dSECS

Including Variometers in Geomagnetic Field Interpolation

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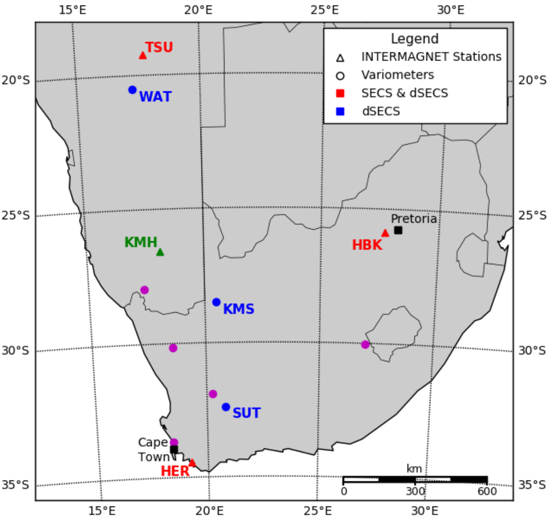
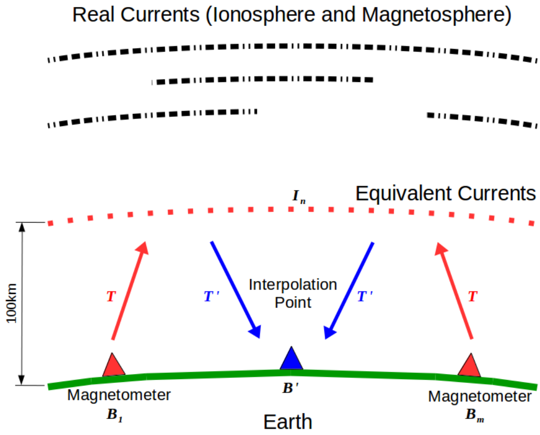
Variometers and Magnetometers



SECS - Spherical Elementary Current Systems

SECS is one of the most physically realistic magnetic field interpolation schemes - it tries to replicate both the internal (induction) and external (ionospheric and magnetospheric) currents in terms of equivalent currents.



SECS - Algorithm Flow

1 - Set up grid of elementary current poles (say n poles). These poles are collected into a vector I.

2 - Use measured magnetic field data (say m stations) to constrain elementary currents. Usually $m < n$. These stations are collected into a vector B.

3 - Set up transfer function matrix T, such that $B = T \cdot I$. The dimensions of T is $m \times n$

4 - Calculate components of I by using SVD to get T^{-1}, i.e. $I = T^{-1} \cdot B$.

5 - Use the elementary current grid to interpolate magnetic field B' to any other point of of interest, i.e. $B' = T' \cdot I$.

The internal component of the measured geomagnetic field needs to be subtracted!

SECS - Planar Approximation

Given a grid with n current elements and assuming m magnetometers, the following matrix equation can be set up for the calculation,

$$\begin{bmatrix} B_{x,y:1} \\ \vdots \\ B_{x,y:m} \end{bmatrix} = \begin{bmatrix} T_{x,y:11} & \cdots & T_{x,y:1n} \\ \vdots & \ddots & \vdots \\ T_{x,y:m1} & \cdots & T_{x,y:mn} \end{bmatrix} \begin{bmatrix} I_1 \\ \vdots \\ I_n \end{bmatrix}$$

where,

$$T_{x,y:ij} = \frac{\mu_0}{4\pi r}\left(1 - \frac{h}{\sqrt{r_{ij}^2 + h^2}}\right).$$

There are a number of factors affecting this planar approximation including an small angle assumption and the resolution of stations.

dSECS - Interpolating dB

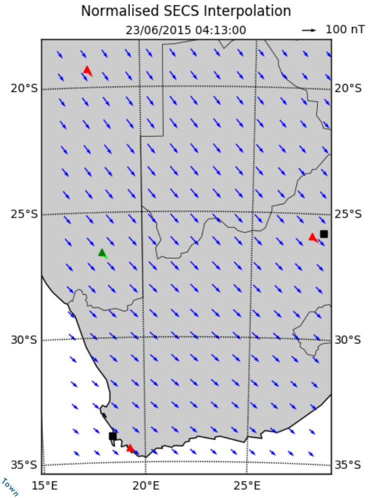
Although variometers are not absolute, they can very accurately measure the change in the magnetic field, i.e. $\Delta B = B(t_i) - B(t_{i-1})$.

