Mathematical modelling of the coefficient of performance of a Carnot's Air source heat pump water heater

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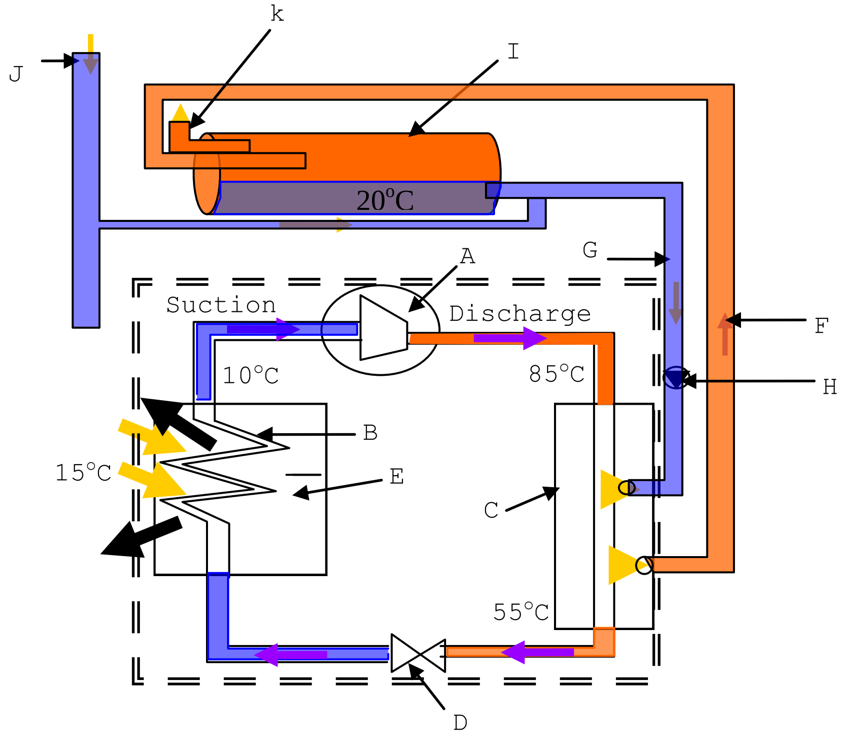
Abstract. In South Africa, there is an ongoing constraint on the electricity supply at the national grid to meet the demand. Eskom is implementing various measures such as the Integrated Demand Management (IDM) and the encouragement of the use of efficient energy devices like air source heat pump (ASHP) water heater for replacement of high electrical energy consumption utility (conventional geysers) in sanitary hot water production. The ASHP water heater market is fast gaining maturity. A critical mathematical model can lead to performance optimization of the system that will further result in the conservation of energy and significant reduction in global warming potential. ASHP is an electro-mechanical device that operates on the principles of vapour compression refrigerant cycle. The ASHP water heater comprises of an ASHP and a hot water storage tank. A data acquisition system (DAS) monitors the temperature at the evaporator, condenser, hot water and the ambient temperature in the vicinity of the evaporator. This work focuses on using the mathematical equation for the Coefficient of Performance (COP) of an ideal Carnot's heat pump (CHP) water heater and writing basic computation in M-file of Matlab and Simulinks software to model this system based on two reservoir temperatures, viz., evaporator temperatures (Tevp) of -10°C to 40°C (equivalent to ambient temperature, Ta) and condenser temperatures (Tcon) at 65°C, 85°C and 95°C (equal set point temperature of hot water in the tank of 50°C, 55°C and 60°C). From the modelling results it can be deduced that at 0°C Tevp, the COP is 4.2 and 2.7 respectively with the set temperatures of 55°C. Above 27°C Tevp, the rate of change of COP increased exponentially for CHP but is constant at 0.01/°C for practical ASHP. Finally the paper will present an analytical comparison of CHP water heater to practical ASHP water heater for hot water temperature set at 55°C.

