Extracting growth rates from a Particle-In-Cell simulation

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Abstract. Using a Particle-In-Cell simulation, the characteristics of electrostatic waves are investigated in a plasma containing 3 electron components (hot, cold and beam electrons) and a cold ion population. Three electrostatic modes are excited, namely electron plasma, electron acoustic and beam driven waves. These modes have a broad frequency spectrum and have been associated with intense broadband electrostatic noise observed in the Earth's auroral zone. The growth rates of the beam mode is studied by constructing a growth rate curve from the electric field data. The beam mode is found to have a high growth rate for an intermediate range of wave numbers while it is damped elsewhere.

1. Introduction

The electron acoustic mode may be present in a plasma containing two electron components of different temperature [1]. An electron beam can drive this mode unstable over a wide range of parameters [2, 3]. The nonlinear properties of the electron acoustic instability for upstream foreshock conditions have been observed in the magnetospheric cusp [4], the upstream bowshock [5, 6], the magnetotail plasma sheet [7], the auroral zone [8] and Saturn's magnetosphere [9]. Free energy sources such as electron beams have also been observed in the polar cap and auroral zone, together with the coexistence of warm and cold plasma populations [10].The electrostatic waves excited in such a system may explain the broadband electrostatic noise observed in this region. In addition to electron acoustic waves, electron plasma waves are also excited by the electron beam [11].

Previously a theoretical study has been made of the electron acoustic mode found containing three electron components [3]. Recently the study has been extended to investigate the nonlinear regime of such a system using a PIC simulation [12]. The system contained warm and cold electron populations with roughly equal densities and a low density electron beam with a large streaming speed compared to the warm and cold electron populations. In such a scenario, the beam mode was found to dominate the wave spectrum being most intense at intermediate wave numbers. Electron plasma waves were found to be weakly damped at low wave numbers while electron acoustic waves were found to be damped over most of the wave number spectrum.

It is quite common to construct a dispersion diagram using the electric field data from a PIC simulation to study the dispersion characteristics of different wave modes. These dispersion

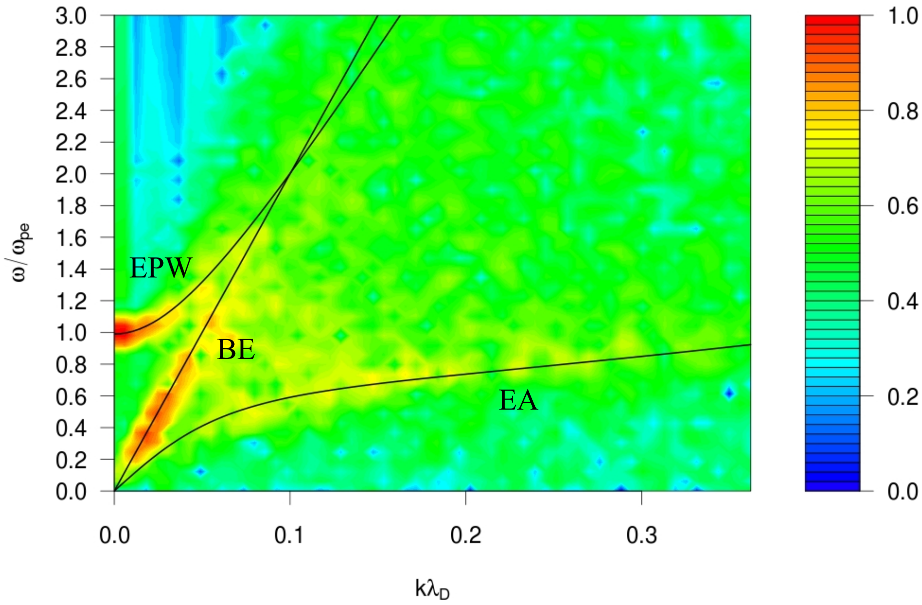

Figure 1. The ω–k diagrams at 1200–1302.4 $\omega_{pe}t$. EPW, EA, and BE curves indicate the theoretical dispersion relations for the electron plasma, electron acoustic, and beam wave modes, respectively

diagrams have been found to successfully reproduce the theoretical curves for different wave modes (Figure 1). The diagram also gives a representation of the damping or growth for the different wave modes. However, these dispersion diagrams, showing the amplitudes for the modes, either gives a snapshot for the whole simulation or a small fixed period within the simulation. Thus, one is limited to only make comparisons of one wave mode's average amplitude to another in most scenarios.

