Core flow Inversions from the Earth's Magnetic Field.

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Abstract

The Earth's core flow at its boundary with the mantle is explored by inverting the magnetic induction equation with respect to the International Geomagnetic Reference Field models, measured in nano-Teslas, and its secular variation, in nano-Teslas per year, for the period 1965 to 2010 divided in five-year intervals. The South Atlantic anomaly and geomagnetic jerks were also investigated. To reduce the solution's non-uniqueness, the Frozen-flux approximation and flow tangential geostrophy is assumed. Further a priori information was added to damp the solution. Both the magnetic field, with its secular variation, and the velocity field's potentials are expanded in terms of spherical harmonics. Inversion proceeds by a linear least-squares method with Choleski decomposition on the resulting Elsasser and Gaunt matrices. Results indicate flow patterns, in kilometers per year, that includes a vortex near the South Atlantic anomaly that stayed relatively stable through this 45-year period. It was also found that accelerations, in kilometers per squared year, are more varied between each five-year interval.

1. Introduction

Since it was determined that the Earth has a magnetic field surrounding it [1], studies were conducted to investigate the origin of this field. Since medieval times explorers also noticed a long-term change in compass bearings at the same location; , a westward drift implying a change in the magnetic field over years and decades. This drift called the field's secular variation. Larmor [2] postulated that a self-sustaining dynamo deep within the Earth's core drove this geomagnetic field. Seismic investigations determined that the core contains a liquid outer core consisting mainly of iron and an inner core of solid iron. Elsasser [3] concluded the geomagnetic field and its variation with time must be generated by currents of fluid flows of iron at the outer core's mantle boundary (CMB). The mantle makes no significant contribution to the magnetic field and the only other sources of interference with the magnetic signals are external: the Earth's crust, the ionosphere surrounding the Earth, the solar stream of charged particles coming from the sun (solar wind), and the solar magnetic field interacting with Earth's field.

2. Theory

The theory governing the dynamics of this flow is called Magnetohydrodynamics (MHD) with the two main differential equations being the Navier-Stokes (NS) equation $\dot{\vec{u}} + (\vec{u}\cdot\nabla)\vec{u} + 2\Omega \times \vec{u} - \nu\nabla^2\vec{u} = \left[\left(\vec{J}\times\vec{B}\right) + \rho' g_e - \nabla p\right]/\rho$ where $\upsilon = \mu/\rho$ and μ are the kinematic & dynamic viscosity and ρ = mass density, Ω = Earth's rotation, p = pressure, g_e= gravity, and $\mu_0\vec{J} = \left(\nabla\times\vec{B}\right)$ = current density; and the magnetic induction (MI) equation $\dot{\vec{B}} = \nabla\times\left(\vec{u}\times\vec{B}\right) + \eta\nabla^2\vec{B}$, with $\nabla\cdot\vec{B} = 0$, and where $\eta = (\mu_0\sigma)^{-1}$ is magnetic diffusion, σ is the electric conductivity [4]. Due to the non-linear nature of the NS equation and the difficulty of obtaining its solutions, focus was mainly concentrated on the MI equation and its solution, which could assist in the first equation.

Backus [5] determined that the solution to the MI equation was non-unique. To reduce this in the search for a solution Roberts & Scott [6] already introduced the Frozen-flux approximation in which the diffusion term were set to zero, thus requiring the currents within the core to be perfectly conducting. This, however, was not enough to eliminate non-uniqueness and further approximations were applied on the core flow. These include the Steady state [7], [8], [9]; Toroidal [10], [11]; Geostrophic [tangential] [12], [13], [14]; [quasi-] [15], [16]; and Helical [17] flow assumptions. Geostrophic flows were adopted here where the horizontal Coriolis force balances the pressure gradients on the core-mantle boundary: $(2\Omega \times \vec{u})_h = -(\nabla_h p)/\rho$.