1. Introduction

Hot water heating constitutes a significant percentage of energy consumption in industrial, commercial and residential sectors worldwide. In South Africa, water heating is the largest residential use of energy, with up to 50% of monthly electricity consumption being used for this purpose (Meyer and Tshimankinda, 1998). Eskom strategic plan outlook for 2010 to 2030 show over 20% reduction of electricity production from coal (Digest of SA Energy statistics, 2009). To achieve this goal, an urgent need of the use of energy efficiency technology such as smart use of heat pump for sanitary hot water is encouraged. Eskom is embarking in rolling out rebate program of 65,580 units of residential ASHP water heater still March 2013(Eskom, 2010) in a bid to promote the use of this technology by residential end users. ASHP can be categorized into integrated or 'drop in' system and retrofit or split type. Each of this type is divided into single or one passed circulation system and a multi-passed or recirculation system. The key components of the heat pump unit are the evaporator coil, vapour compressor, heat rejection condenser and an expansion valve. ASHP water heater provides hot water at 40 to 100 percent of the rate of electric resistance units and 30 to 50 percent of the rate of gas units, but require warm ambient temperatures and a large heat pump or storage tank to provide a constant flow of hot water (Bodzin, 1997). The characteristics of a heat pump that enables it provide such a very high efficiency of 300% is called coefficient of performance (COP) (De Swardt *et al.,* 2000). The COP of ASHP water heater depends on various parameters including components design, heating load cycle, thermo-physical properties of the working fluids, relative humidity and air speed through the duct space. The instantaneous, seasonal or annual COP can be calculated using simulation with TRNSYS software package (Kline *et al.,* 1999), components testing (Morrison, 1977) and mathematical modelling (Itoe *et al.,*1999).

2. Fundamental principles of ASHP water heater

The schematic diagram in figure 1 shows the full designed and components of the ASHP water heater



Key features
A= Vapour compressor
B=Evaporator coil
C=Condenser
D=Expansion valve
E=Fan
F=ASHP outlet pipe
G=ASHP inlet pipe
H=water pump
I=Storage tank
J=Incoming cold water
K=Hot water supply

Figure 1: A shows a schematic diagram of an ASHP water heater

During a heating load cycle, the ASHP undergo a vapour compression refrigerant cycle (VCRC) and this cycle can only be achieved by electrical energy input to drive the compressor for a CHP and for a practical ASHP energy is needed to run the pump and the fan showed in figure 1. The low pressure and temperature refrigerant extract aero-thermal energy from the ambient air and this low pressure vapour flow to the compressor where it is compressed and discharged as super-heated gas. The thermal energy absorbed by the gas is rejected at the condenser unit while incoming cold water is heated to hot water set temperature. The fluid finally flow passed an expansion valve where it pressure and temperature drop. The complete process involved in the cycle is showed in the temperature and entropy as well as pressure and enthalpy graphs. From the graphs the COP of the heat pump is calculated. At the evaporator the process is isothermal, while at the compressor is isentropic. Also, for the condenser the process is isobaric and at the expansion valve is isenthalpic. The figure 2 and 3 illustrated the graphs of the various processes provided the thermo-physical fluid is a pure refrigerant.

