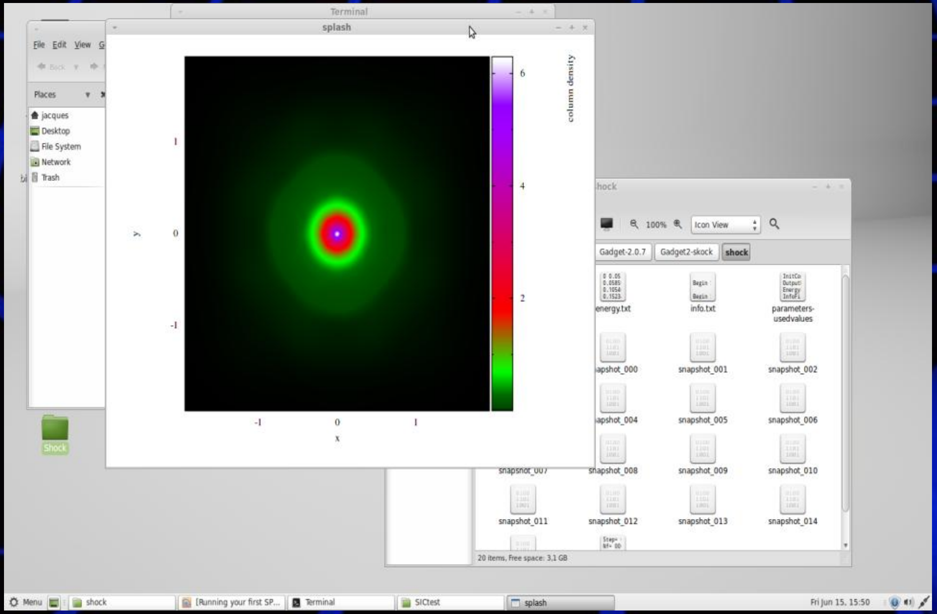
Periodic Sources of GW:



JM MARITZ (UFS), PJ MEINTJES

The financial assistance of the South African Square Kilometre Array Project towards this research is hereby acknowledged. Opinions expressed and conclusions arrived at, are those of the author and are not necessarily to be attributed to the NRF.

Outline:

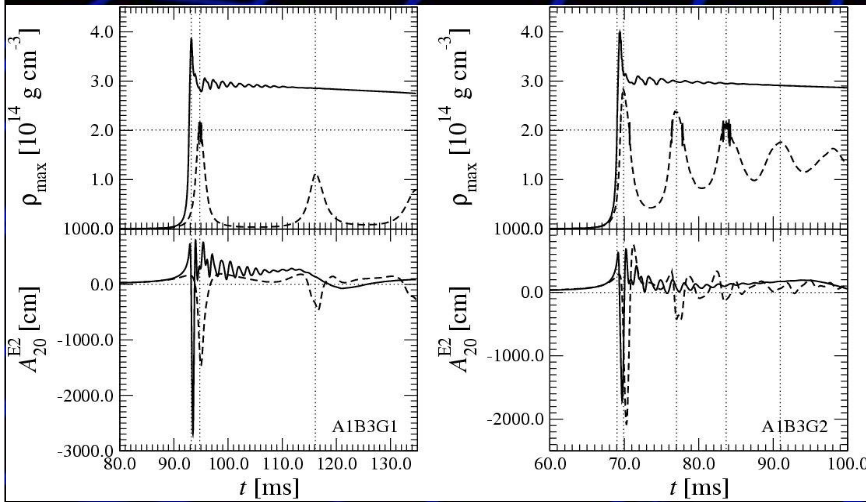
-Supernovae Core Bounce: Signal Extraction

-A GW Magnetar Model for several systems

-Initializing SPH

Periodic sources of GW: Supernovae

Vulcan 2D code



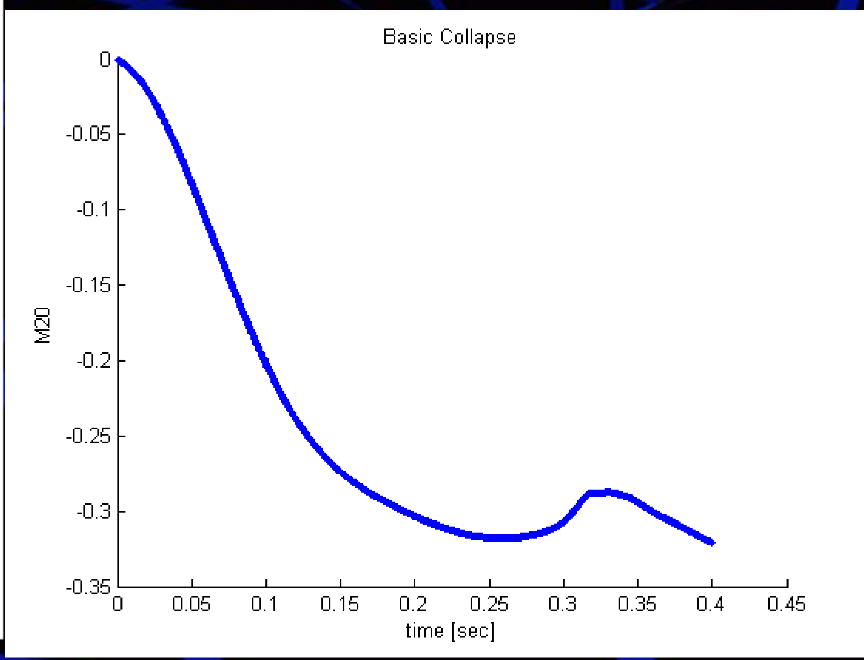
$$\Delta E_{GW} = \frac{1}{32\pi} \sum_{L,M} \int \left\{ \left| \frac{\partial A_{LM}^{\ E2}}{\partial t} \right|^2 + \left| \frac{\partial A_{LM}^{\ B2}}{\partial t} \right|^2 \right\} dt$$

$$\frac{dE_{GW}}{d\omega} = \frac{1}{16\pi} \sum_{L,M} \left| \tilde{A}_{LM}^{\ E2} \right|^2 + \left| \tilde{A}_{LM}^{\ E2} \right|^2 \omega^2$$

$$A_{20}^{\ E2} = \frac{d^2}{d^2 t} M^{E2}_{\ 20}$$

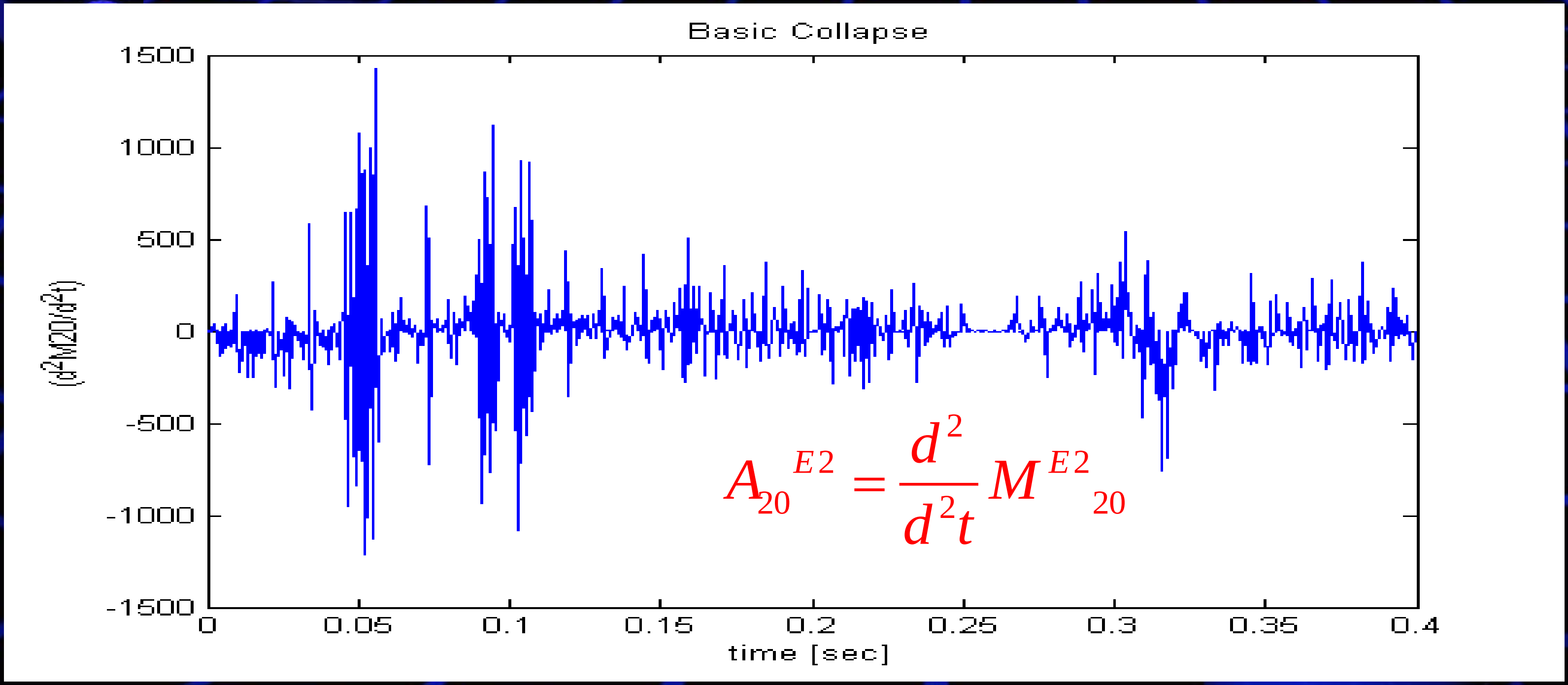
$$A_{30}^{\ B2} = \frac{d^3}{d^3 t} M^{B2}_{\ 30}$$

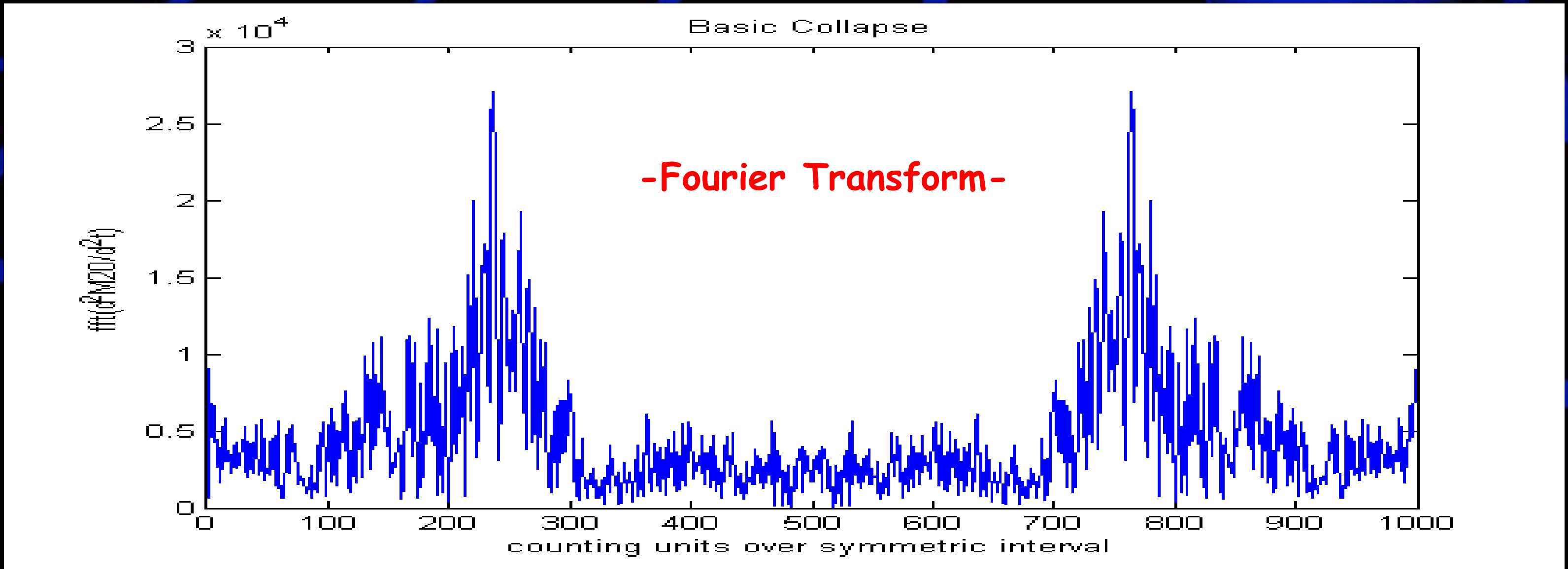
Thorne 1978
Wagoner 1977
E. Muller
1982+ 1990

