC simulation. This package uses different random walkers, each starting at a different initial parameter value and each converging on the most probable parameter values[1].

3. Results

3.1. Concordance model $\sim \Lambda CDM$ model

We use the ΛCDM model to calibrate our MCMC simulation. We will use the ΛCDM model as the "true" model to which we can compare the *f(R)*-gravity models against, to find if they are viable for being alternative models. We assume a flat universe $\Omega_k = 0$, as well as, neglecting the radiation density of the Universe, since the expected value is in the range of $\Omega_r h^2 = 2.47 \times 10^{-5}$

[1] This entire section including the MCMC simulation code is similar to work done in the conference proceedings paper by [10], where they used the code developed in the masters dissertation [11], which this paper is based on, to test their model. The final results of this proceedings are published in [12]

[13] (they assumed $h = 0.73$). Using these assumptions, we obtain a Friedmann equation in terms of redshift as

$$h(z) = \sqrt{\Omega_m(1+z)^3 + 1 - \Omega_m}, \tag{10}$$

where $h(z) = \frac{H(z)}{H_0}$ is dimensionless parameter, and making the substitution $\Omega_\Lambda = 1 - \Omega_m$. When we execute the MCMC simulation for the ΛCDM model, we obtain the results in Figure (1).

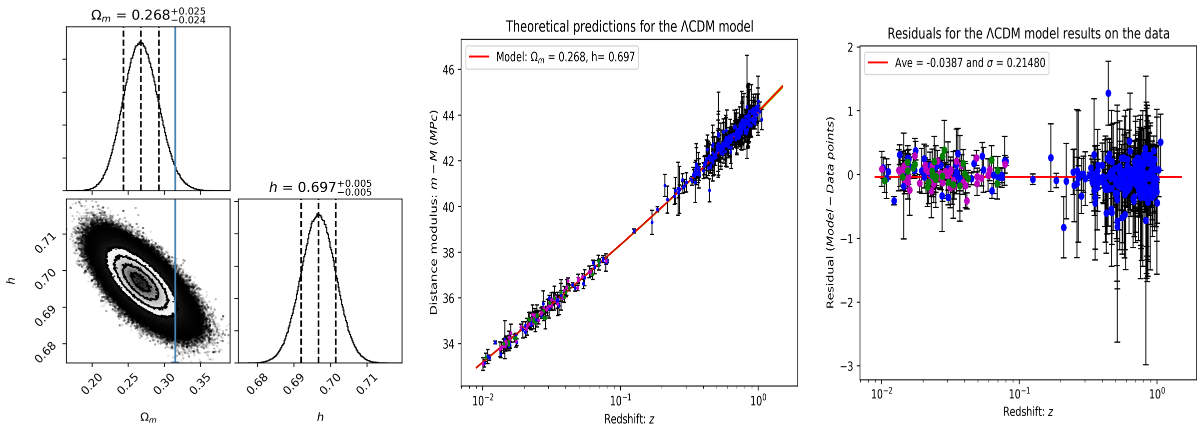

Figure 1. MCMC simulation results (Panel: 1) and the corresponding model fitted to the Supernovae Type 1A data obtained from JLA (Panel: 2). Furthermore, the residuals between the model prediction and the actual data points are also shown (Panel: 3).

From Figure (1), we can confirm that the MCMC simulation works, even though the predicted parameter values are not within 1σ from the Planck2018 results (blue line in Panel: 1), that were found to be $\Omega_m = 0.315 \pm 0.007$ and $H_0 = 67.4 \pm 0.5\text{km.s}^{-1}.\text{Mpc}^{-1}$ [2]. The reason for this is due to the Planck results being determined on Cosmic Microwave Background (CMB) radiation data, and it has been shown that the Supernovae Type 1A data predicts a higher Hubble constant value than the CMB results, namely $H_0 = 73.24 \pm 1.74\text{km.s}^{-1}.\text{Mpc}^{-1}$ [14] (they used a different calibration method). So our results, where we used previously calculated absolute magnitudes, therefore not needing to calibrate the distance modulus as they have done in [14], we were able to find a best-fitting H_0 closer to the CMB results [2], even though we were not able to remove the discrepancy.

3.2. f(R)-gravity model results

Since Figure (1) confirmed that our MCMC simulation works, we can go ahead and test different *f(R)*-gravity models in a similar fashion as done for the concordance model. Thus, we derive a distance modulus equation for each of the chosen *f(R)*-gravity models. These models include 2 toy models ($f(R) = \beta R^n$ and $f(R) = \alpha R + \beta R^n$), the Starobinsky model and the Hu-Sawicki model, where the latter 2 models are considered as the more realistic models:

- $f(R) = R + \beta R_c\left[\left(1 + \frac{R^2}{R_c^2}\right)^{-n} - 1\right]$ -Starobinsky model,
- $f(R) = R - \alpha R_c\left[\frac{\left(\frac{R}{R_c}\right)^n}{1+\left(\frac{R}{R_c}\right)^n}\right]$- Hu-Sawicki model.