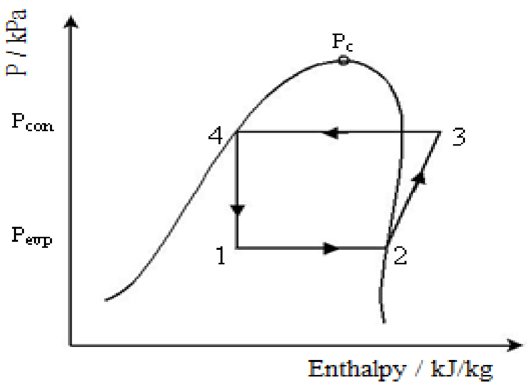


Figure 2: Pressure versus enthalpy graph

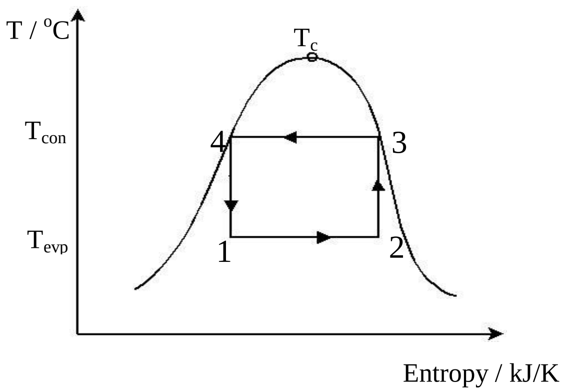


Figure 3: Temperature versus entropy graph

The analysis of the processes involved in each section of the two figures is explained below: where h and s are specific enthalpy and specific entropy of the system, respectively, E_{in} is input electrical energy and Q_{out} is the useful heat gain. In order to understand and mathematically represent the processes taking place in the evaporator, compressor, condenser and expansion valve sections of the heat pump, a set of equations (1- 8) were deduced. Analyzing figure 2, the process 1 to 2 occurred in the evaporator and the heat gain was calculated using equation 1.

$$\Delta Q = mh_2 - mh_1 \tag{1}$$

Process 2 to 3 occurred at the compressor and heat gain was calculated as shown in equation 1

$$\Delta Q = mh_3 - mh_2 \tag{2}$$

Process 3 to 4 occurred in the condenser and the heat rejected was calculated using equation 3

$$\Delta Q = mh_3 - mh_4 \tag{3}$$

Process 4 to 1 occurred in the expansion valve and enthalpy change was calculated as shown in equation 4

$$\Delta Q = mh_4 - mh_1 = 0 \tag{4}$$

Using the definition of COP in terms of energy factor, equation 5 was obtained

$$COP = \frac{Q_{out}}{E_{in}} \tag{5}$$

From equation 5, both energies (Q_{out}, output thermal energy and E_{in}, input electrical energy) can be expressed in terms of h to give equation 6.

$$COP = \frac{m(h_3 - h_1)}{m(h_3 - h_2)} = \frac{h_3 - h_4}{h_3 - h_2} \tag{6}$$

The COP could also be derived from figure 3 and by using the first law of thermodynamics. The COP can be defined by equation 7

$$COP = \frac{s_2 T_{con} - s_1 T_{con}}{(s_2 T_{con} - s_1 T_{con}) - (s_2 T_{evp} - s_1 T_{evp})} \tag{7}$$

Simplifying equation 7 gives equation 8

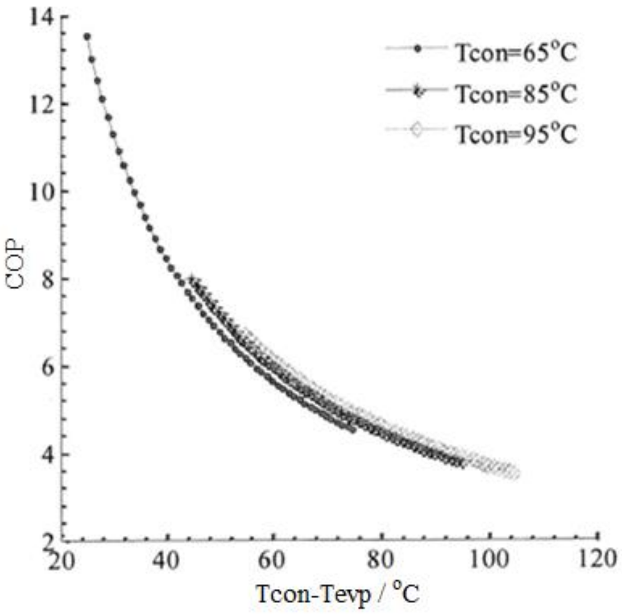
$$COP = \frac{T_{con}}{T_{con} - T_{evp}} \tag{8}$$

Both the evaporator and condenser temperatures were expressed in Kelvin (K). Equation 8 was the COP formulation applied to the CHP water heater. Based on this equation, a computational programme which modeled the COP variation with temperature lift (T_{con}-T_{evp}) was generated from the M-file script of the MATLAB software.