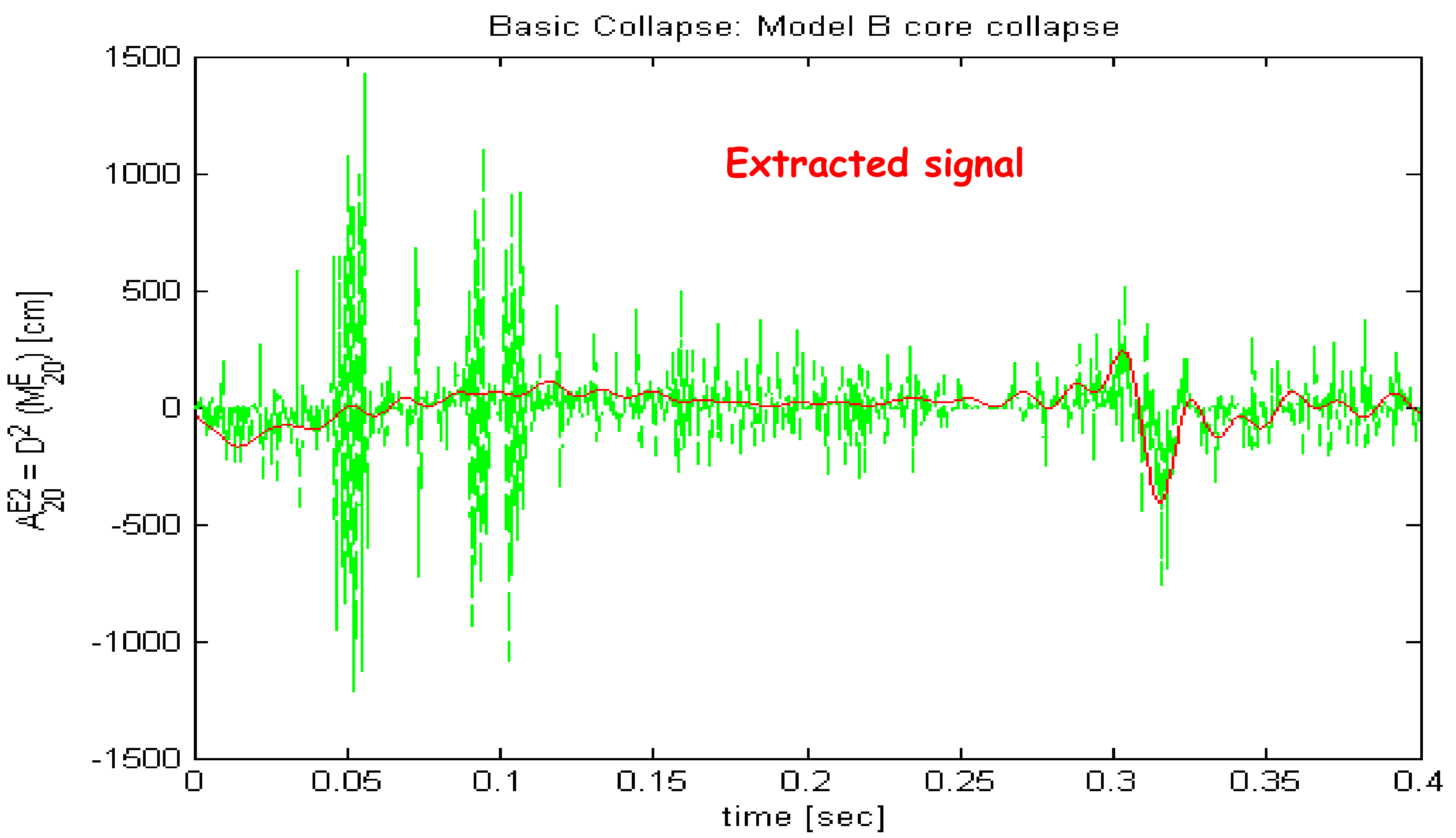


Model		A	B	C	D
ρ_c^b	$[10^{14}\ \mathrm{gcm}^{-3}]$	2.57	0.36	2.11	1.51
ρ_c^{eq}	$[10^{14}\ \mathrm{gcm}^{-3}]$	2.10	0.21	1.30	0.75
M_{ic}^b	$[M_\odot]$	0.97	1.12	1.01	1.02
K_{ic}^b	$[10^{51}\ \mathrm{erg}]$	3.3	0.7	2.1	1.7
E_{rot}^{eq}	$[10^{51}\ \mathrm{erg}]$	9.3	7.3	8.3	6.5
$E_{rot}^{eq,ic}$	$[10^{51}\ \mathrm{erg}]$	6.5	5.2	7.0	5.3
β^{eq}		0.085	0.140	0.094	0.096
T_{osc}^{I}	[ms]	3.1	15.0	4.9	14.0
T_{osc}^{II}	[ms]	2.5	10.0	4.0	9.0
t^b	[ms]	189	308	202	221

Algorithm Results







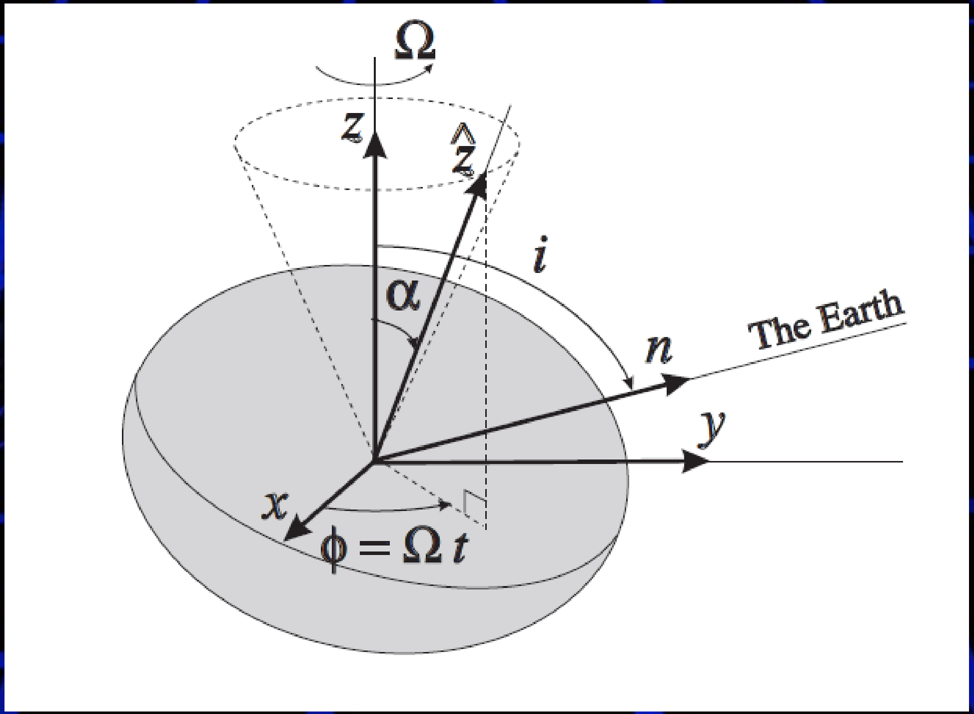
The extracted signal composes of core bounce and a ring-down following the bounce, for model B that uses a central mass of $\sim 1.12 M_{\odot}$, the maximum amplitude produced is $\sim 400cm$ and a strain of $h \sim 3.54 \times 10^{-21}$ at a distance of $10kPc$. The energy emitted per unit frequency corresponds to that that could be observed by LIGO,

Introducing a model for predicting the GW from Magnetars.

Magnetars are the strongest magnets in the universe, they are Super-Nova remnant and their magnetic field are produced by strong dynamo rotation, magnetic fields range between $10^{14}G - 10^{17}G$ (L.Stella 2005). The massive magnetic field induces an slight deformation in the object, and under these circumstances it becomes unstable(Chandrasekhar 1953) and oblate...

Basic Geometry of the Model

S. Bonazzola (1996)



The model uses a maximum GW configuration, where the wobble angle is, $\alpha = \frac{\pi}{2}$
And the line of sight is, $\theta = 0$

Structure of the model..

$$h_\times = h_0[2\sin(2\Omega t)],\ for..h_0 = 10^{-27}\left(\frac{I}{10^{38}\,kg.m^2}\right)\left(\frac{10kPc}{d}\right)\left(\frac{\Omega}{100Hz}\right)\left(\frac{e}{10^{-6}}\right)$$

$$\dot{\Omega}_{total} = \dot{\Omega}_{GW} + \dot{\Omega}_{dipole} + \dot{\Omega}_{MW}[\sec^{-2}]$$

$$= -\frac{2GIe^2\Omega^5}{5c^5}\sin^2(\alpha)(1+15\sin^2(\alpha)) - \frac{1}{6}\frac{B_d{}^2R^6}{Ic^3}\Omega^3\sin^2(\alpha) - \frac{B_{pole}{}^2R^6\Omega^3}{3c^3I}\left(\frac{R_{LC}}{r_{open}}\right)^2$$

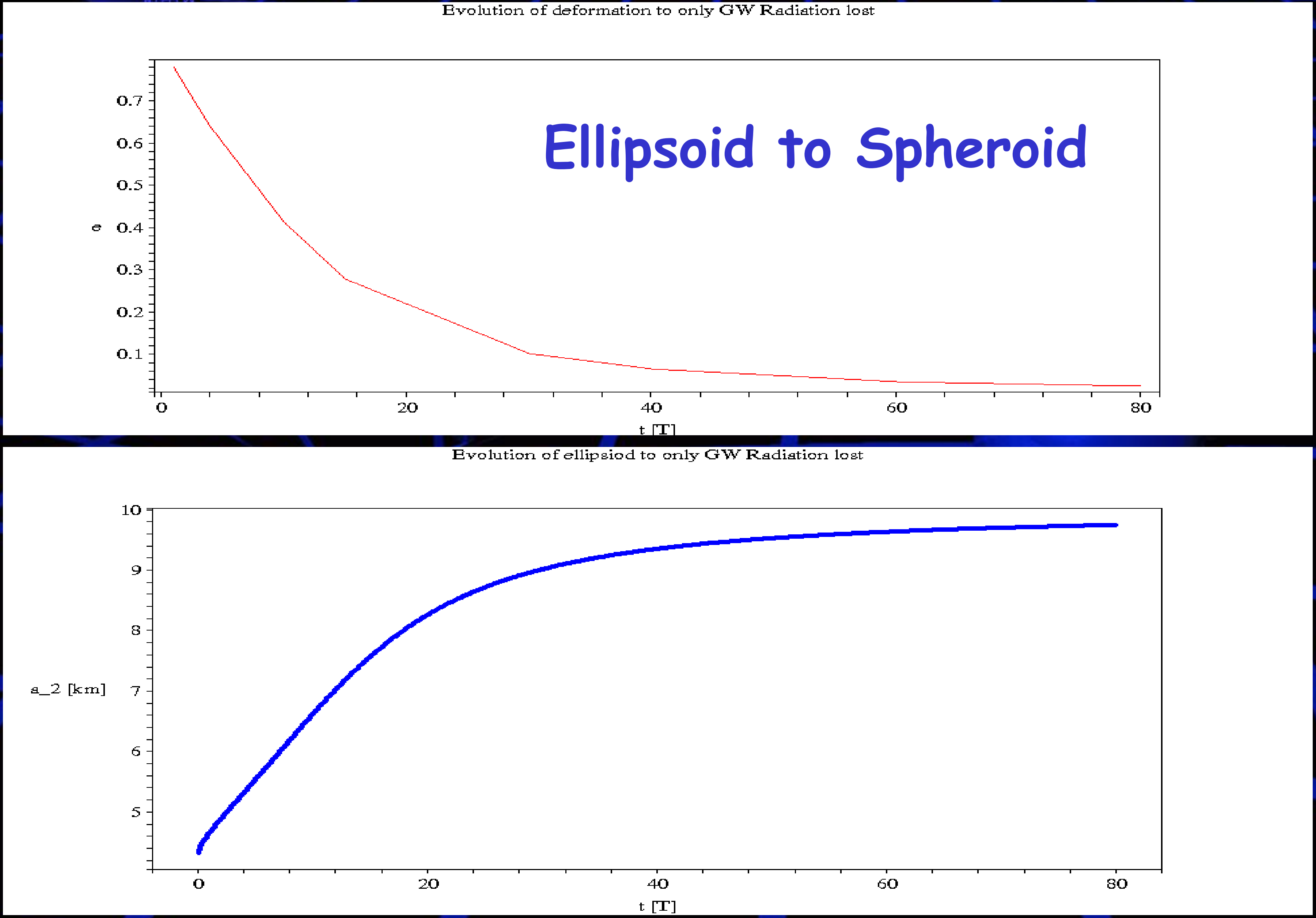
With the geometry and the inertia tensor...