3. Methodology

In this study, in addition to the magnetic field $\vec{B}(t,\vec{r})$ and its change with time (secular variation) $\partial_t\vec{B}(t,\vec{r})$, the velocities were expanded by spherical harmonics, $\vec{u} = \vec{u}_p + \vec{u}_t$:

$$\begin{matrix} \vec{u}_p = \nabla_h(rS) \\ \vec{u}_t = \nabla_h \times (\vec{r}T) \end{matrix} \qquad ; \qquad \begin{pmatrix} S(\theta,\phi) \\ T(\theta,\phi) \end{pmatrix} = \sum_{n=1}^{\infty}\sum_{m=0}^{n}\begin{pmatrix} s_n^m \\ t_n^m \end{pmatrix} Y_n^m(\theta,\phi)$$

where (s_n^m, t_n^m) is the velocity coefficients, governing the poloidal and toroidal potentials $S(\theta,\phi)$ and $T(\theta,\phi)$, and in turn the velocities $\vec{u}_p$ and $\vec{u}_t$ respectively. $Y_n^m(\theta,\phi)$ are the spherical harmonics.

The radial MI equation, with no diffusion, is then inverted. Inversion for flow velocity solutions proceeds with a linear least-square method with normal damping and Cholesky decomposition performed on normal symmetric matrices formed from a system of equations involving the Elsasser E and Gaunt G matrices. The latter matrices results from the above scheme of harmonic expansions. Here $\partial_t\vec{B}(t,\vec{r}) = \vec{b}(t,\vec{r}) = A\vec{m} = E\vec{t} + G\vec{s}$, with model $\vec{m} = (s_n^m, t_n^m)$, is solved via an objective function

$$F(\vec{m}) = \left(\vec{b} - A\vec{m}\right)^T {C_{SV}}^{-1}\left(\vec{b} - A\vec{m}\right) + \lambda\vec{m}^T {C_v}^{-1}\vec{m}$$

where ${C_{SV}}^{-1}$ and ${C_v}^{-1}$ are the covariance matrices of geomagnetic change and velocity respectively, and λ are the damping coefficient in the regularisation term.

The International Geomagnetic Reference Frame (IGRF) model was used to supply the Gaussian coefficient values up to the 13th degree needed, not only to calculate the magnetic field and its secular variation, but also to invert the MI equation for the flow Gaussian coefficients. Secular variation coefficients were calculated as the difference of the field coefficients at two different epochs (years) divided by the time-units (year difference) between the epochs. The magnetic field is measured in nano-Tesla (nT) units. This forms the basis for the Finite-difference method of numerically determining a derivative of a function. In the IGRF-model, coefficients were provided only for 5-year periods beginning in the year 1900 and ending in 2005. Years after 2005 are coefficient estimates only. Secular variation data are determined from one 5-year period to the next and is the same for an epoch within each period. The field for an epoch within this period is determined linearly between the 5-year epochs at its ends.

4. Results

Velocities and their accelerations (see the plots), in units of kilometres per year and kilometres per squared year respectively, over five-year periods from 1965 to 2010 were plotted for the whole globe with the Hammer map projection. Results show smooth flow patterns that changed gradually over the 45-year period. Seven vortices can be discerned. From 1965 to 2010 one south of Africa remained strong and stable, one under Siberia moved to the North Pole, the North Pacific one remained strong and moved south, the South Pacific one became stronger, the New Zeeland one moved from east to west of it, a new one formed under Sudan/Ethiopia in Africa, and another one formed in the Central Atlantic Ocean . The greatest speed was found under the central Pacific. This maximum appeared to double in magnitude during this period. Vortices were identified where velocities were low or zero (blue spots). It appears in the larger blue regions that there might be smaller more numerous vortices not easily resolved under the current numerical surface grid density. Under Hermanus the magnitude increased from 1965 to 1975, then decreased until 1990 before climbing again.

The flow accelerations show much more variability and this are attributed to different secular variation conditions to the magnetic field for each 5-year period as mentioned above. When a uniform mass distribution is assumed over the CMB, this field would be proportional to the distribution of forces over the surface. Under Hermanus (see table below) from 1970 to 1990, the force direction changed from westerly through north to easterly. It then suddenly made an almost 180 degree turn to easterly again in 1995.