3. Methodology

3.1 Development of programme code to modelled Carnot's ASHP water heater COP

The basic programme is computed upon key assumptions that evaporator and ambient temperature are equal and the thermodynamic fluid is a pure refrigerant. All comments are expressed with % and are not executed by the programme. The programme is run for condenser set temperatures at 65 ^oC, 85 ^oC and 95 ^oC corresponding to hot water temperatures of 50 ^oC, 55 ^oC and 60 ^oC for a domestic ASHP water heater. This programme used the "for loop" and show that Carnot's COP increases linearly with ambient temperature from 0 °C to 20 °C at 0.04/ °C and above this range increases exponentially. The code is shown below.



% COP of Carnot's heat pump as function of evaporator and condenser temperature
% Ambient temperature range (T_a) is approximated to T_{evp}.
% T_{evp} = Evaporator temperature in K
% T_{con} = Condenser temperature in K
% T_h = Hot water set point temperature in °C
T_0 = 273; % Temperature conversion factor
Tcons = ['r.-';'b*-';'gd-';'mx-']; % set color for Tcon
x=1; % Initializing color to start count
T_a = -10:1:43; % Annual ambient temperature in °C
T_{evp} = T_a+T_0; % Tevp range in K
for T_H = 85:5:100; % condenser temperature in °C
Tcon = T_H+T_0;
hold on % generate various graphs
COP = T_{con}/(T_{con}-T_{evp});
Plot((T_{con}-T_{evp}), COP,Tcons(x,1:end))
x = x+1;
end

Figure 4: COP versus temperature lift

From graphs generated in figure 4, the COP increases with a decreased in temperature lift.

3.2 Mathematical modelling of practical ASHP water heater

The equation 9 and 10 are the Morrison COP correlation with temperature that was used to mathematically modelled the practical ASHP water heater.

$$COP = a_1 + a_2(T_t - T_a)\left[\frac{T_t - T_w}{T_t - T_a}\right] \quad (9)$$

$$COP = a_1 + a_2(T_t - T_a) \quad (10)$$

Equation 10 is a constrain equation to optimized the COP, and the constants a_1 and a_2 were determined for T_t = 50 °C, T_t = 55 °C, and T_t = 60 °C. Figure 5 presents the modelled COP for these different temperatures.

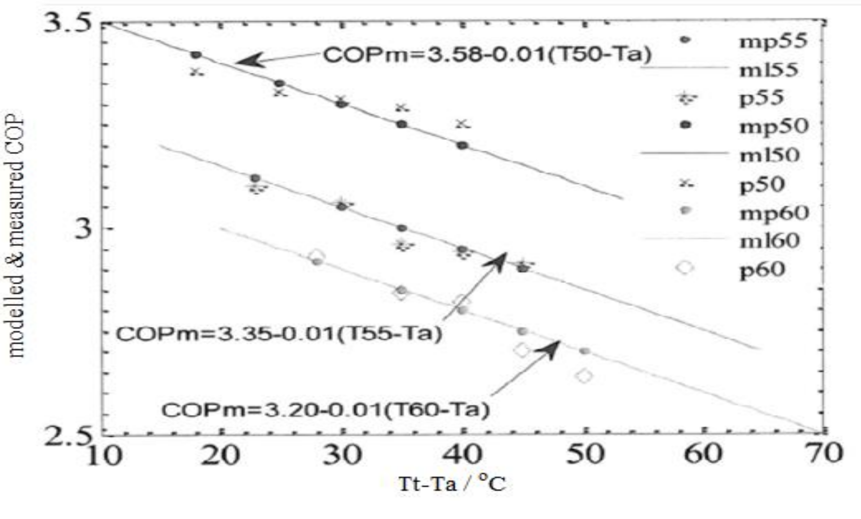


Figure 5: Shows measured and modelled COP against (Tt-Ta)

The line plots with solid markers show the modelled equations of the COP while the scatter marker points represent the calculated COP at the specific hot water temperature as well as relative humidity and ambient temperature.

