Dissipative Dynamics of a Spinless Electron Strongly Interacting with an Environment of Spinless Electrons

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Abstract. We consider the dissipative dynamics of a spinless electron (fermion) strongly interacting with a finite bath of fermions. The fermionic environment is embedded in a bosonic Markovian bath. The master equation for the fermion interacting with the fermionic bath is derived. Based on the master equation for this system, the reduced dynamics and thermalization of the spinless electron is studied.

Understanding thermalization in complex quantum systems plays an important role in modern quantum statistical physics. The description of the mesoscopic system is important from both the experimental and the theoretical points of view. Recently, there has been a lot of interest in modeling steady state transport through quantum dots [1, 2, 3]. Typically, electrons in the quantum dot strongly interact with electrons in the surroundings (finite number degrees of freedom). In order to describe the interaction of the electron with the environment of electrons without restrictions on the strength of the electron-electron interactions we embed the whole system into a bosonic Markovian bath which will thermalize the subsystem of the electron interacting with the bath of electrons. In the present work we will not consider the spin-spin interaction between electrons, so that we will describe all electrons in this model as spinless fermions.

The studied system consists of a fermion interacting with a mesoscopic fermionic bath which is embedded into a Markovian bosonic bath. The total Hamiltonian of the system reads,

$$H = H_S + H_B + H_{SB}, \tag{1}$$

where H_S is the Hamiltonian of a fermion interacting with fermionic bath, i.e.,

$$H_S = \omega d^\dagger d + \sum_{i=1}^{N} \left(\epsilon c_i^\dagger c_i + g d c_i^\dagger + g d^\dagger c_i \right), \tag{2}$$

where $d^\dagger, d$ are creation and annihilation operators of the fermion of interest and $c_i^\dagger, c_i$ are creation and annihilation operators of fermions in the mesoscopic bath. All the operators $d^\dagger, c_i^\dagger$ satisfy standard anticommutation relations. The Hamiltonian of the bath H_B reads,

$$H_B = \sum_n \omega_n b_n^\dagger b_n, \tag{3}$$

where $b_n^\dagger, b_n$ are standard bosonic creation and annihilation operators. The Hamiltonian of interaction of the fermonic bath with the bosonic Markovian environment is denoted by H_{SB} and given by,

$$H_{SB} = \sum_{i=1}^{N} \sum_{n} g_n b_n c_i^\dagger + g_n^* b_n^\dagger c_i. \tag{4}$$

In order to derive the quantum master equation for the system it is convenient to diagonalize the Hamiltonian of the system H_S, as

$$H_S = \sum_{i=0}^{N} \lambda_i \xi_i^\dagger \xi_i, \tag{5}$$

where $\xi_i^\dagger, \xi_i$ are creation and annihilation operators of a new set of quasi-fermions satisfying standard anti-commutation relationships, i.e., $\{\xi_i, \xi_j^\dagger\}_+ = \delta_{i,j}$. The explicit expressions for $\xi_i^\dagger$ and λ_i read,

$$\xi_0^\dagger = \cos\theta d^\dagger + \frac{\sin\theta}{\sqrt{N}} \sum_{i=1}^{N} c_i^\dagger, \quad \lambda_0 = \frac{\omega+\epsilon}{2} + \frac{\Omega_N}{2}, \tag{6}$$

$$\xi_1^\dagger = -\sin\theta d^\dagger + \frac{\cos\theta}{\sqrt{N}} \sum_{i=1}^{N} c_i^\dagger, \quad \lambda_1 = \frac{\omega+\epsilon}{2} - \frac{\Omega_N}{2}, \tag{7}$$

and for $i = 2 \dots N$

$$\xi_i^\dagger = \frac{1}{\sqrt{i(i-1)}} \sum_{k=1}^{i-1} c_k^\dagger - \sqrt{\frac{i-1}{i}} c_i^\dagger, \quad \lambda_i = \epsilon, \tag{8}$$

where the coefficients $\Omega_N, \cos\theta, \sin\theta$ read,

$$\Omega_N = \sqrt{4g^2 N + (\epsilon-\omega)^2}, \quad \cos\theta = \frac{2g\sqrt{N}}{\sqrt{\Omega_N\left(\Omega_N + (\epsilon-\omega)\right)}}, \quad \sin\theta = \sqrt{\frac{\Omega_N + (\epsilon-\omega)}{2\Omega_N}}. \tag{9}$$