In theoretical studies of plasma waves, a dispersion curve together with a growth rate curve is required to give a complete picture of the wave characteristics. The growth rate curve gives information on the damping or growth of the wave mode whereas the dispersion diagram gives information on the dispersion characteristics. The aim of this work is to extend the analysis of the beam mode previously performed [12] by constructing a growth rate curve from the electric field data.

2. Theoretical Model

An unmagnetised plasma with one ion and three electron components is considered. Ions are treated as massive (therefore motionless) and are only included to form a neutral background. The electron components consist of hot, cold, and beam electrons. The initial velocity distributions of the particles are assumed to be Maxwellian with the beam electrons having a streaming speed v_b. The total unperturbed density is $n_{e0} = n_{c0} + n_{h0} + n_{b0}$ where n_{c0}, n_{h0}, and n_{b0} are the cold, hot, and beam electron equilibrium densities, respectively. The plasma frequency for component j is

$$\omega_{pj} = \left(\frac{n_j e^2}{m_e \epsilon_0}\right)^{1/2}. \tag{1}$$

Similarly, the Debye length is

$$\lambda_{Dj} = \left(\frac{\epsilon_0 k_B T_j}{n_j e^2}\right)^{1/2}, \tag{2}$$

where T_j is the temperature of component j.

In such a plasma, the dispersion relation takes the form [13]

$$1 + \frac{2}{k^2\lambda_{Dc}^2}\left[1 + \zeta_c Z(\zeta_c)\right] + \frac{2}{k^2\lambda_{Dh}^2}\left[1 + \zeta_h Z(\zeta_h)\right] + \frac{2}{k^2\lambda_{Db}^2}\left[1 + \zeta_b Z(\zeta_b)\right] = 0, \tag{3}$$

where $\zeta_c = \omega/kv_{Tc}$, $\zeta_h = \omega/kv_{Th}$ and $\zeta_b = (\omega - ku_d/kv_{Tb})$ with v_{Tj} being the thermal velocity of component j. $Z(\zeta)$ is the plasma dispersion function [14].

When the streaming speed of the beam electrons becomes sufficiently large and $n_b \ll n_{e0}$, Eq. (3) can be approximated by [15]

$$\omega = \frac{kv_b}{1 + n_b/n_{e0}} \approx kv_b, \tag{4}$$

which is the beam wave mode. This is also the mode of interest for this study of the construction of the growth rate curve.

3. Simulation

A 1D electrostatic PIC simulation was used to solve Maxwell's equations [16]. Particle charge densities, ρ, were projected onto a grid. The fields were updated based on the particle distribution using the discrete form of Poisson's equation.

The particle positions and velocities were then advanced by a small time step using the most recent fields. Periodic boundary conditions were applied. The densities were normalized to the total unperturbed electron density n_{e0}. Velocities were expressed in units of thermal velocity of cold electrons v_{Tc}. The space and time (x and t) were normalized to cold electron Debye length $\lambda_{Dc} = (\epsilon_0 k_B T_c/n_{c0}e^2)^{1/2}$ and to the inverse of the total electron plasma frequency $\omega_{pe} = (n_0 e^2/m_e\epsilon_0)^{1/2}$. The electric field and potential were normalized to $m_e\omega_{pe}v_{Tc}/e$ and $k_B T_c/e\epsilon_0$.

The combined number of particles employed for the hot and cold electrons was 2×10^6 and the number of beam electrons was allocated by the ratio of beam to cold electrons, n_b/n_c. The simulation uses 1024 grid cells with a grid spacing of λ_D (the total electron Debye length). A fixed time step of $0.01\omega_{pe}^{-1}$ is used throughout the simulation.

Fig. 1 shows a ω–k (dispersion) diagram, which was obtained by Fourier transforming the electric field E_x in space and time for the quasi-equilibrium stage $\omega_{pe}t$ = 1200–1302.4. The frequencies are normalised to ω_{pe}. The theoretical dispersion relation curves overlaid on the ω–k diagram are based on parameters of the unperturbed plasma. The waves and particles interact through Landau damping, changing the unperturbed parameters of the plasma and waves.