3.3 Methodology for determination of input parameters

The computation of the COP using mathematical modelling from Matlab and Simulink software was performed for a typical week baseline profile of a 3 kW convectional 150 L geyser installed in a domestic residence (occupied by four adults and two children) of middle class at Fort Bueafort, South Africa. The geyser was installed to operate without interruption for 24 h on a daily basis. Figure 6 and 7 illustrate the DAS and a full schematic set up. Figure 6 and 7 illustrate the DAS and the full schematic block diagram of the metering transducers and sensors employed in the set up parameters. The main electric power consumption and total current drawn to the building was measured by power track 1. Power track 1 was installed on the main distribution board with the positive voltage cable (red) and the negative voltage cable (black) connected to the live and neutral lines of the mains. The current transformer (CT) of the power tack was placed on the live. Power track 2 was placed to the line supplying current to the geyser and measured the current and total power utilized by the 150 L, 3 kW combinational geyser. The power track is configured to log every 1 minute. All the temperature sensors were thermistor resistance sensors. Temperature sensor 1 was placed at the cold water inlet pipe to the geyser well insulated. Similarly, the temperature sensors 2, 3 and 4 measured the hot water temperatures for the outlet hot water from the geyser, hot water to the bathroom and kitchen respectively. Flow meter (T-Minol 130) was placed in close proximity to temperature 2 on the hot water pipe and measured the flow rate of the hot water drawn in number of counts per minute and was logged. The relative humidity and ambient temperature sensor measured the relative humidity and ambient temperature. The Hobo NRC-U30 data logger was used to logged counts for the volume of water drawn, various temperatures and the relative humidity and ambient temperature.