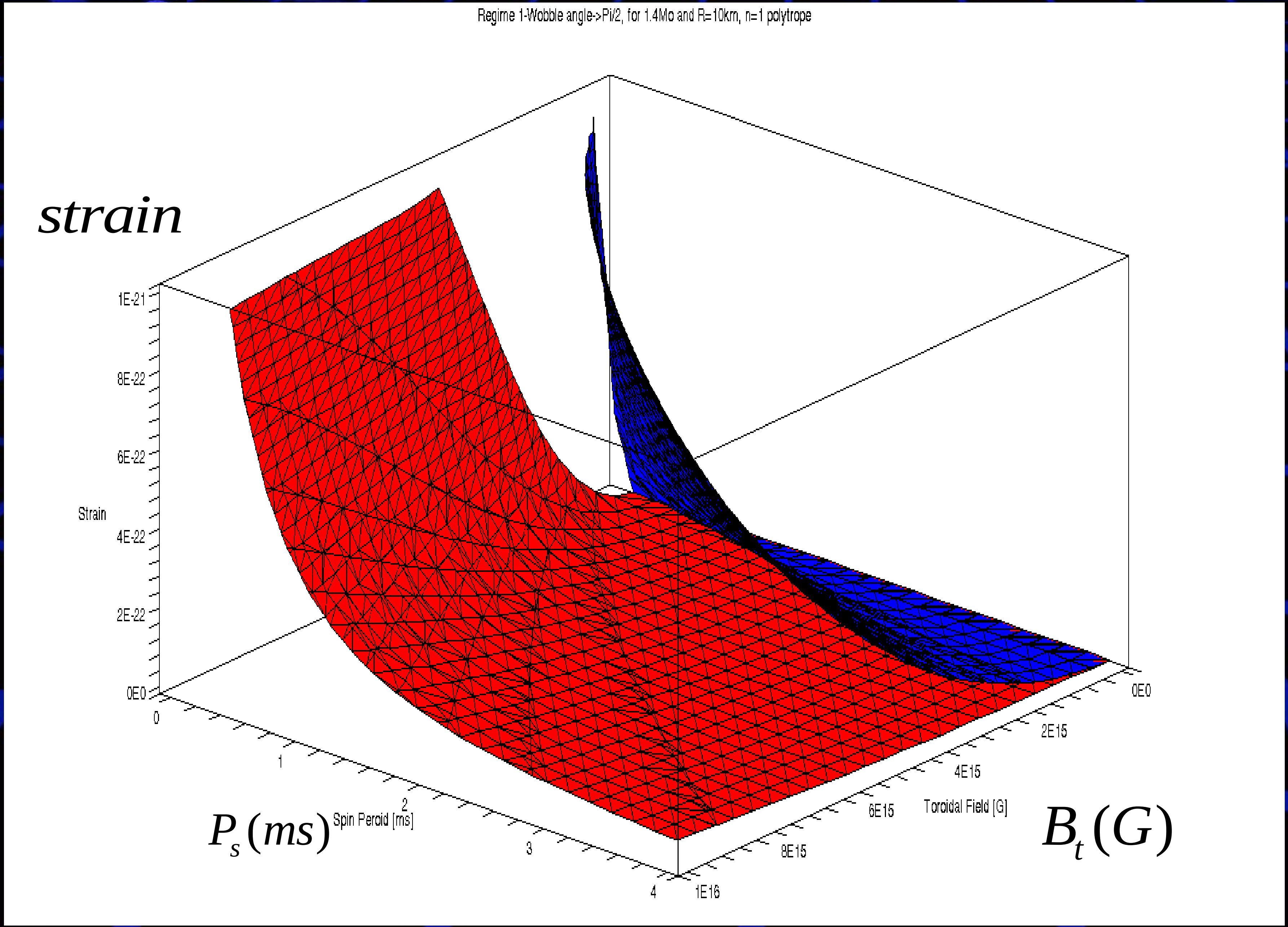
$$e = \frac{a_2 - a_1}{\sqrt{a_1 a_2}}$$

$$I = \frac{1}{5}M(a_1{}^2 + a_2{}^2 + a_3{}^2)$$

Lifshitz, 1962
Misner, Thorne, Wheeler, 1972
B. Haskell, 2007
K.Harding, 1999

Background effects: Just GW Torque





Regime 1-Wobble angle->Pi/2, for 1.4Mo and R=10km, n=1 polytrope
strain
1E-21
8E-22
6E-22
Strain
4E-22
2E-22
0E0
0
1
2
Spin Peroid [ms]
3
4
$P_s(ms)$
1E16
8E15
6E15
Toroidal Field [G]
4E15
2E15
0E0
$B_t(G)$

Assumptions:
- Orthogonal rotator
- Deformations are caused by B_t (Haskell)
- The main torque mechanism is Dipole radiation
- Glitches are ruled out(like Star Quakes)

Possible configurations:

$$B_t \sim 10^{15}, B_p \sim 10^{14}, e \sim 10^{-6}$$

$$B_t \sim 10^{16}, B_p \sim 10^{15}, e \sim 10^{-4}$$

$$B_t \sim 10^{17}, B_p \sim 10^{16}, e \sim 0.01$$

GW –VS- Dipole mechanisms:

$$\dot{E}_{tot} = \dot{E}_{GW} + \dot{E}_{dipole} + \dot{E}_{MW} = I\Omega\,\dot{\Omega}_{GW} + I\Omega\,\dot{\Omega}_{dipole} + I\Omega\,\dot{\Omega}_{MW}$$

$$= -\frac{2GI^2e^2\Omega^6}{5c^5}\sin^2(\alpha)(1+15\sin^2(\alpha)) - \frac{1}{6}\frac{B_d{}^2R^6}{c^3}\Omega^4\sin^2(\alpha) - \frac{B_{pole}{}^2R^6\Omega^4}{3c^3}\left(\frac{R_{LC}}{r_{open}}\right)^2$$

$$for.....\alpha = \frac{\pi}{2} \Rightarrow \dot{E}_{dipole} >> \dot{E}_{GW}$$

For the first configuration $with..(\Omega_0 = 1000\pi, 1M_\odot)$

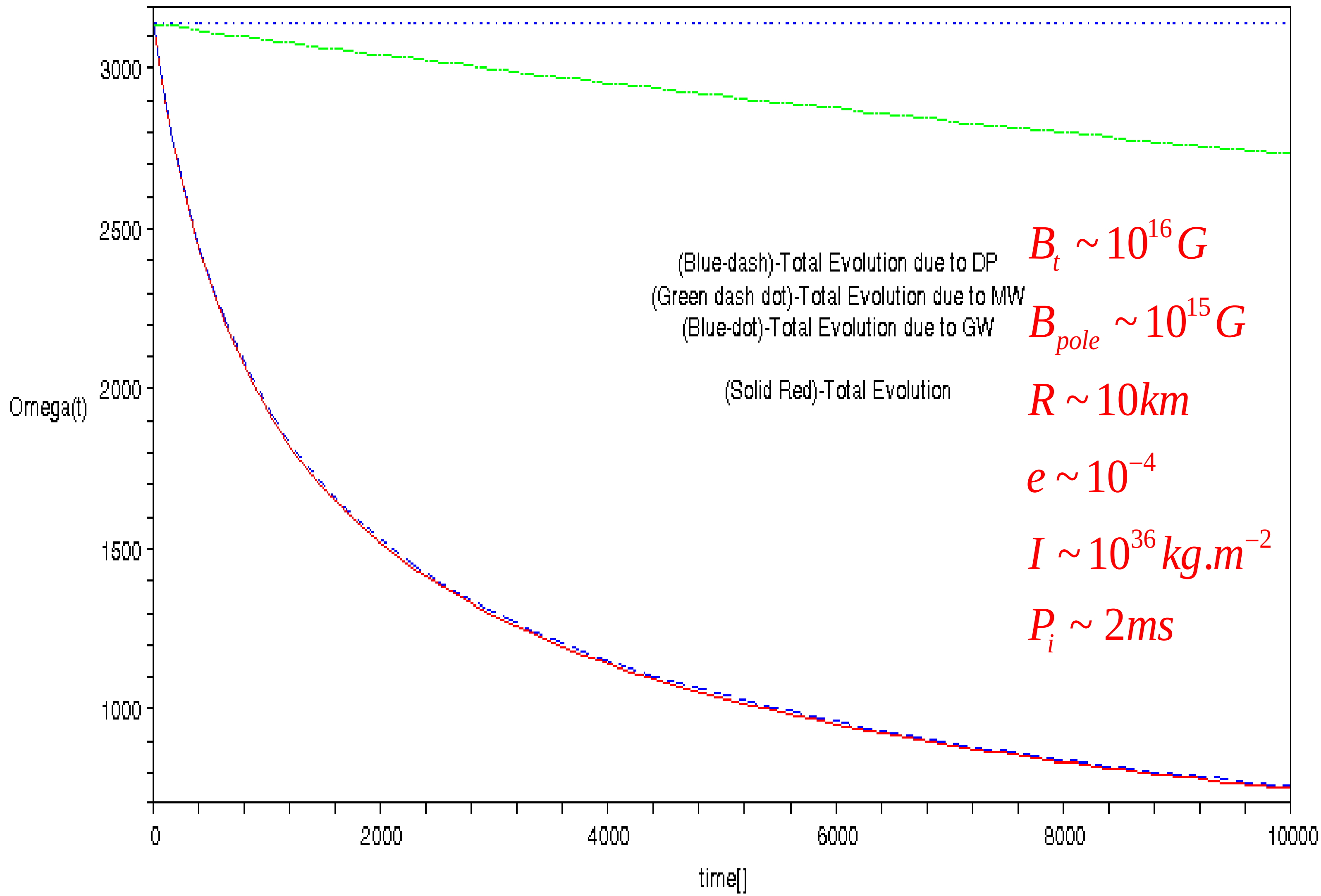
$$E_{GW} <<< 10^{-2} E_{rot}$$

Thus, there exist several mechanisms orders larger then the GW mechanisms....The model could be observed as a upper limit for GW energy from a magnetar.