Using the explicit expression for the diagonalized Hamiltonian of the system, the quantum Markov equation can be obtained from the general expression [4],

$$\frac{d}{dt}\rho_S(t) = -\int_0^\infty d\tau \mathrm{Tr}_B[H_{SB}^{(I)}(t), [H_{SB}^{(I)}(t-\tau), \rho_S(t) \otimes \rho_B(0)]]. \tag{10}$$

By the direct substitution of the interaction Hamiltonian into Eq. (10) and assuming that $|\lambda_0 - \lambda_1| \gg 1$ (this holds for $N \gg 1$), we obtain the following quantum master equation,

$$\frac{d}{dt}\rho_S = \sum_{i=0}^{1} \gamma_i^+ \left(\xi_i \rho_S \xi_i^\dagger - \frac{1}{2}\{\xi_i^\dagger \xi_i, \rho_S\}_+\right) + \gamma_i^- \left(\xi_i^\dagger \rho_S \xi_i - \frac{1}{2}\{\xi_i \xi_i^\dagger, \rho_S\}_+\right), \tag{11}$$

where the damping rates $\gamma_i^\pm$ are given by,

$$\gamma_0^\pm = \pi N \sin^2\theta J(\lambda_0) \left(\coth\frac{\beta\lambda_0}{2} \pm 1\right), \tag{12}$$

$$\gamma_1^\pm = \pi N \cos^2\theta J(\lambda_1) \left(\coth\frac{\beta\lambda_1}{2} \pm 1\right), \tag{13}$$

where $J(\omega)$ is the spectral density and β is the inverse temperature of the bosonic Markovian bath.

The obtained quantum master equation can be solved exactly and its solution can be presented with help of the Kraus representation,

$$\rho_S(t) = \sum_{i=0}^{1}\sum_{k=1}^{4} E_k^i(t)\rho_S(0)E_k^{i\dagger}(t), \tag{14}$$

where the Kraus operators are given by

$$E_0^i(t) = \frac{\cos\alpha_i}{\sqrt{2}}\left(\xi_i^\dagger\xi_i + f_i(t)\xi_i\xi_i^\dagger\right), \tag{15}$$

$$E_1^i(t) = \frac{\cos\alpha_i}{\sqrt{2}} g_i(t)\xi_i^\dagger, \tag{16}$$

$$E_2^i(t) = \frac{\sin\alpha_i}{\sqrt{2}}\left(\xi_i\xi_i^\dagger + f_i^*(t)\xi_i^\dagger\xi_i\right), \tag{17}$$

$$E_3^i(t) = \frac{\sin\alpha_i}{\sqrt{2}} g_i(t)\xi_i, \tag{18}$$

and

$$\cos\alpha_i = \sqrt{\frac{\gamma_i^-}{\gamma_i^+ + \gamma_i^-}}, \quad \sin\alpha_i = \sqrt{\frac{\gamma_i^+}{\gamma_i^+ + \gamma_i^-}}, \tag{19}$$

$$f_i(t) = \exp\left(-\frac{\gamma_i^+ + \gamma_i^-}{2}t - i\lambda_i t\right), \quad g_i(t) = \sqrt{1 - |f_i(t)|^2}. \tag{20}$$