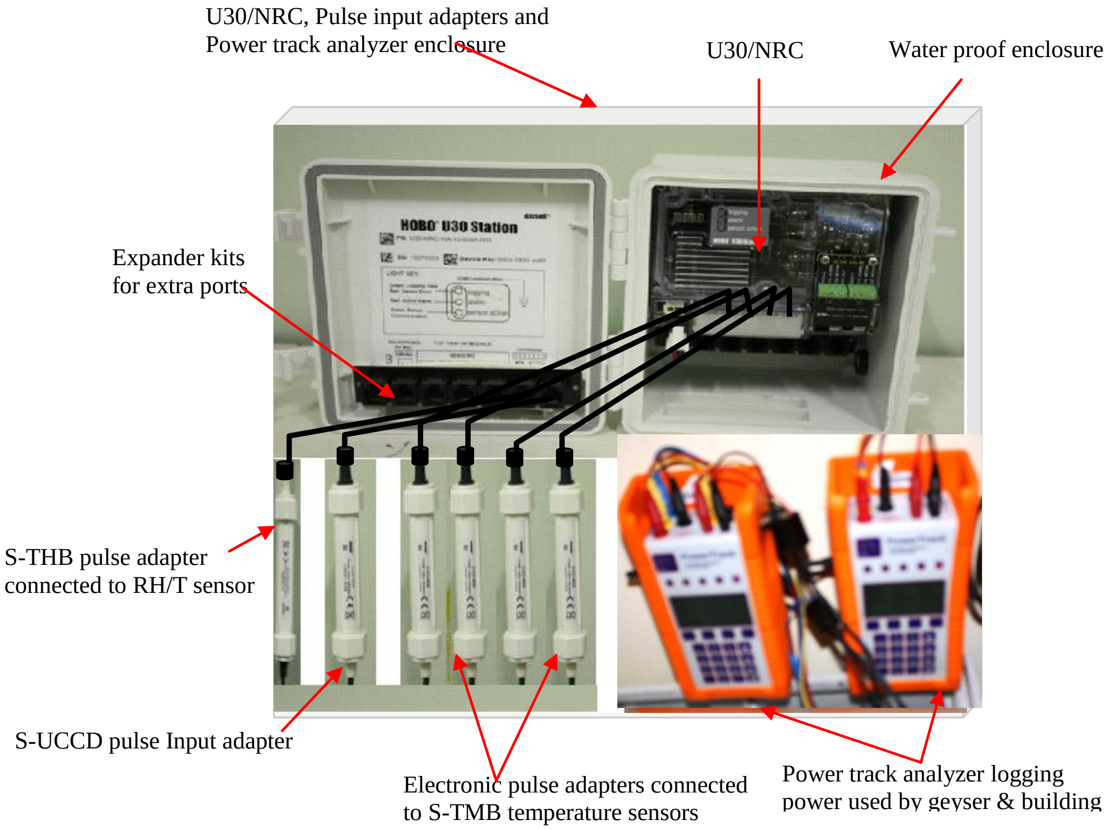


Figure 6: DAS for the experimental set up.

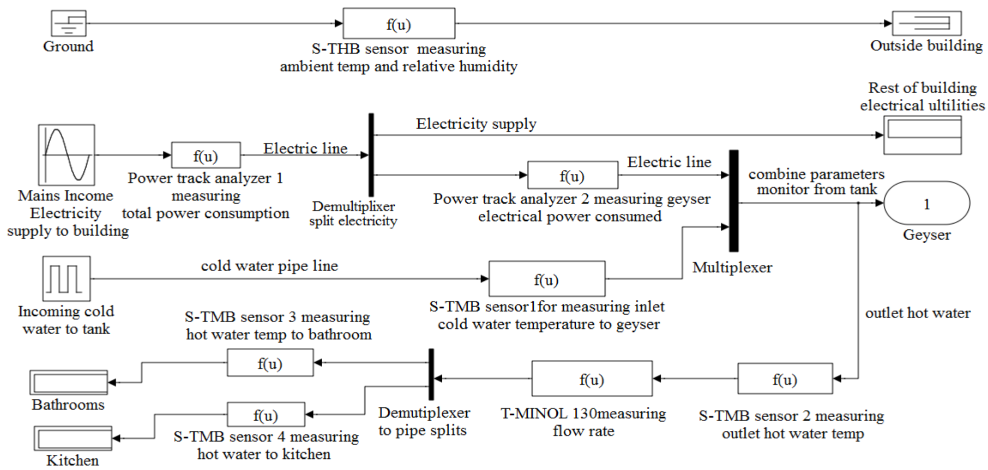


Figure 7: Schematic layout of the full installation set up

3.3.1 Data acquisition system

The research actually employed power track analyzers which recorded and logged voltages, current and power consumption in 1 minute interval. The U30-NRC logger was used to log all the temperature sensors, flow meter and the relative humidity and ambient temperature sensors measured data.

4.0 Results and discussion

4.1 Experimental determination of the baseline

A baseline is needed as a real reference to justified the energy savings when retrofitting geyser with ASHP. Again is needed to calculate the input energy to be used in the simulation model. A typical week profile was considered and used in the development of the modelled. From the data, the building power usage, geyser power consumes and hot water drawn off was determined for a typical week over a 24 h shown in table 1.