L.Stella 2008

Evolution of Omega- All Mechanisms

For the different mechanisms



$B_t \sim 10^{16} G$

$B_{pole} \sim 10^{15} G$

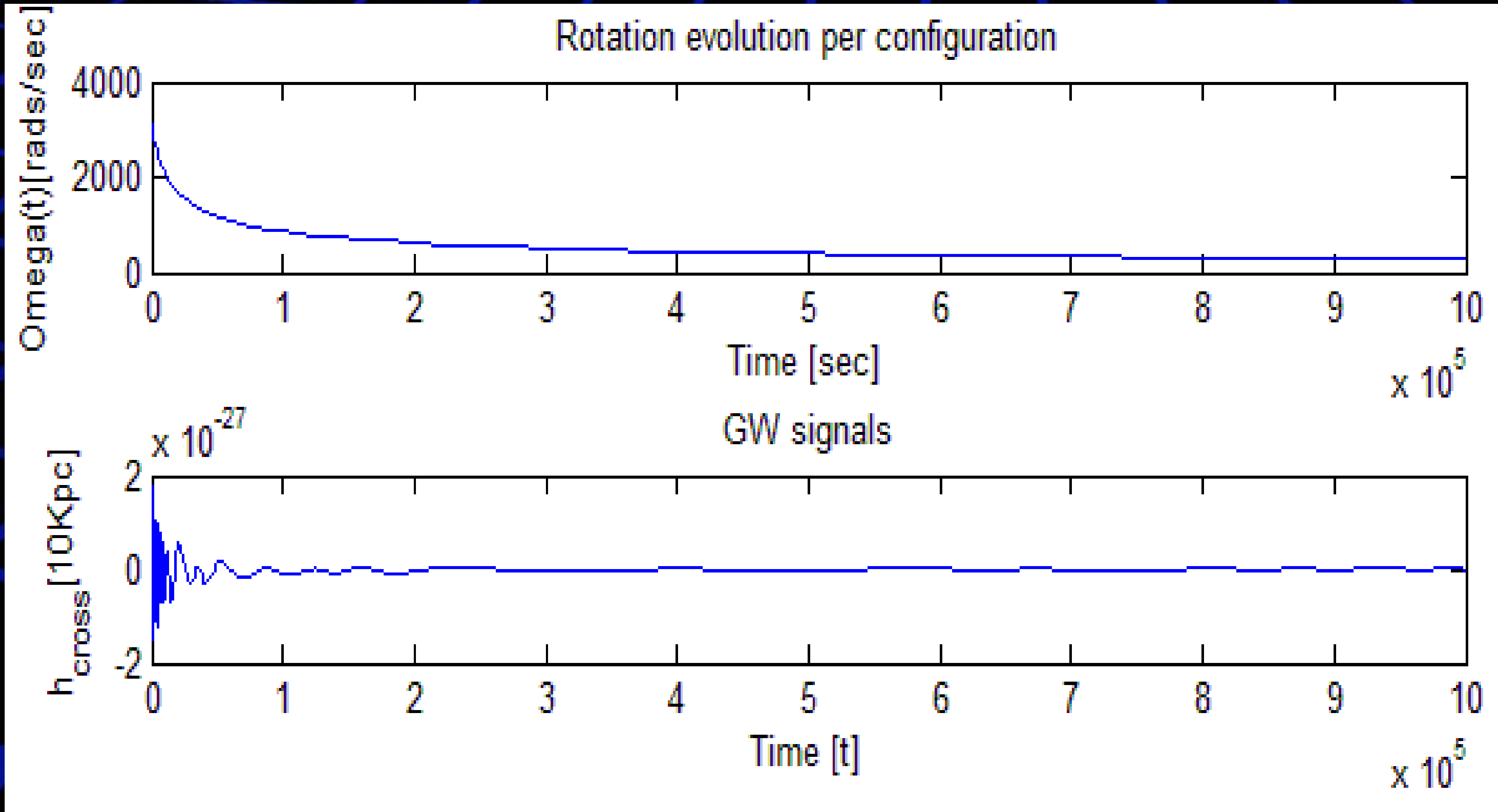
$R \sim 10 km$

$e \sim 10^{-4}$

$I \sim 10^{36} kg.m^{-2}$

$P_i \sim 2 ms$

$$B_t \sim 10^{15}, B_p \sim 10^{14}, e \sim 10^{-6}$$



$P_i = 2ms$

$\bar{a} = 10km$

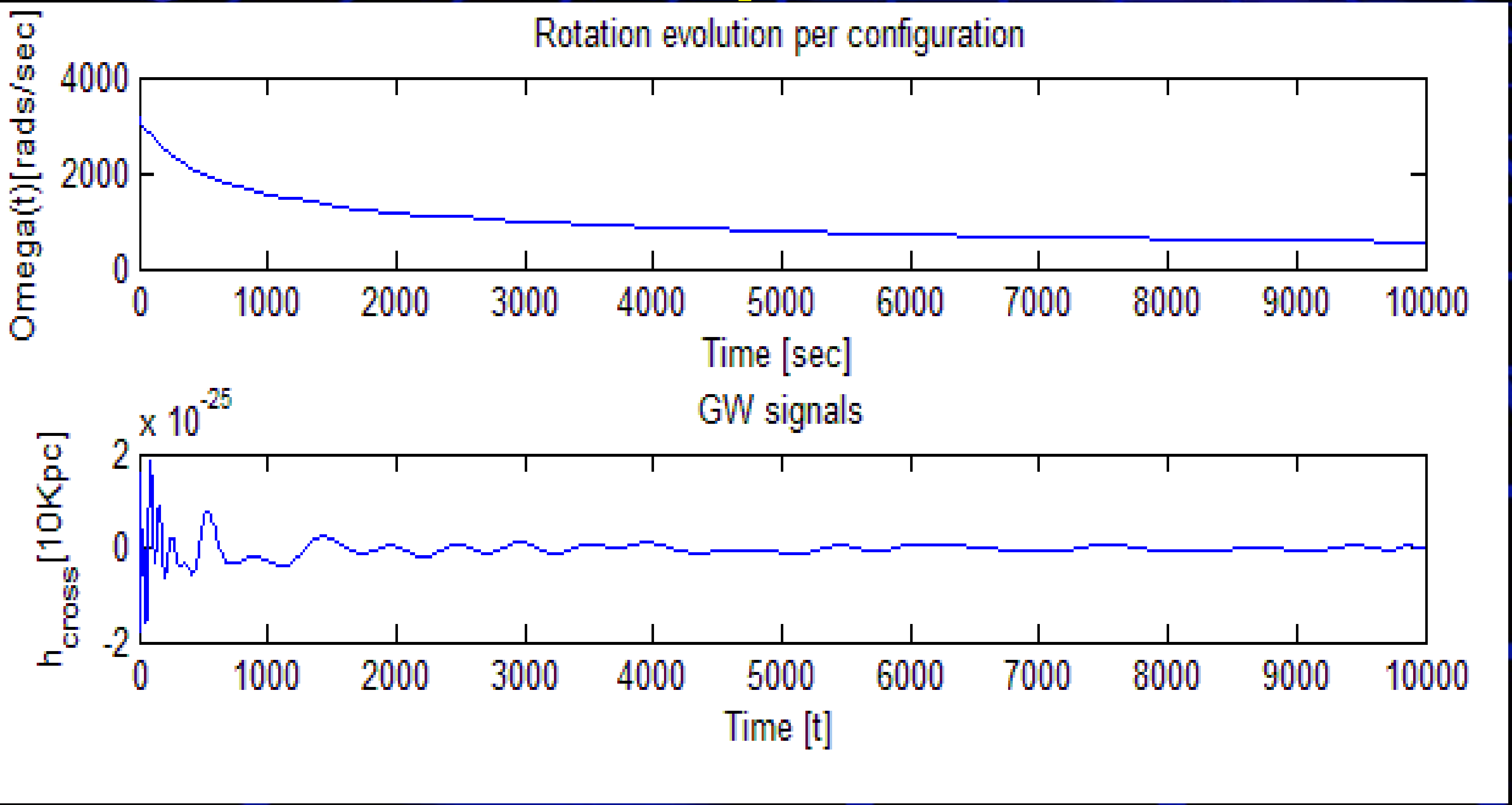
$1M_\odot$

$10kPc$

Spin-down($\rightarrow 10Hz$)= 0.85 yr

L.Stella (2005)

$$B_t \sim 10^{16}, B_p \sim 10^{15}, e \sim 10^{-4}$$



$P_i = 2ms$

$\bar{a} = 10km$

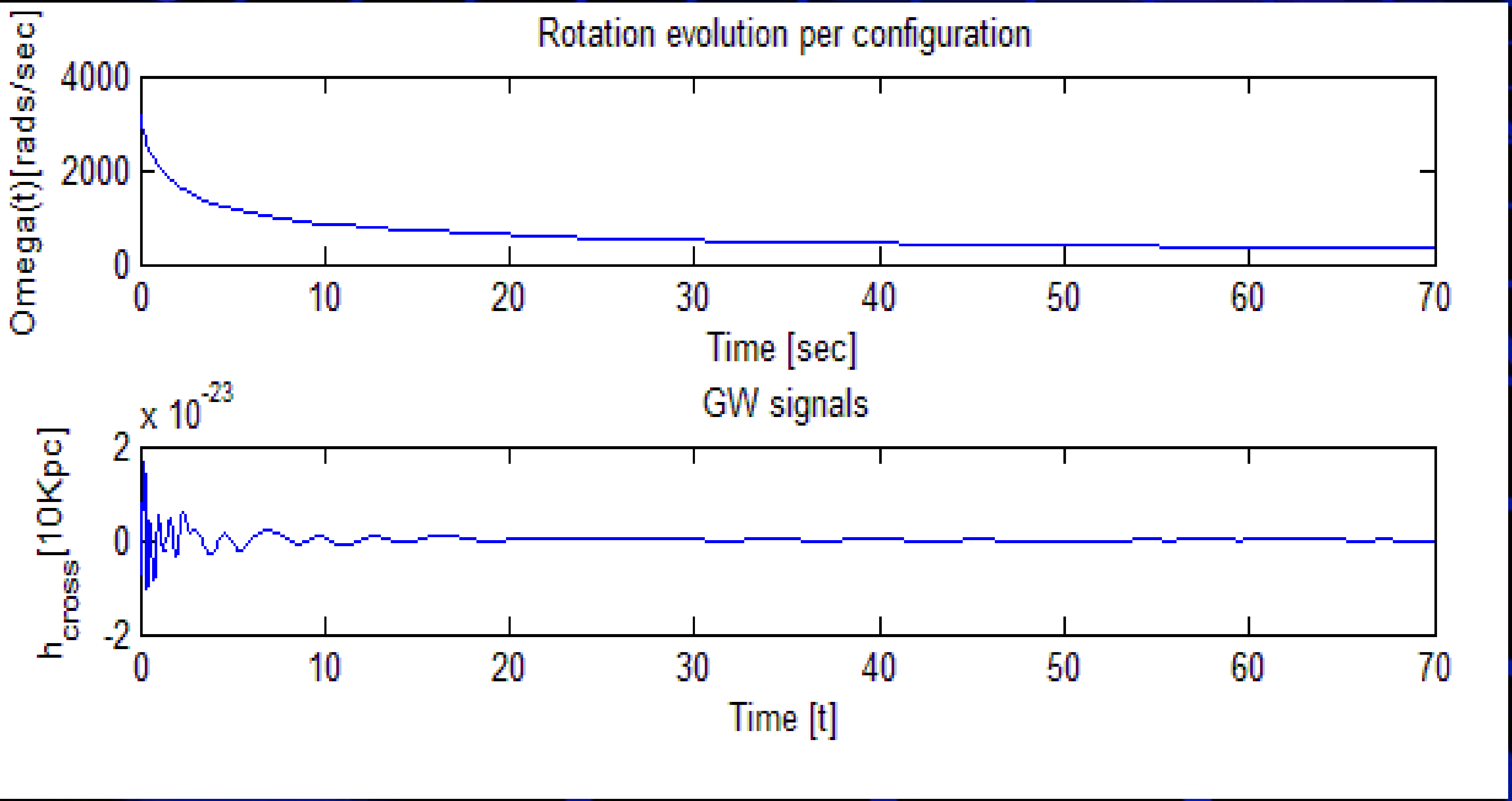
$1M_\odot$

$10kPc$

Spin-down($\rightarrow 10Hz$)= 3 days

L.Stella (2005)

$$B_t \sim 10^{17}, B_p \sim 10^{16}, e \sim 0.01$$



Spin-down($\rightarrow 10Hz$)= 0.77 hours

L.Stella (2005)

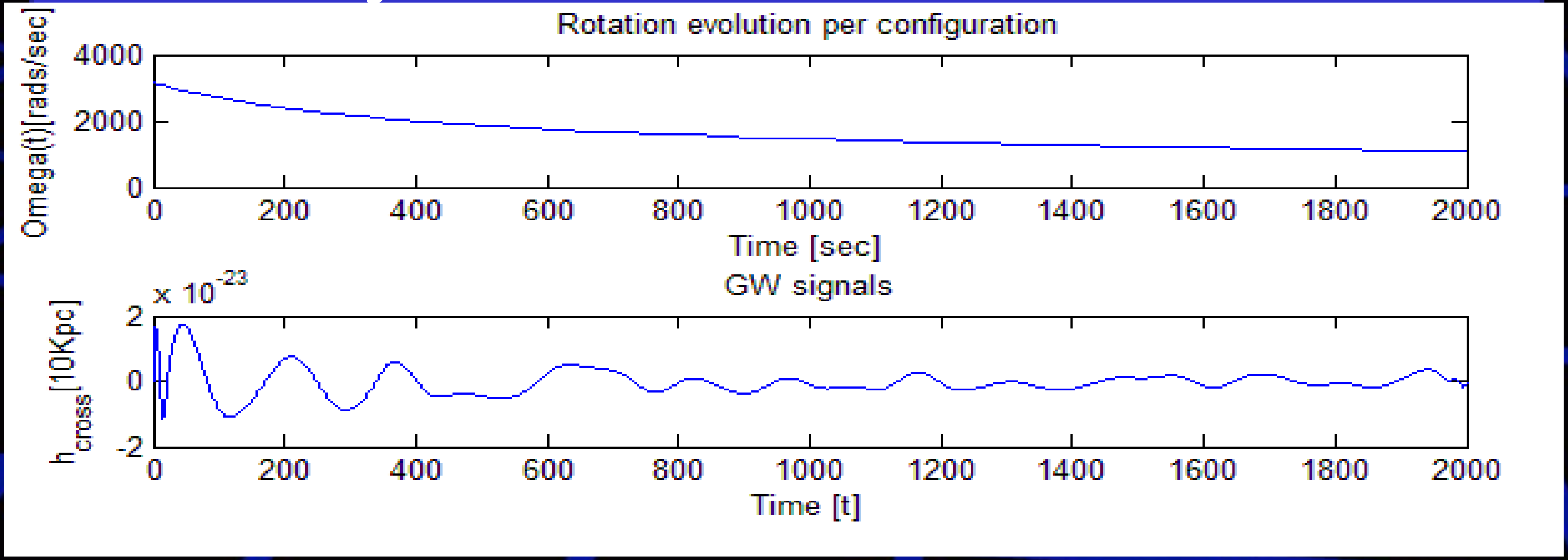
$P_i = 2ms$

$\bar{a} = 10km$

$1M_\odot$

$10kPc$

SGR 1806-20 is a Magnetar, a particular type of neutron star. It was discovered in 1979 and has been identified as a soft gamma repeater. SGR 1806-20 is located about 14.5 kPc (50,000 light-years) from Earth on the far side of our Milky Way galaxy in the constellation of Sagittarius. It has a diameter of no more than 20 kilometers (12 mi) and rotates on its axis every 7.5 seconds (30,000 km/h rotation speed at the surface). As of 2012, SGR 1806-20 is the most highly magnetized object ever observed, with a magnetic field over 10^15 gauss



Spin-down($\rightarrow 200Hz$)= 17 hours

L.Stella (2005)

$P_i = 2ms$

$\bar{a} = 12km$

$1.4M_{\odot}$

$14.5kPc$

Modifying the model to construct a binary neutron star toy model.