4. Growth rate analysis

The growth rate curve was constructed by analysing unique ω–k areas every time step along the beam mode's dispersion curve. The time window for each dispersion diagram at each time step was $\omega_{pe}t = 50$. When analysing a portion of the dispersion diagram it produces a series of amplitudes with time. From this series of amplitudes an exponential curve is fitted by using

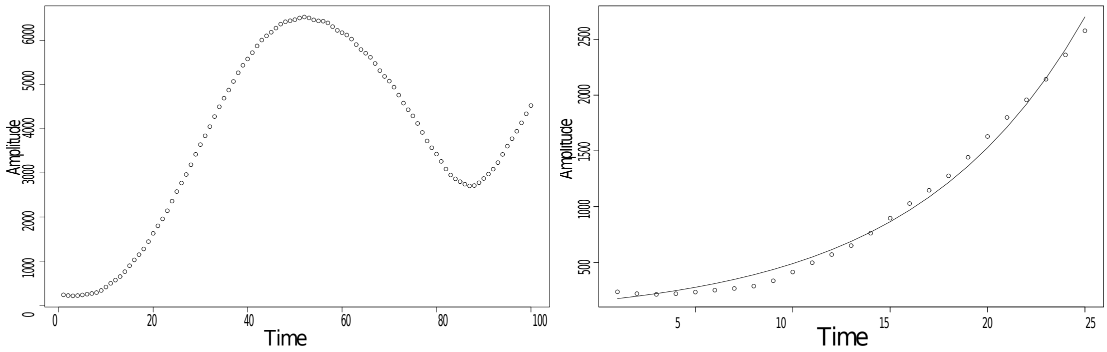

Figure 2. Example of time series of amplitudes extracted for $\omega/\omega_{pe} = 0.4$ and $k\lambda_D = 0.02$ for the whole simulation (Left) and the initial part of the same simulation with an exponential function fitted to the data (Right).

the nonlinear least squares method (Figure 2). The exponential curve has the form $y = Ae^{\gamma t}$ where γ refers to the growth rate parameter in the complex wave function $\omega = \omega_r + i\gamma$. Here γ corresponds to the growth rate of the wave mode.

Figure 3 shows the growth rate curve for a range of wave numbers for the beam mode. The range of wave numbers extends from $k\lambda_D = 0.006$–0.072. This range was chosen to exclude the coupling effects of the electron acoustic mode with the beam mode. Figure 3 shows that there is a region, $k\lambda_D = 0.018$–0.048 for which the beam mode will be weakly damped. At greater and smaller wave numbers the beam mode will be damped.

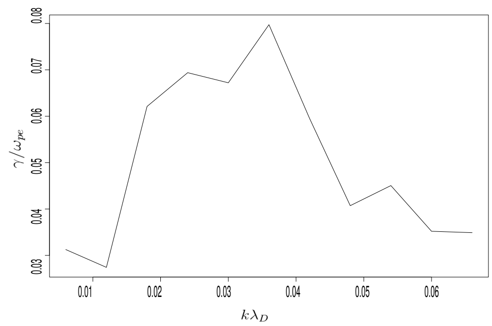

Figure 3. Growth rate diagram constructed from coefficients of exponential curves fitted to a series of amplitudes for different wave numbers.

Summary and conclusions

PIC simulations is a simulation tool based on kinetic theory which enables one to study wave dispersion by constructing a dispersion diagram from the electric field data. These diagrams give information of the dispersion characteristics of the wave modes at a specific time for a time window. By using a fixed time window and advancing the time of analysis, a growth rate diagram can be constructed that gives the growth rates of a given wave mode at different wave number intervals. The growth rate diagram together with the dispersion diagram now gives a more complete picture of both the dispersion and growth characteristics of the wave modes.

The beam mode in a plasma containing hot, cold and beam electrons was investigated. It was found that the beam mode has a strong growth rate in the region of $k\lambda_D = 0.006$–0.072 while the mode is damped at smaller and larger wave numbers.

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