We use the best-fit parameter values for the ΛCDM model to set appropriate priors for the *f(R)* models, to ensure that the resulting cosmological values are close to those found by the ΛCDM model. The best-fit model results (without the MCMC results) are shown in Figures 2 - 5 in the same order as given above. The Starobinsky and Hu-Sawicki model results are preliminary due to having non-solvable Friedmann equations (executing a numerical method). We will only be able to compare the Starobinsky and Hu-Sawicki models to the literature. In the paper by [15], they found the cosmological parameters for these two models to be $\Omega_m = 0.269^{+0.050}_{-0.042}$, $\bar{h} = 0.714^{+0.030}_{-0.028}$ and $\Omega_m = 0.264^{+0.059}_{-0.055}$, $\bar{h} = 0.722^{+0.042}_{-0.033}$, respectively.

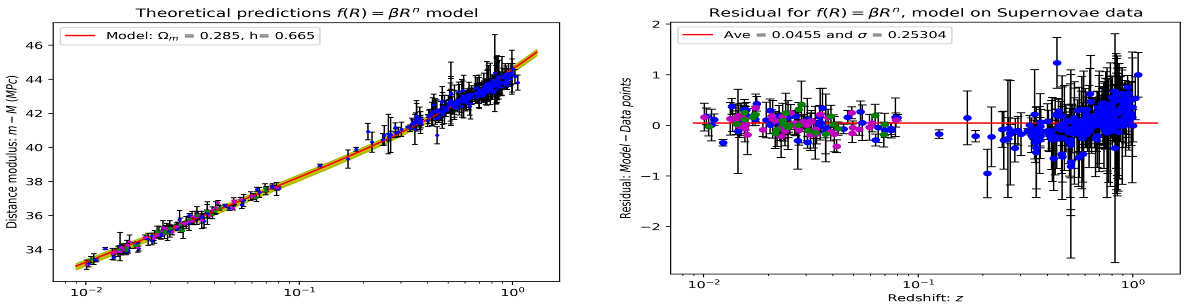

Figure 2. First toy model's best fit to the Supernovae Type 1A data, with cosmological parameter values $\Omega_m = 0.285^{+0.082}_{-0.105}$, $\bar{h} = 0.665^{+0.054}_{-0.045}$, and $q_0 = -0.011^{+0.001}_{-0.002}$. The *f(R)*-model free parameter values are $\beta = 2.687^{+0.968}_{-0.999}$ and $n = 1.270^{+0.000}_{-0.000}$.

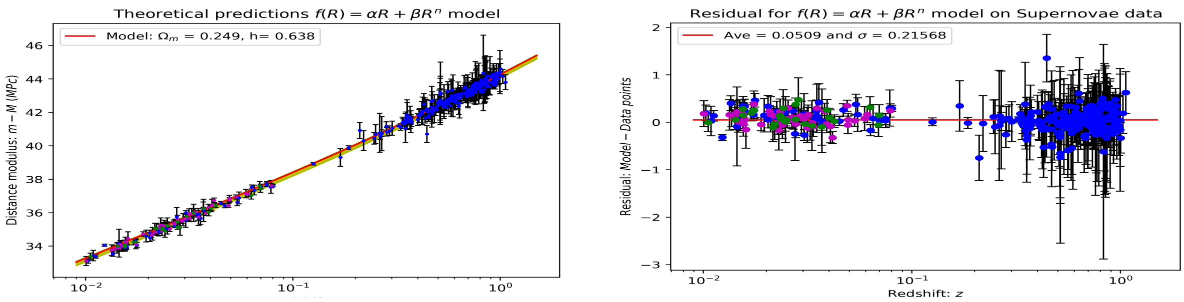

Figure 3. Second toy model's best fit ($n = 2$ and the (-) solution) to the Supernovae Type 1A data, with cosmological parameter values $\Omega_m = 0.249^{+0.102}_{-0.101}$, $\bar{h} = 0.638^{+0.046}_{-0.027}$, and $q_0 = -0.575^{+0.040}_{-0.046}$. The *f(R)*-model free parameter values are $\alpha = 19.642^{+2.967}_{-1.753}$ and $\beta = 0.903^{+0.070}_{-0.107}$.