M. Shibata 2011

$$\dot{\Omega}_{total} = \dot{\Omega}_{GW} + \dot{\Omega}_{matter} + \dot{\Omega}_{dipoleRadiation} [\sec^{-2}]$$

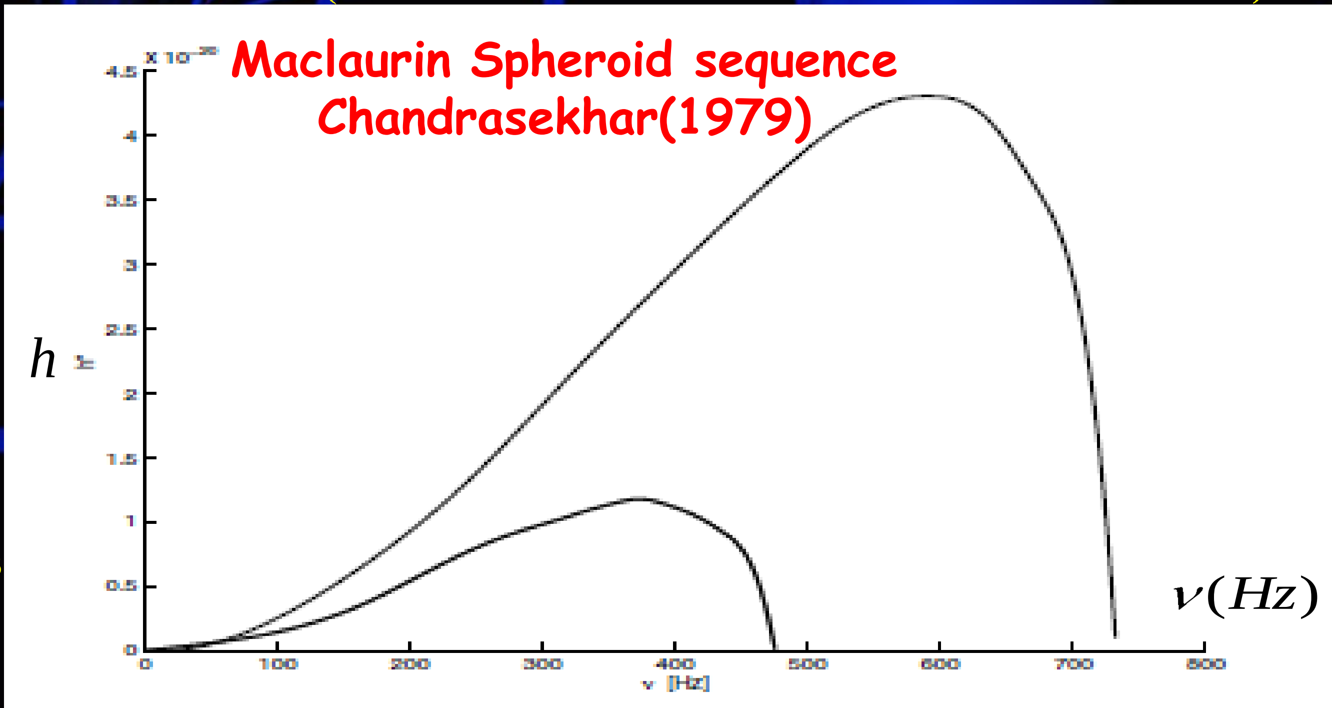
$$= -\frac{2GIe^2\Omega^5}{5c^5}\sin^2(\alpha)(1+15\sin^2(\alpha)) - I^{-1}\left(10^{41}\left(\frac{B_{pole}}{10^{13}}\right)^2\left(\frac{R}{10km}\right)^3 + 10^{40}\left(\frac{B_{pole}}{10^{13}}\right)^2\left(\frac{R}{10km}\right)^3\right)$$

$$e = \frac{a_2 - a_1}{\sqrt{a_1 a_2}}$$

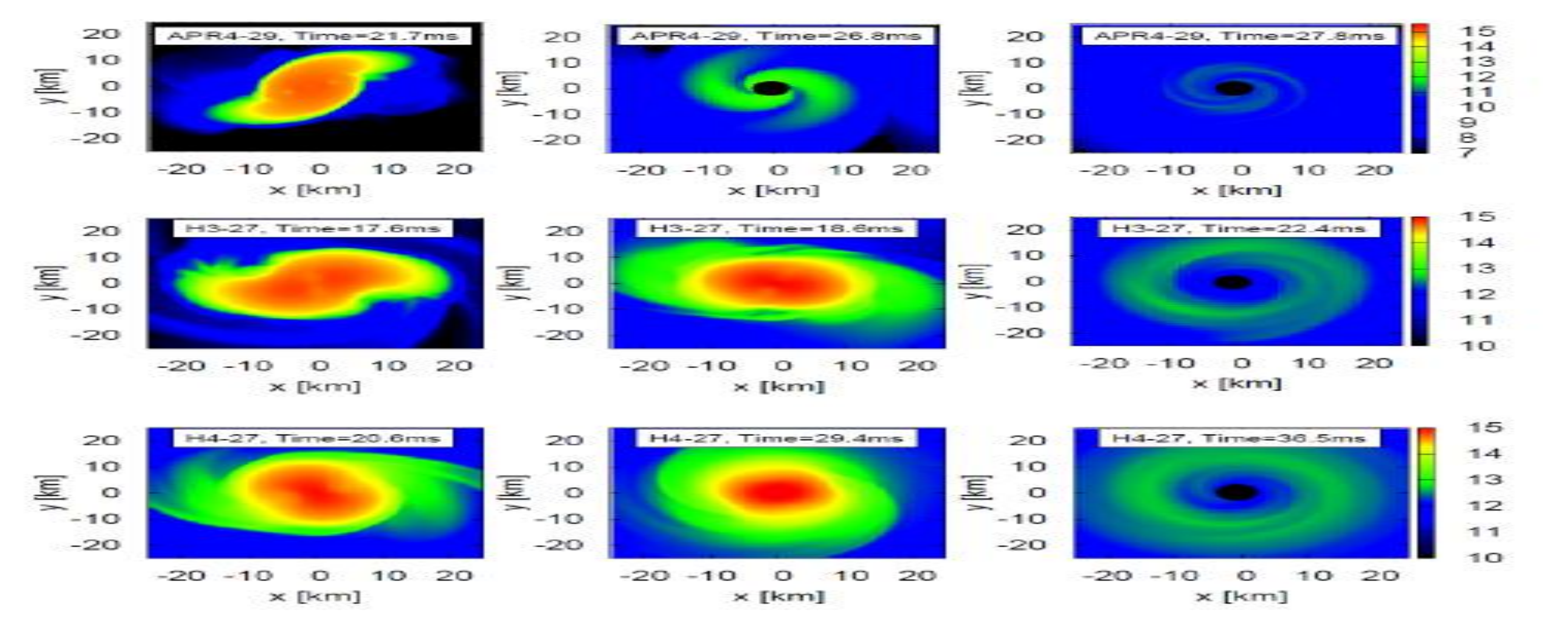
$$I = \frac{1}{5}M(a_1{}^2 + a_2{}^2 + a_3{}^2)$$

$$-2A_1 I_{11} + \Omega^2 I_{11} = -2A_3 I_{33}$$

$$\Omega^2 = 2e^2 B_{13}$$



Setup and initial conditions



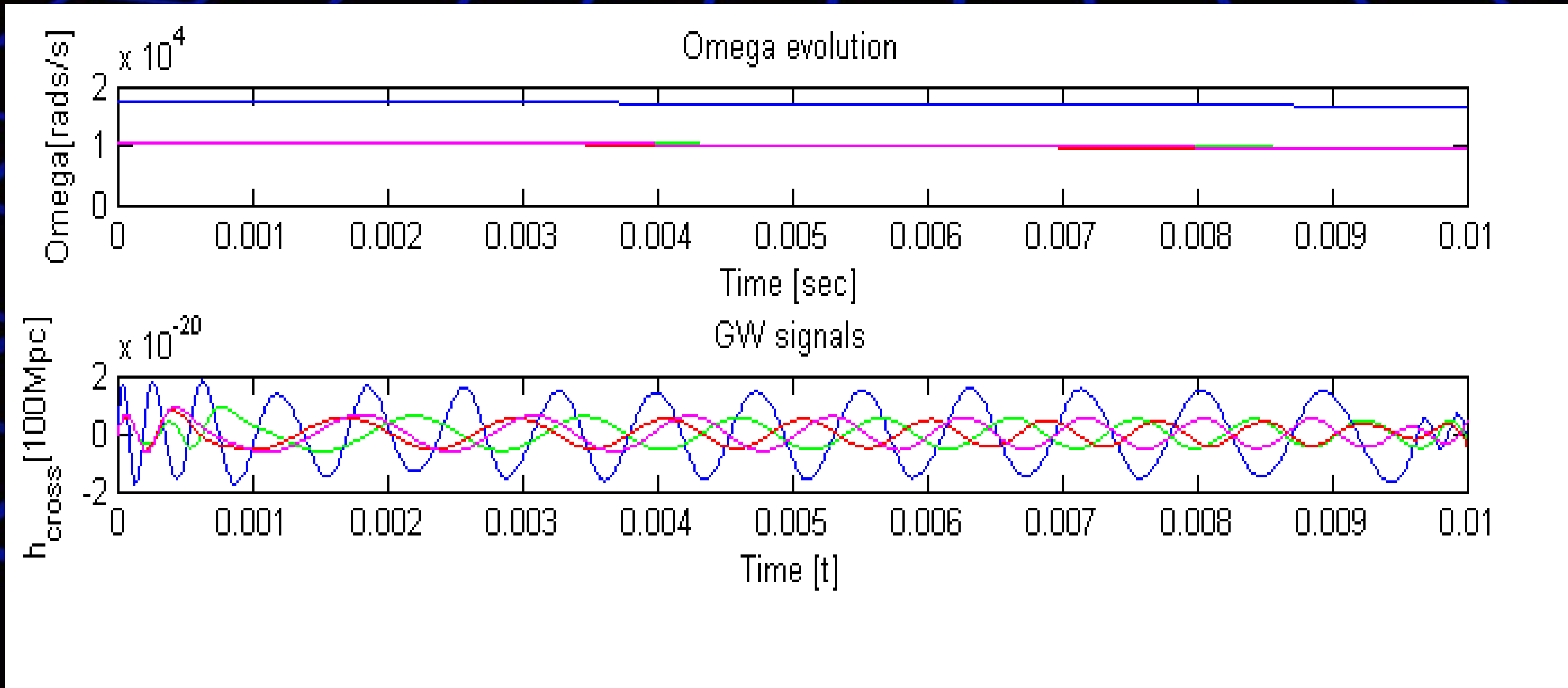
EOS	$\log P_1$ (dyne/cm^2)	Γ_1	Γ_2	Γ_3	$M_{max}(M_\odot)$	$R_{1.4}$(km)	Approach	composition
APR4	34.269	2.830	3.445	3.348	2.213	11.428	Variational-method	np
SLy	34.348	3.005	2.988	2.851	2.049	11.736	Effective-one-body potential	np
H3	34.646	2.787	1.951	1.901	1.788	13.840	Relativistic mean field	npH
H4	34.669	2.909	2.246	2.144	2.032	13.759	Relativistic mean field	npH
ALF2	34.055	4.070	2.411	1.890	2.086	13.188	APR+Quark matter	npQ
PS	34.671	2.216	1.640	2.365	1.755	15.472	Pion condensation	$n\pi^0$

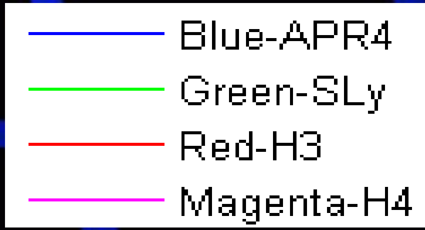
$e \sim 0.8$

$100 MPc$

Kenta Hotokezaka 2011

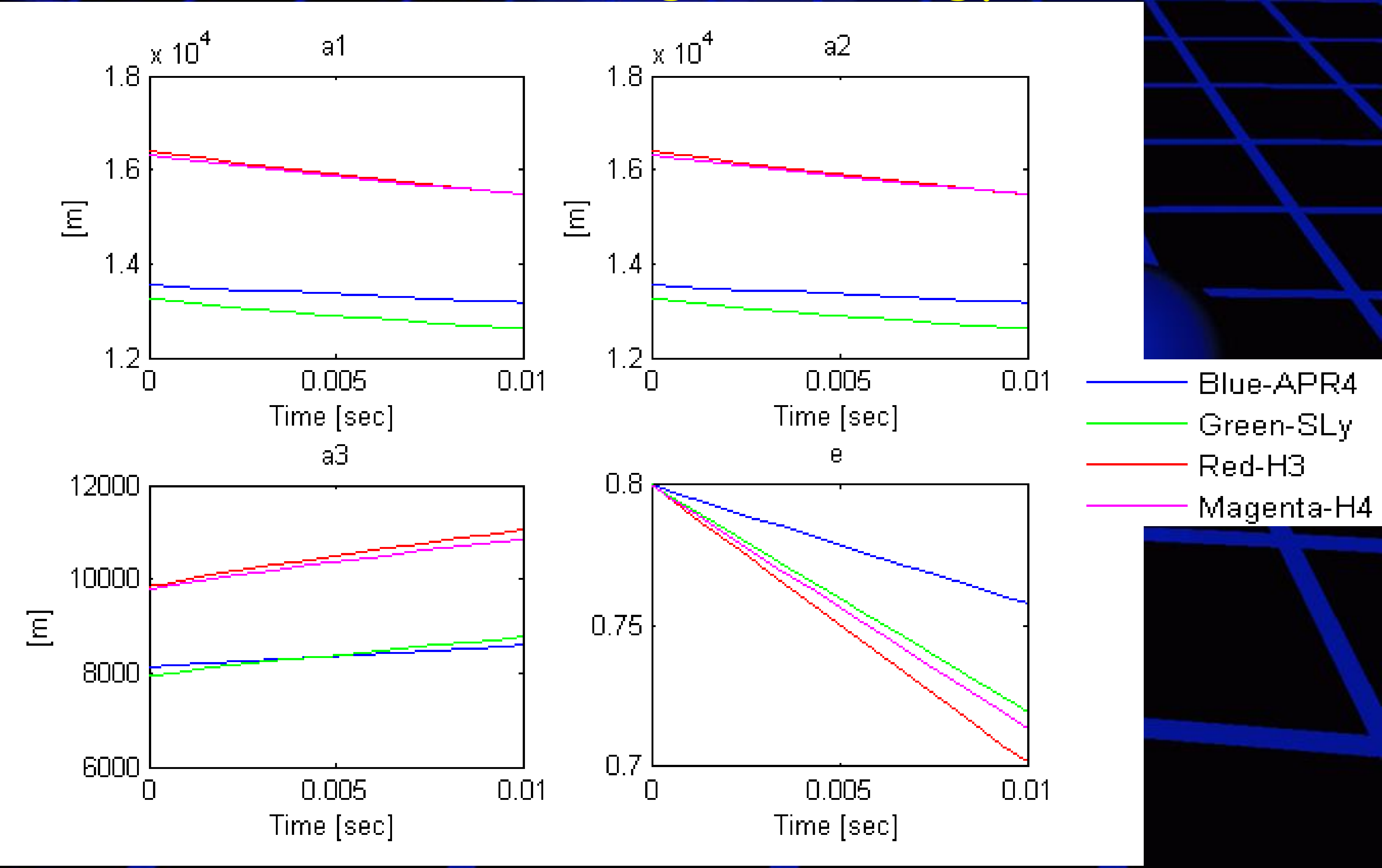
Looking at the first 10ms of such event...