Table 1: Average typical week energy and hot water consumption

Days of the week	Volume of water drawn off (L)	Average maximum power (kw)		Total energy consumption (kWh)	
		Building	Geyer	Building	Geyer
Monday	287.690	5.305	2.063	28.696	17.424
Tuesday	283.905	2.850	2.013	28.499	15.742
Wednesday	405.038	3.125	2.040	34.176	21.376
Thursday	344.471	4.051	2.092	31.839	21.375
Friday	348.257	2.551	2.003	30.737	19.604
Saturday	370.969	3.655	2.043	38.774	20.002
Sunday	393.682	3.119	2.034	33.177	19.582

From the table 1, the capacity of daily hot water drawn for the average week day and average week end was 333 L and 382 L. This indicates significant electrical energy consumption and hence favoured the retrofit of geyser with ASHP.

4.2 Simulation model using Simulink

Figure 8 shows the simulation application used to modelled the COP of the propose retrofit geyser with an ASHP.

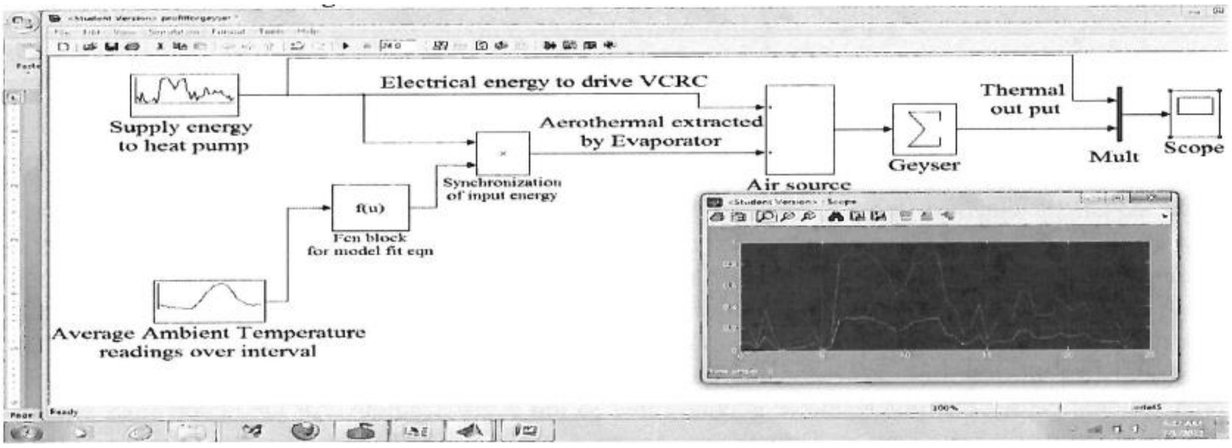


Figure 8: Illustrate the simulation architectural and programming sequence used in modelling the system COP

The input parameters (energy used by ASHP and ambient temperature) over 30 minutes interval are loaded to the two sources. The function block (f(u)) contain the modelling equation while the summation block act as heat pump extracting aero-thermal energy at the evaporator and electrical energy via compressor for CHP and additional energy used by fan and pump for ASHP water heater. The both input and output are displaced on the oscilloscope.

4.3 Average week energy and ambient condition variation

The figure 9, 10, 11 and 12 show the energies, ambient temperature and relative humidity for an average week day and week end profiles.