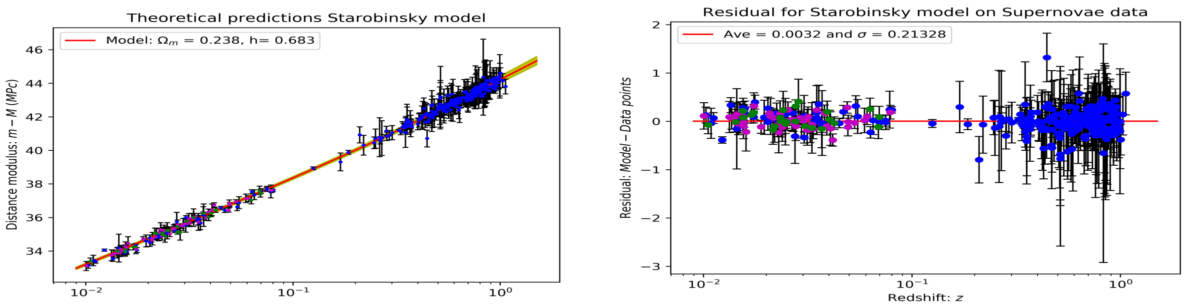

Figure 4. The Starobinsky model's best fit to the Supernovae Type 1A data, with cosmological parameter values $\Omega_m = 0.238^{+0.089}_{-0.087}$, $\bar{h} = 0.683^{+0.026}_{-0.024}$, and $q_0 = -0.494^{+0.298}_{-0.278}$. The *f(R)*-model free parameter values are $\beta = 4.588^{+3.668}_{-2.683}$ and $n = 3.493^{+3.705}_{-2.057}$.

4. Conclusion

From these results, we can see that these models do fit the data, although each model has a disadvantage in some sense. In the first toy model, the predicted cosmological values are close to the Planck2018 results [2], thus minimizing the discrepancy between CMB and Supernovae Type

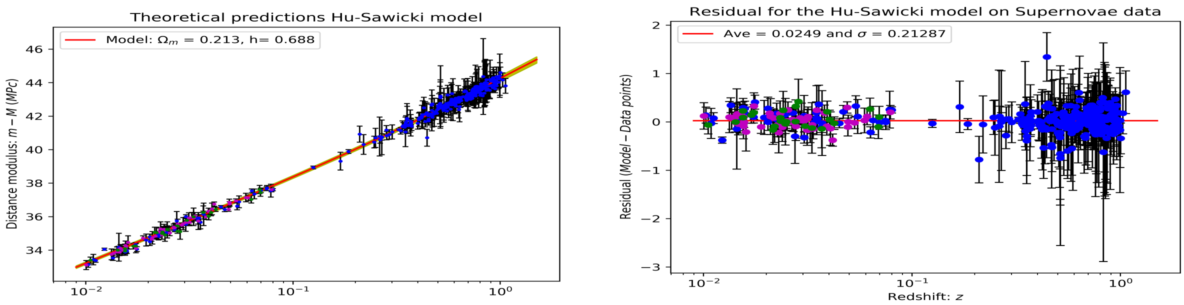

Figure 5. The Hu-Sawicki model's best fit to the Supernovae Type 1A data, with cosmological parameter values $\Omega_m = 0.213^{+0.076}_{-0.038}$, $\bar{h} = 0.688^{+0.024}_{-0.024}$, and $q_0 = -0.538^{+0.144}_{-0.179}$.The *f(R)*-model free parameter values are $\alpha = 4.823^{+0.070}_{-0.092}$, $\beta = 5.012^{+0.087}_{-0.087}$, and $n = 3.500^{+0.038}_{-0.049}$.

1A results, but struggled with predicting the period before the acceleration started ($z > 0.5$), as well as with the deceleration parameter value (q_0) that is expected to be close to ~ -0.5 based on observations. The second toy model fitted the data better, but gave a Hubble constant that is lower than any observed value. The Starobinsky and Hu-Sawicki models (preliminary results) found realistic Hubble constant values compared to the CMB observations, but in both cases found matter densities that were lower than observations suggest. Both of these also gave deceleration parameters close to the expected values.

Future work will include doing a statistical analysis on each individual best-fit values by calculating the χ^2-value. Furthermore, we will also calculate the AIC and BIC criterion values of each for the *f(R)*-gravity models and compare them to the ΛCDM model to check if some of these *f(R)*-gravity models are cosmologically viable alternatives or not. With the statistical analysis results, we will be able to see whether or not a particular model is viable for further testing on various different data sets, such as the CMB data. This will enable us to obtain a better insight into these models and whether or not the can be considered to be alternative models to avoid the dark energy problems.

Acknowledgements
RH acknowledge funding through a National Astrophysics and Space Science Program (NASSP) and National Research Foundation (NRF) scholarship (Grant number 117230). AA and SF acknowledge that this work is based on the research supported in part by the NRF (with grant numbers 109257/112131 and 109253 respectively).

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