APR4 EOS creates the most favourable merger object for GW

How the instability reacts to matter shedding and energy lost..



General results from the model

For Magnetars...

-At the right distances Magnetars can produce some observable LIGO GW signals.
-The Window of GW activity is a fraction of spin-down time.
-Spin-down induces a damping and decrease in frequency of GW signal.
-Model represents Realistic spin-down time scales(L.Stella 2005)
-The magnitude of the B-field determines detectability and spin-down rate, thus it determines the GW signature directly
-The initial frequency, size, and deformation influence the dynamic system.

For Binary Neutron stars..

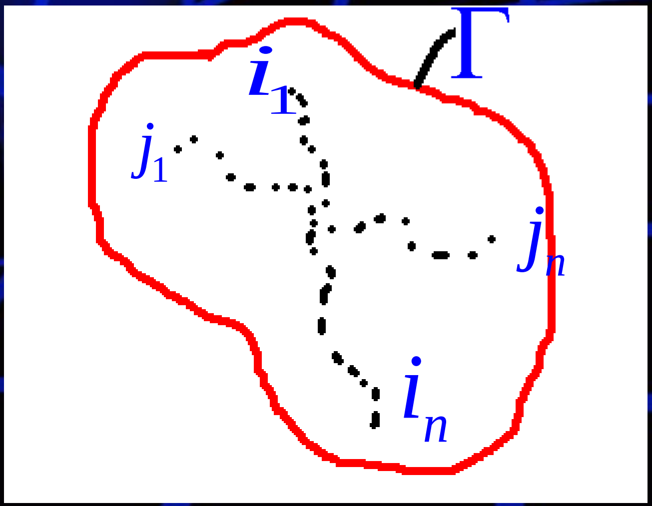
-Clearly the EOS plays a mayor role in the maximum strain produced by such an event, it allows the existence of unique objects (HMNS) and objects with unique size, rotation and life cycles, also depicted in Kenta Hotokezaka 2011
-There exist distinct GW signatures for each EOS, and the evolution of the instability evolve uniquely.

Initializing SPH simulations via GADGET 2.

-Running GADGET 2 on several nodes, visualizing with splash.

$$\rho_i = \sum_i m_i W_{ij}$$

$$W_{ij} = Kernel$$



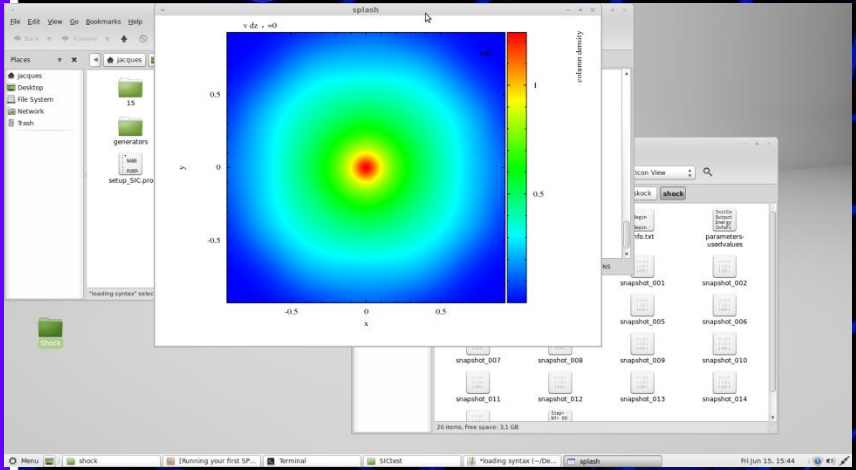
$$Dens(\Gamma) = \sum_i m_i W_{ij}$$

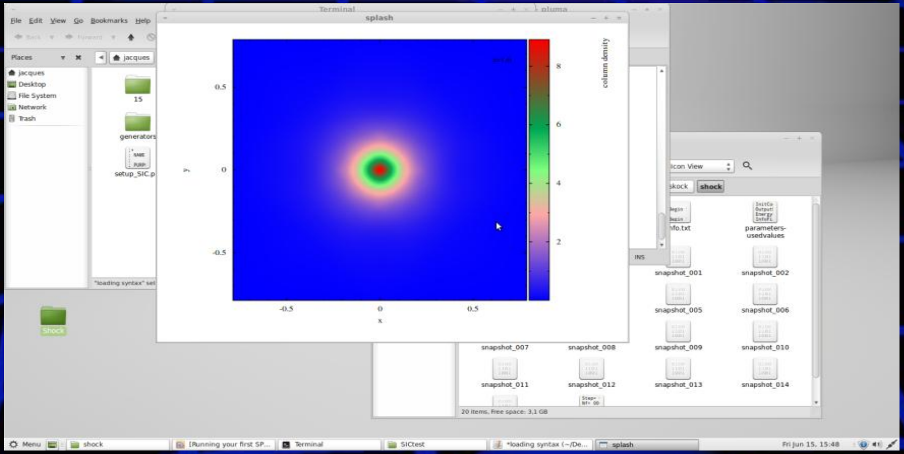
Aim: This method allows for extracting $\dot{E}_{GW}$ Without full relativistic simulations:

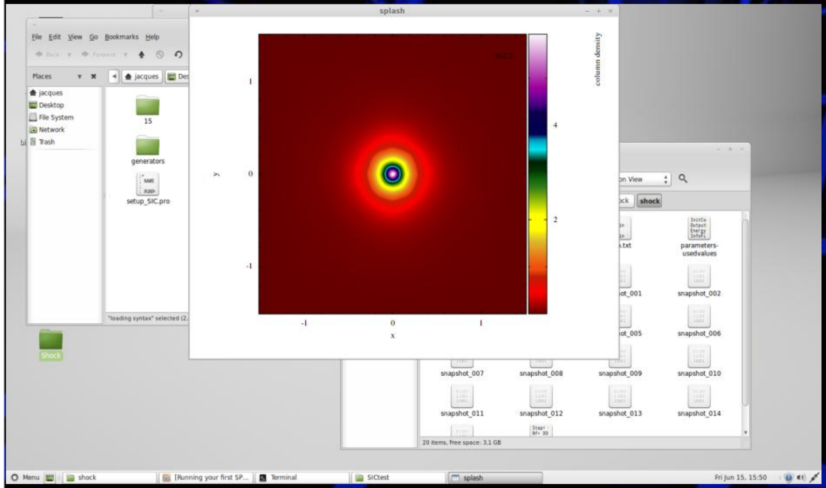
Using ....

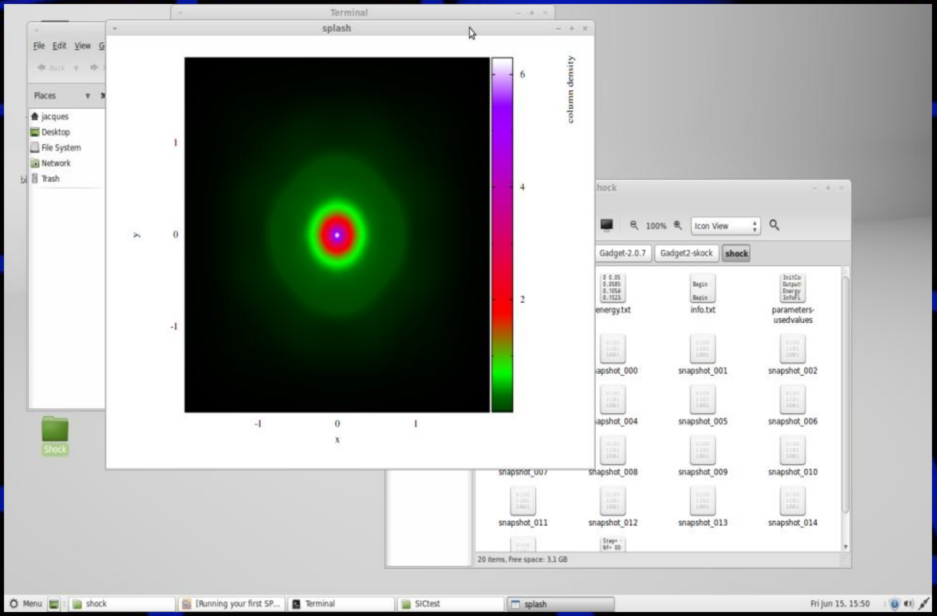
$$\frac{dE}{dt} = \frac{1}{5}\frac{G}{c^5}\left\langle \dddot{I}_{lm}\dddot{I}_{lm} \right\rangle$$

Gas sphere collapse with shock front









If the formulation of $\frac{dE}{dt} = \frac{1}{5}\frac{G}{c^5}\langle \dddot{I}_{lm}\dddot{I}_{lm} \rangle$ could be successfully implemented all these events could have a GW signal that describes the event.

Code test model

Thank you!!!

Albert Van Eck-Custer UFS
Jon Smit (Maths UfS)-optimization advice

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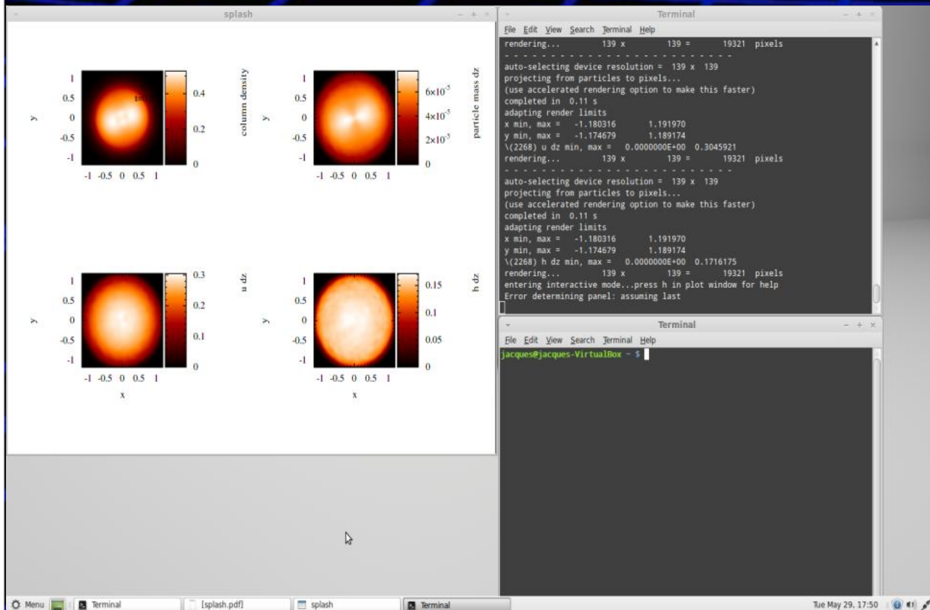
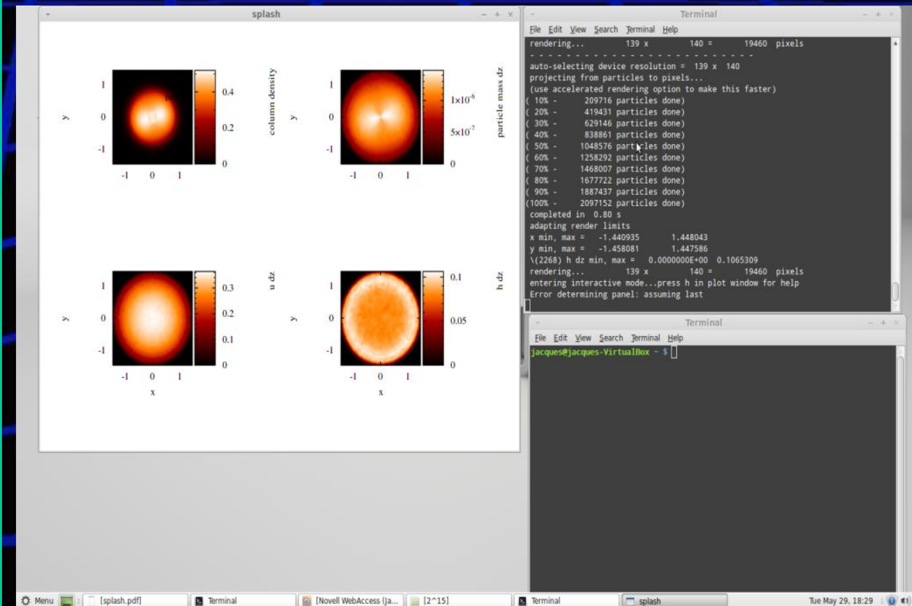
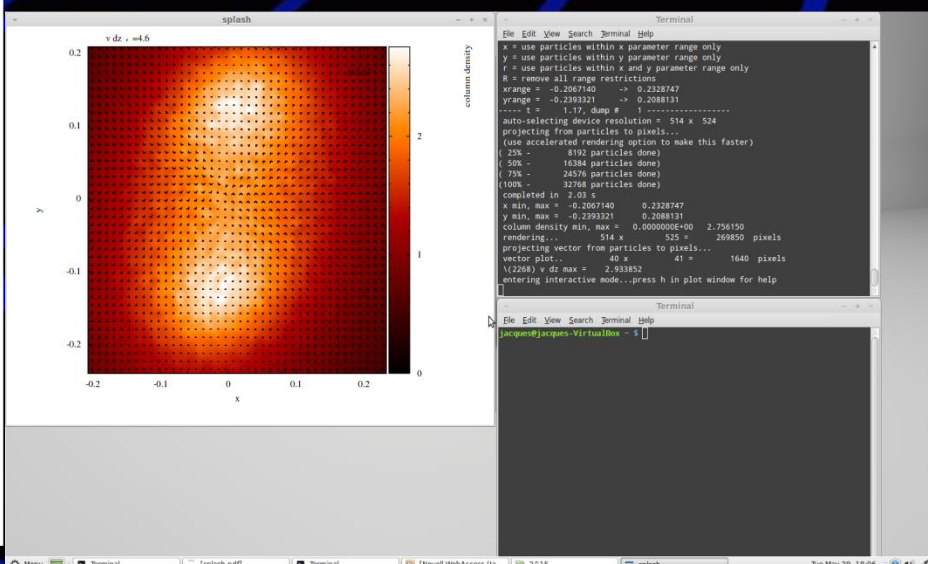
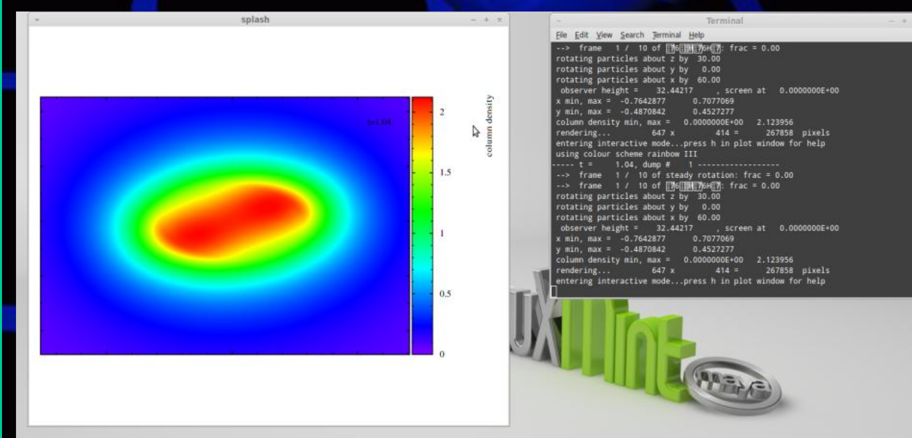
$$R = 5\times10^{16}\,cm, 1M_{\odot}, \Omega = 7.14\times10^{-13}\,s^{-1}, \rho = \rho_0(1+\alpha\cos(2\phi))$$