In the present work we will consider that initially there is a fermion of interest and no fermions in the mesoscopic bath, i.e.,

$$\rho_S(0) = d^\dagger|0\rangle\langle 0|d = \cos^2\theta\xi_0^\dagger|0\rangle\langle 0|\xi_0 + \sin^2\theta\xi_1^\dagger|0\rangle\langle 0|\xi_1 - \sin\theta\cos\theta\left(\xi_1^\dagger|0\rangle\langle 0|\xi_0 + \xi_0^\dagger|0\rangle\langle 0|\xi_1\right). \tag{21}$$

Using the explicit form of the Kraus operators and the initial conditions for the system, the density matrix can be obtained in the quasi-fermionic picture. After transformation to the original fermionic picture and tracing out the mesoscopic bath we obtain the explicit expression for the reduced dynamics of the electron of interest,

$$\rho_e(t) = \kappa(t)d^\dagger|0\rangle\langle 0|d + (1-\kappa(t))|0\rangle\langle 0|, \tag{22}$$

where

$$\kappa(t) = \cos^2\theta c_{00}(t) + \sin^2\theta c_{11}(t) + \sin^2\theta\cos^2\theta \mathrm{Re}\left(f_0(t) + f_1(t)\right) + w(t), \tag{23}$$

$$c_{00}(t) = \frac{\cos^2\theta}{2}\left(\cos^2\alpha_0 + \cos^2\alpha_1|f_1(t)|^2 + \sin^2\alpha_0|f_0(t)|^2 + \sin^2\alpha_1\right), \tag{24}$$

$$c_{11}(t) = \frac{\cos^2\theta}{2}\left(\cos^2\alpha_1 + \cos^2\alpha_0|f_0(t)|^2 + \sin^2\alpha_1|f_1(t)|^2 + \sin^2\alpha_0\right), \tag{25}$$

$$w(t) = \frac{1}{2}\left(\cos^2\alpha_0\sin^2\theta g_0^2(t) + \cos^2\alpha_1\cos^2\theta g_1^2(t)\right). \tag{26}$$

Based on the exact expression for the reduced density matrix we calculate the mean number of fermions, and it is clear that $\langle d^\dagger d\rangle = \kappa(t)$.

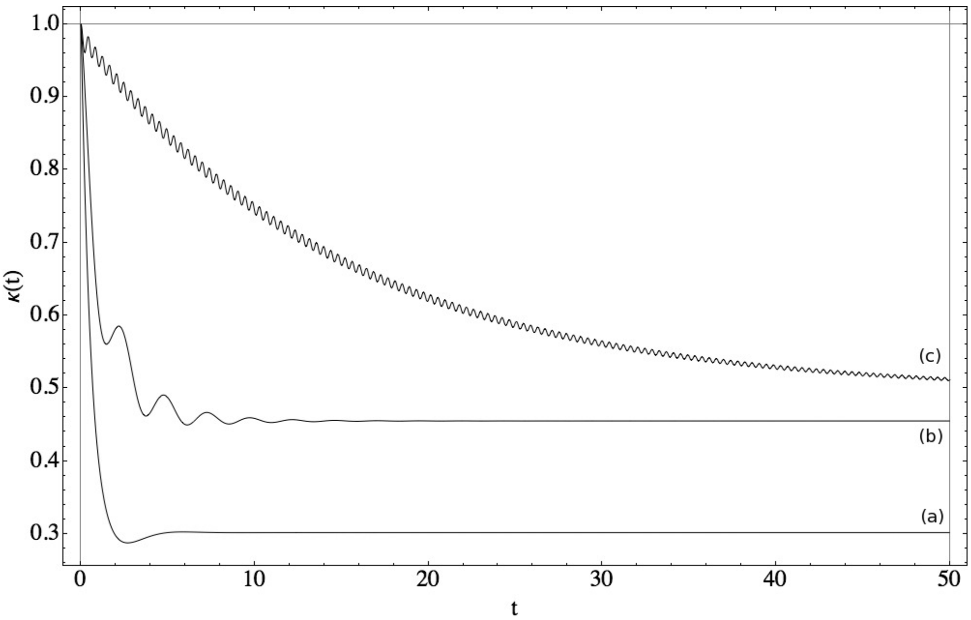

Figure 1. Time dependance of the mean number of fermions as a function of coupling strength to the mesoscopic bath. Curves (a), (b) and (c) correspond to 0.01, 0.1 and 1 values of the coupling strength g, respectively. The rest of the parameters are chosen to be the same for all three curves: $\epsilon = 1$, $\omega = 1.3$, $N = 200$, $J(\lambda_0)$=$J(\lambda_1)$=0.01 and $\beta = 10$.