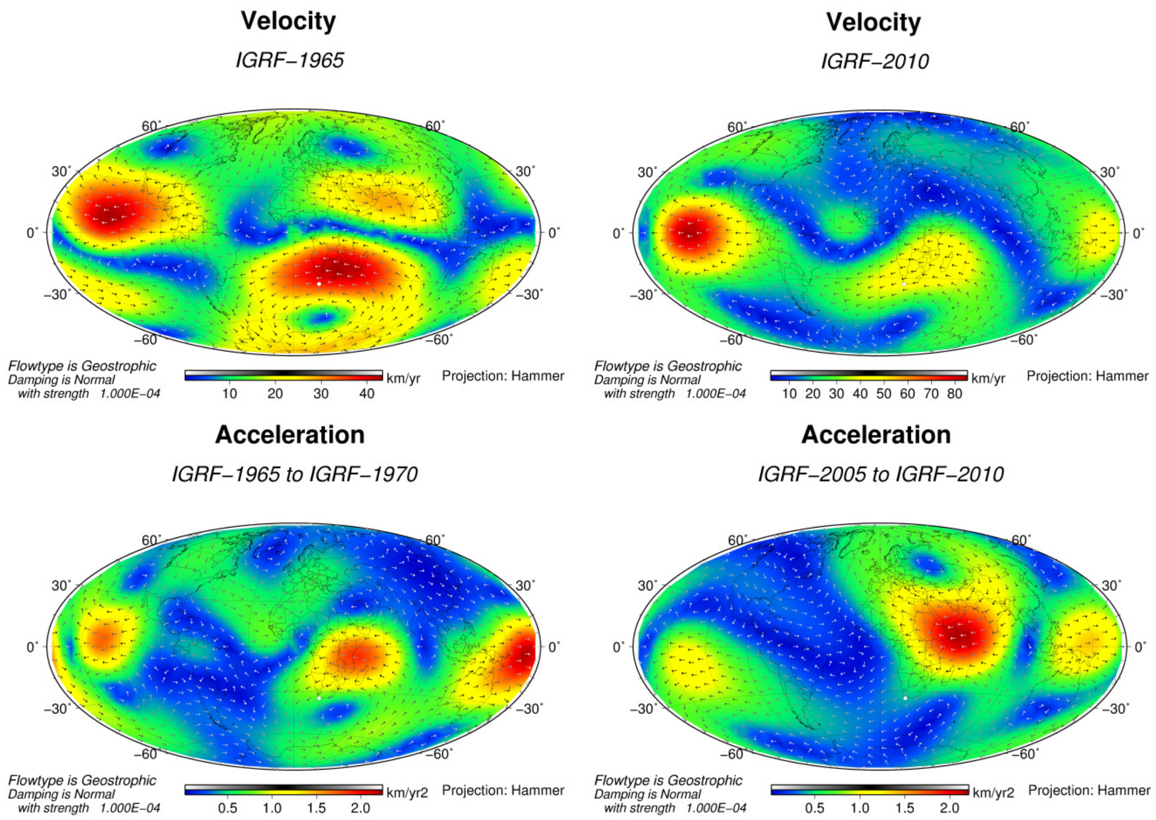

Table: *Hermanus (19.225°E, 34.425°S)*

IGRF	Velocity (km/year)	Bearing (° E from N)	Acceleration (km/year^2)	Bearing (° E from N)
1965	38.140	260.222		
1970	41.154	257.673	**0.698**	228.610
1975	41.403	258.591	***0.141***	327.479
1980	*39.934*	260.267	0.379	40.523
1985	*35.524*	259.794	**0.885**	84.083
1990	*33.362*	259.237	0.436	**88.292**
1995	35.746	258.595	0.482	**249.655**
2000	37.613	258.304	0.377	252.602
2005	39.163	255.720	0.464	208.839
2010	40.942	254.069	0.423	221.854

5. Conclusions

As indicated in the table the emboldened and font typed values, of trends at Hermanus, could be linked to local magnetic jerks under South Africa, first in 1981 and then 1993.

Possibilities for further studies may include toroidal and other flow types, inclusion of magnetic diffusion, changing to a better geomagnetic model, using live satellite magnetic data and tweaking the underlying theory.

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