2^15 particles | **2^21 particles**





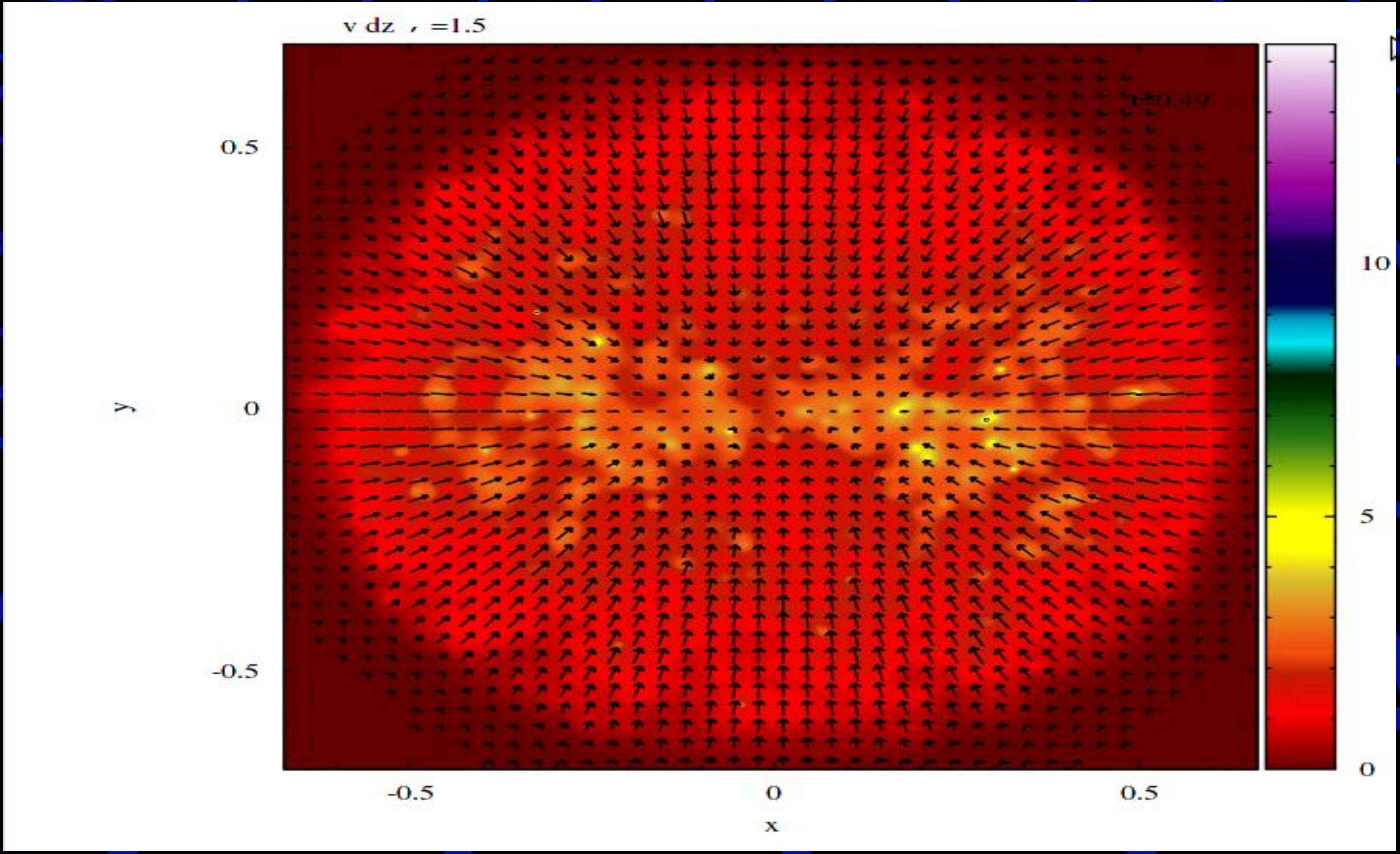




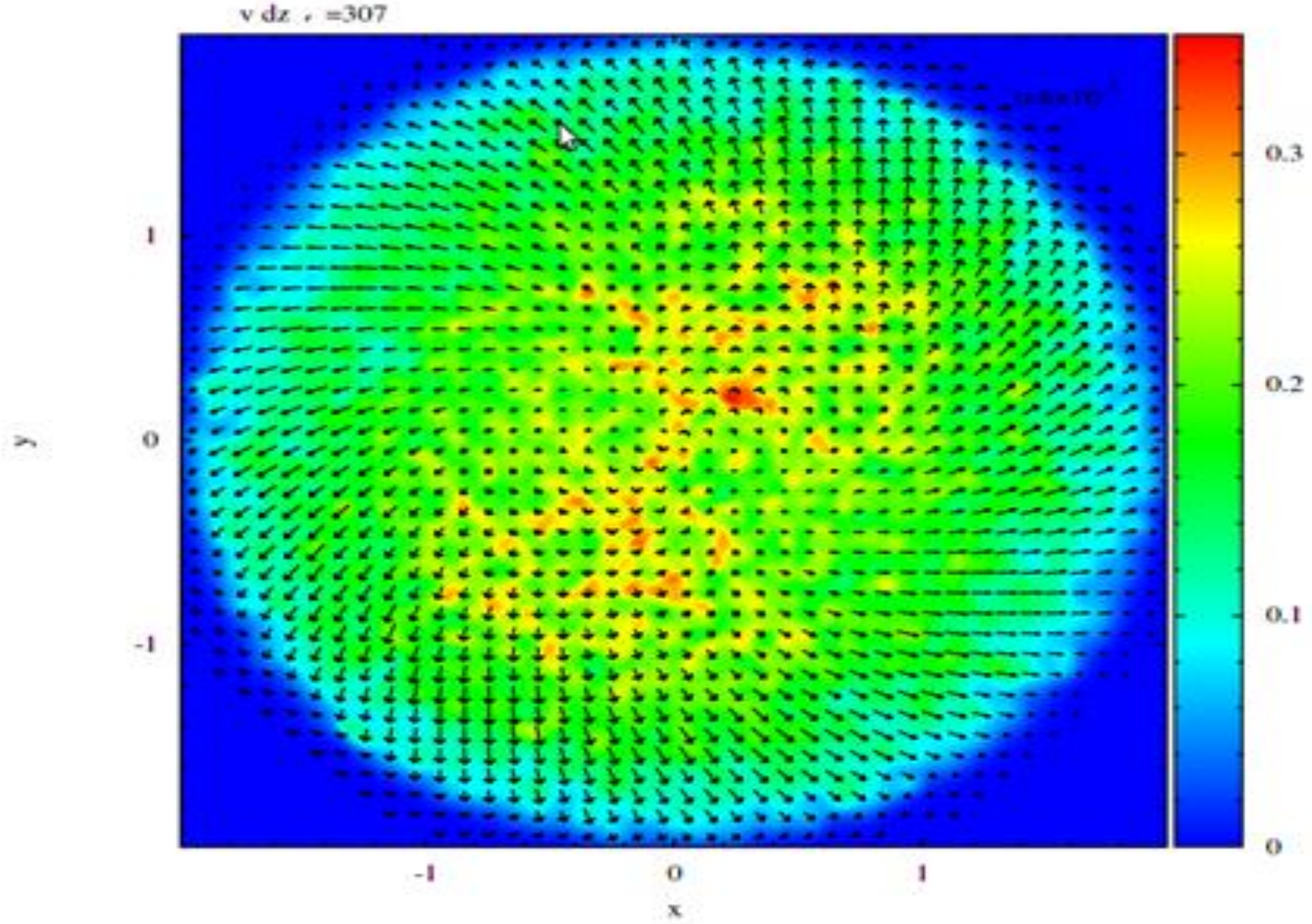
No equilibrium reached

Rotating neutron star like objects

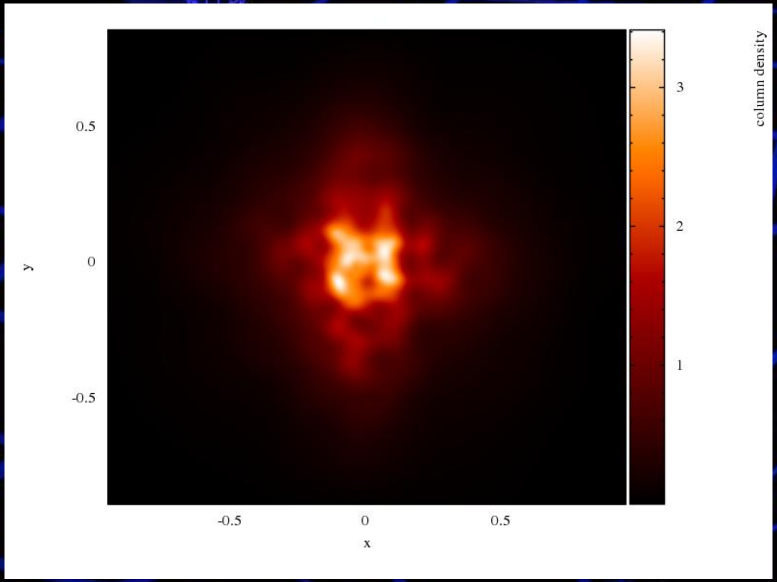
No Rotational instability,
invoking a collapse..