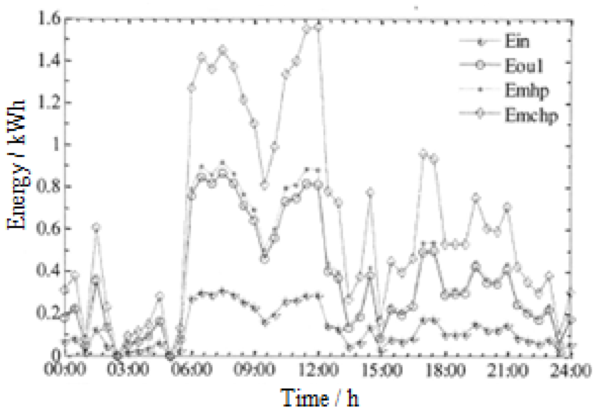


Figure 9: A typical week day energy profiles

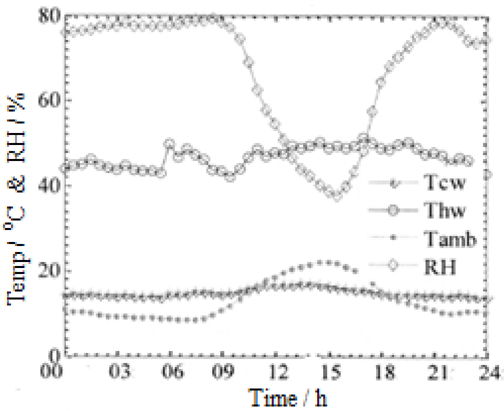


Figure 10:A typical week day energy profiles

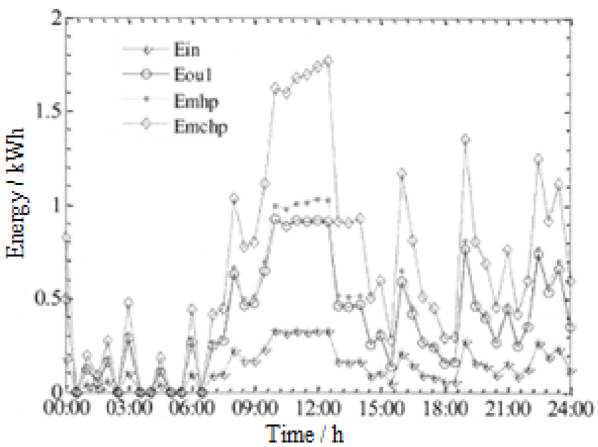


Figure 11: A typical week end energy profiles

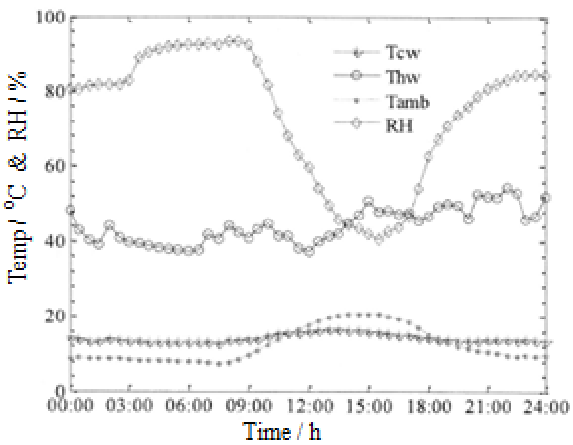


Figure 12: A typical week end energy profiles

The both energy profiles for the typical week day and week end follows a common dynamics and is classified into three time zones. Between 00:00 to 05:30, there is no hot water drawn, and energy (8% of total) used up is to compensate for standby losses, and hot water set temperature is almost constant. During the period the relative humidity is very high since ambient temperature is low. Between 06:00 to 11:00, the demand is high and over 60% of total energy is utilized and average COP of the modelled system is 3.008 and the CHP COP is 5.110. From 12:00 to 18:00, an average COP of 3.011 is achieved due to the increase in ambient temperature although the relative humidity is low. From the profiles of week day and week end, it was depicted that there was negligible variation of the system COP (above 3) and CHP COP (about 5) owing to the relative constant ambient temperature recorded throughout the week.

5.0 Conclusion

The modelled COP of the practical ASHP water heater was comparable to the experimental ASHP water heater with less than 0.03% error. Seasonal and annual COP can be accurately determined for ASHP water heater provided transient ambient temperatures and hot water temperatures are known. The ASHP water heater is an energy efficient technology for sanitary hot water production but compared to Carnot's system is below 65 % efficiency. Mathematically modelling and simulation of COP can be determined without actually measuring the primary factors (load cycle, inlet cold water temperature).

6.0 References

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