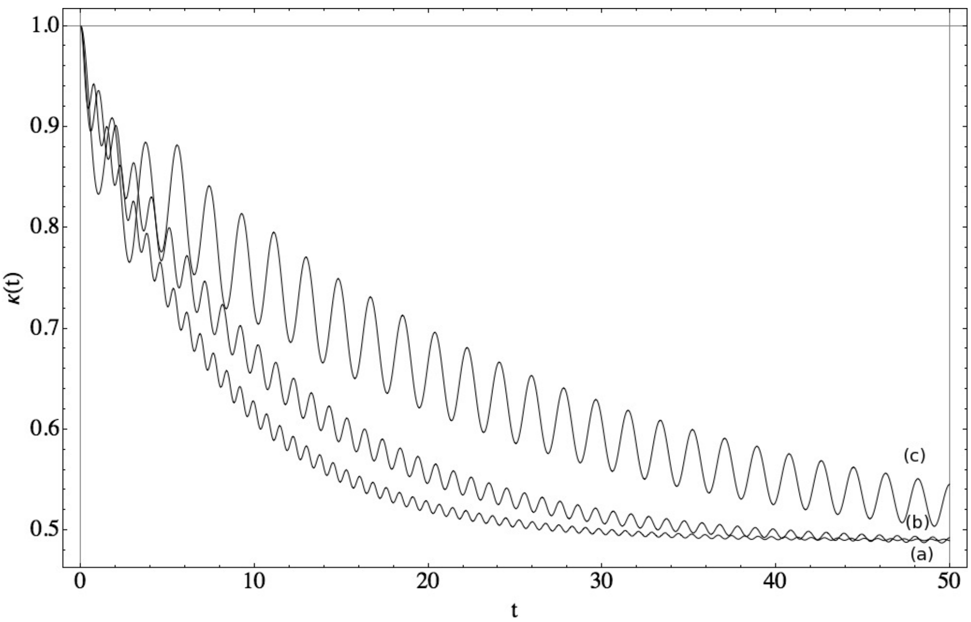

Figure 2. Time dependance of the mean number of the fermions as a function of the number of fermions in the mesoscopic bath. Curves (a), (b) and (c) correspond to 200, 100 and 20 number of fermions in the mesoscopic bath (N), respectively. The rest of the parameters are chosen to be the same for all three curves: $\epsilon = 1$, $\omega = 1.3$, $g = 0.5$, $J(\lambda_0)$=$J(\lambda_1)$=0.01 and $\beta = 10$.

In Fig. 1 and Fig. 2 the dynamics of the mean number is shown. In Fig. 1 we analyze different regimes of the interaction between fermion and meso-reservoir of fermions. It is clear that in the weak coupling case (Fig. 1a) Markovian dissipation is observed, however, increasing the interaction strength (g) (Fig. 1b, Fig. 1c), the process of thermalization shows clear signs of non-Markovian behaviour. In Fig. 2 we analyze the influence of the number of fermions in the mesoscopic bath on the dynamics of the fermion. It is clear from the Fig. 2 that decreasing the number of fermions strongly influences the frequency of oscillations ($\sim \sqrt{N}$).

In conclusion, we derive and solve analytically the quantum master equation for the spinless electron interacting with a mesosopic bath of spinless electrons with restrictions on the system-bath interaction. We further analyze the dynamics of the reduced system. In the future we plan to take into account spin-spin interactions and consider more general initial conditions.

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