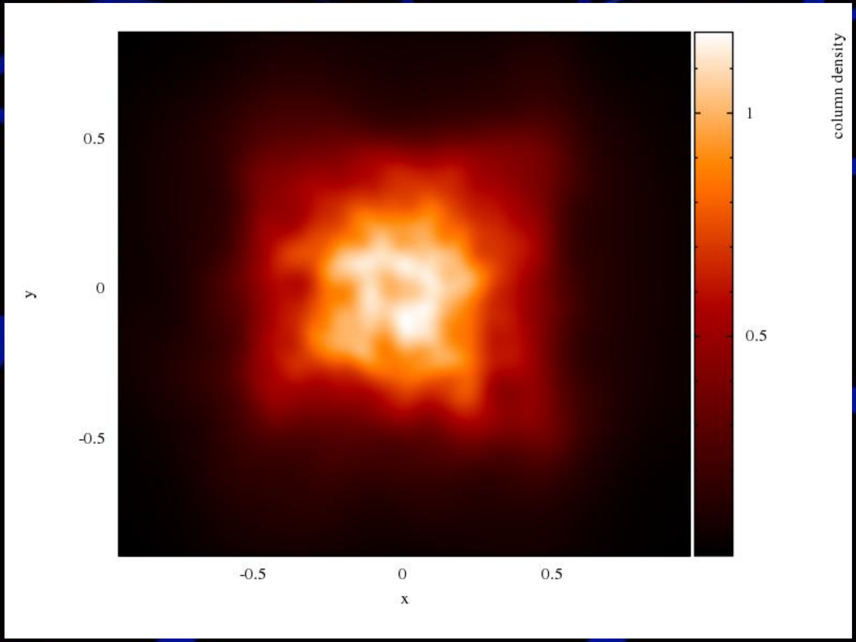
Rotational instability

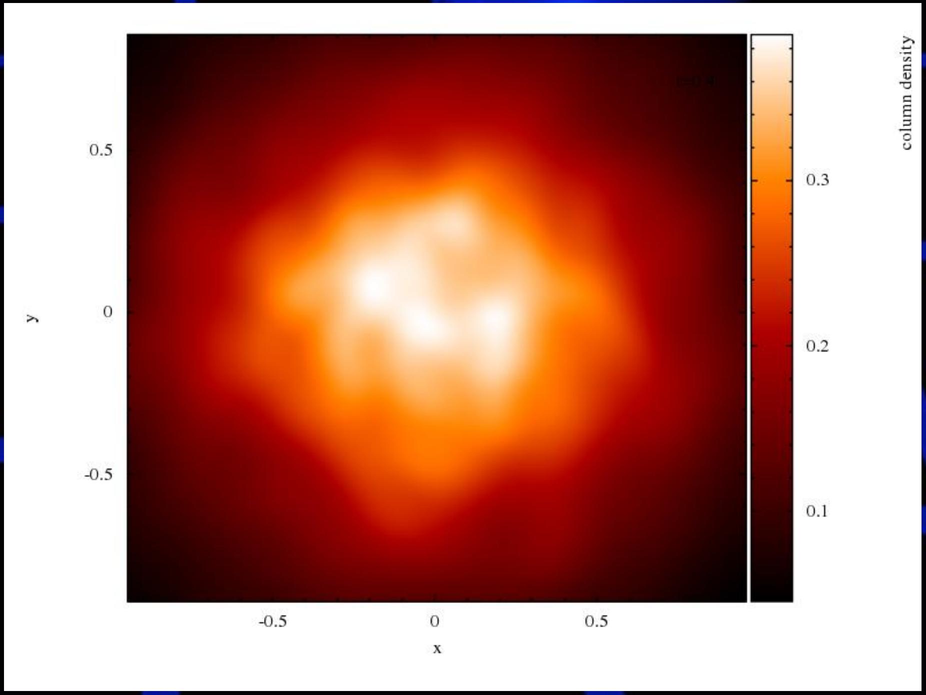


Primitive model of Particles in a Circle

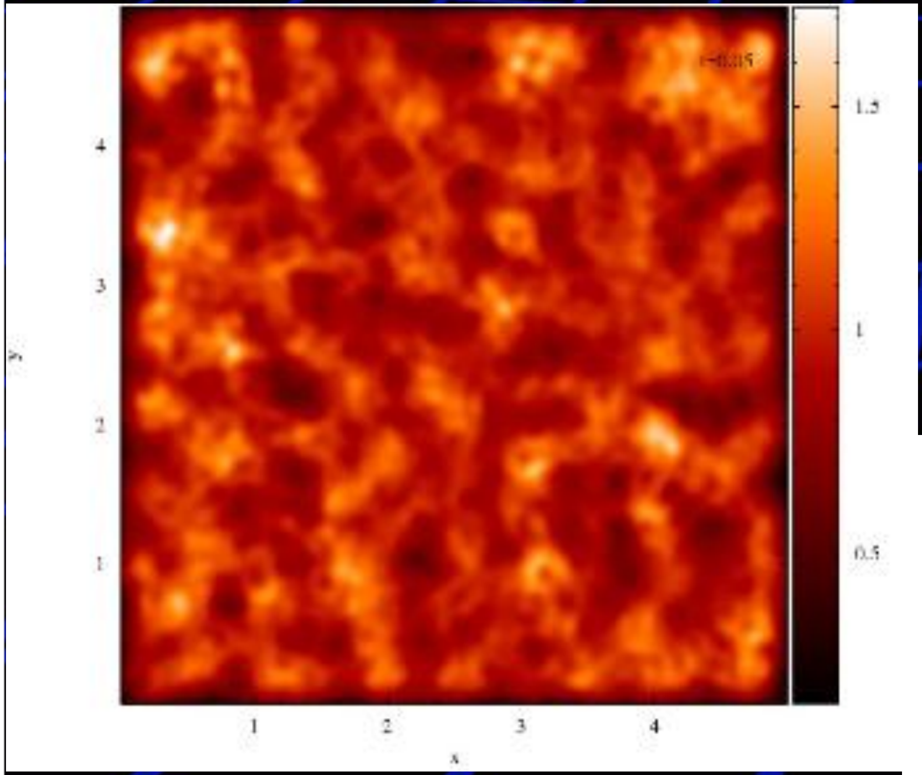


Using a unity-model
Pressure=1
Box size=1
Density=1

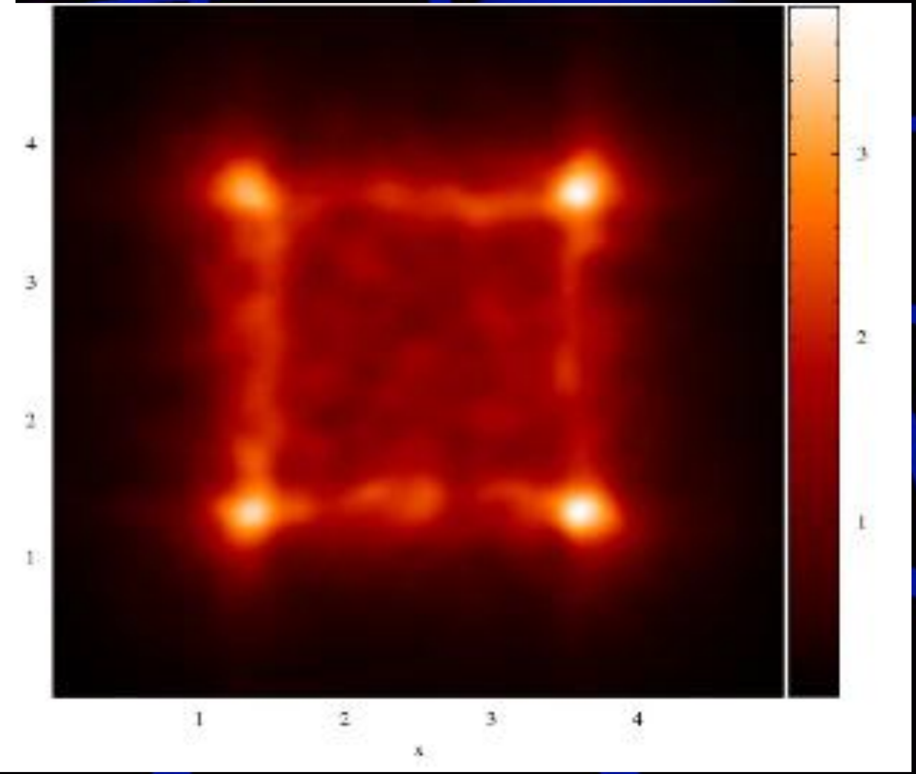




Primitive model of Particles in a Square



Using a unity-model
Pressure=1
Box size=1
Density=1



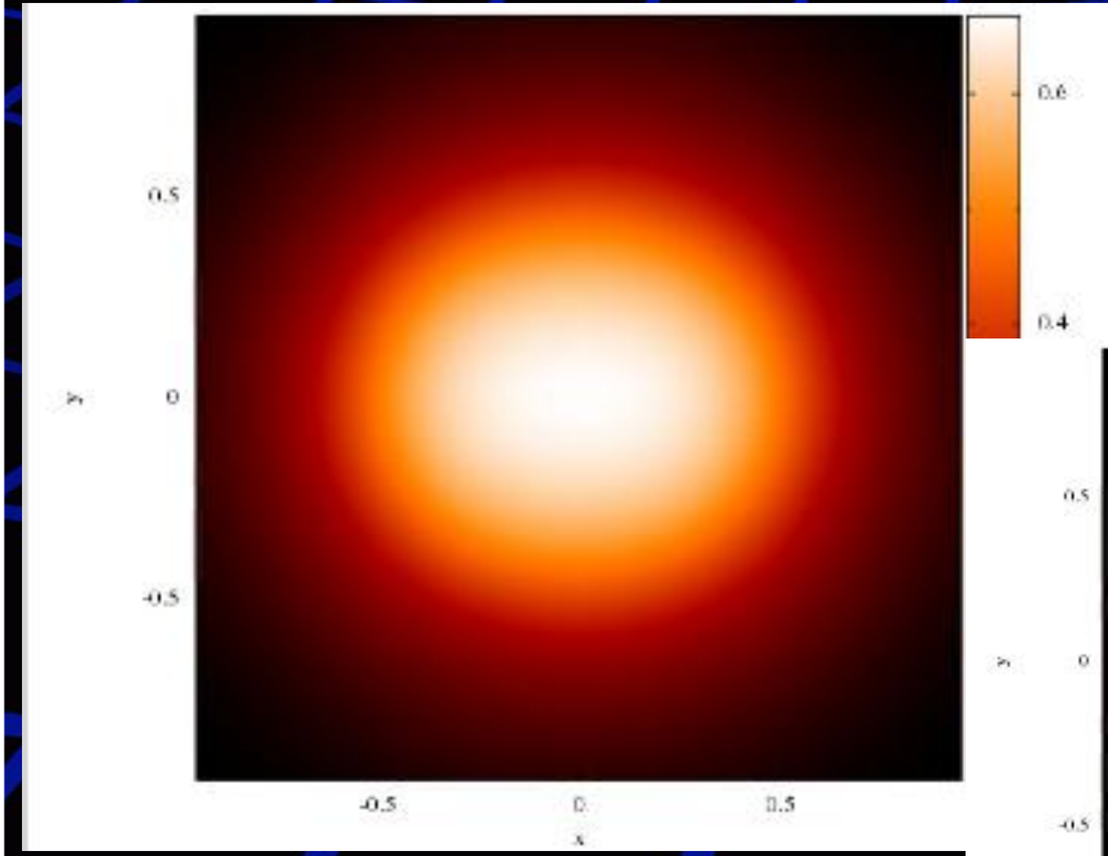
More Realistic Collapse without Pressure Mechanism enabled in GADGET 2

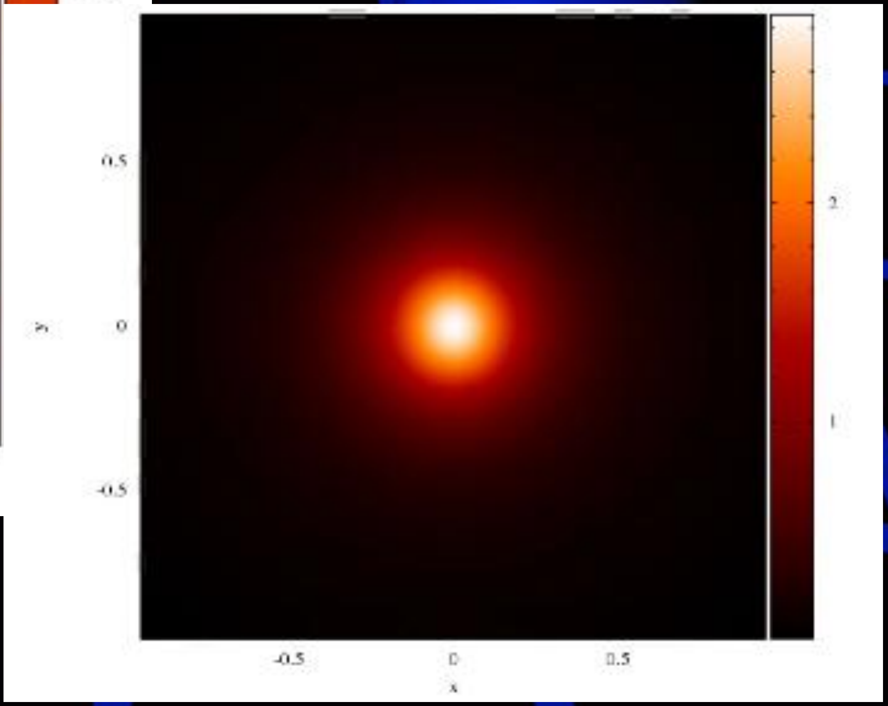
$$\Omega = 0$$

$$\rho_0 \sim 10^{14}\, g.cm^{-3}$$

$$\Gamma = \frac{5}{3}$$

$$r_{gas} \sim 10^{7}\, cm$$





Dense core-like collapse