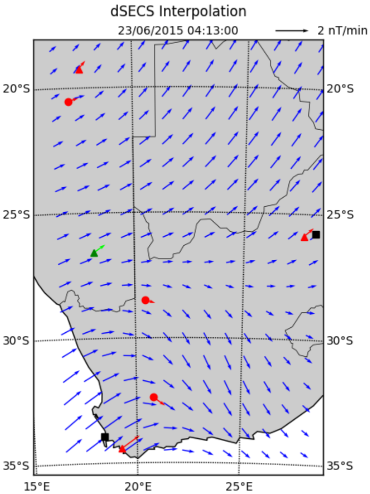
Since the SECS method is entirely linear, ΔB can be used in the same way as B (T is purely a spatial constant). The only difference is that I becomes ΔI in this scheme.

$$\begin{aligned} B(t_i) - B(t_{i-1}) &= T \cdot I(t_i) - T \cdot I(t_{i-1}) \\ &= T \cdot (I(t_i) - I(t_{i-1})) \\ \Delta B &= T \cdot \Delta I \end{aligned}$$

With more variometers than absolute magnetometers, the confidence in ΔB interpolation is much higher than that for B. ΔB is also what is typically used for GIC studies.

Merging SECS and dSECS





Merging SECS and dSECS

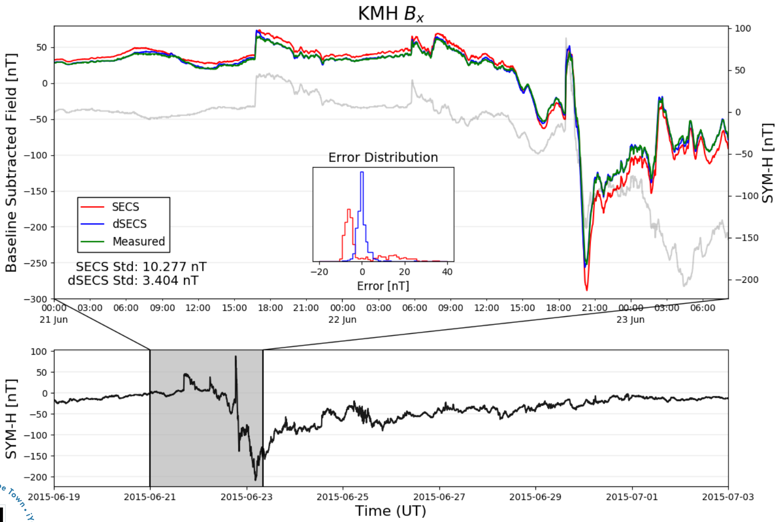
We can now use the greater amount of information about ΔB to improve the interpolation of B. Given a set of perturbations ϵ, the two results can be equated,

$$\Delta B_1 = (B_2 + \epsilon_2) - (B_1 + \epsilon_1)$$
$$\dots$$
$$\Delta B_N = (B_{N+1} + \epsilon_{N+1}) - (B_N + \epsilon_N).$$

This can then be rewritten in a matrix equation of the form $A\vec{x} = \vec{b}$,

$$\underbrace{\begin{bmatrix} -1 & 1 & 0 & \cdots & 0 \\ 0 & -1 & 1 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & -1 & 1 \end{bmatrix}}_{N \times (N+1)} \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \vdots \\ \epsilon_{N+1} \end{bmatrix} = \begin{bmatrix} \Delta B_1 + B_1 - B_2 \\ \Delta B_2 + B_2 - B_3 \\ \vdots \\ \Delta B_N + B_N - B_{N+1} \end{bmatrix}.$$

KMH B_x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SYM-H [nT]
SECS
dSECS
Measured
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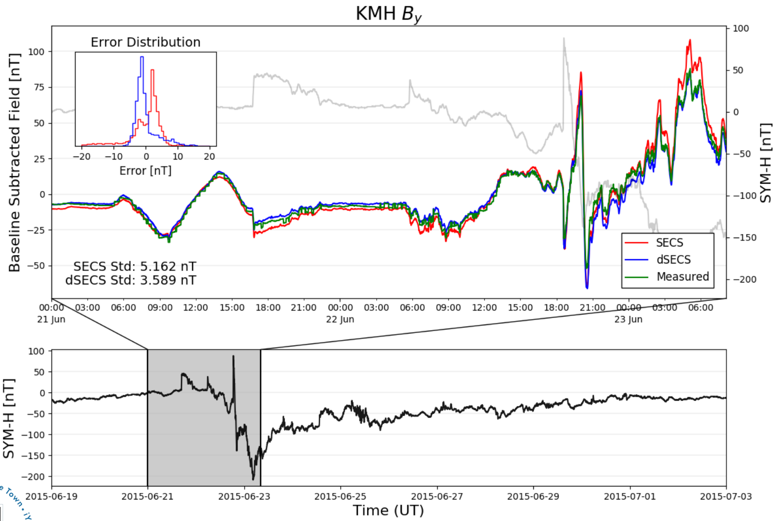
dSECS Std: 3.404 nT
Error Distribution
Error [nT]
21 Jun
22 Jun
23 Jun
2015-06-19
2015-06-21
2015-06-23
2015-06-25
2015-06-27
2015-06-29
2015-07-01
2015-07-03
Time (UT)

KMH B_y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SYM-H [nT]
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SECS
dSECS
Measured
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22 Jun
23 Jun
2015-06-19
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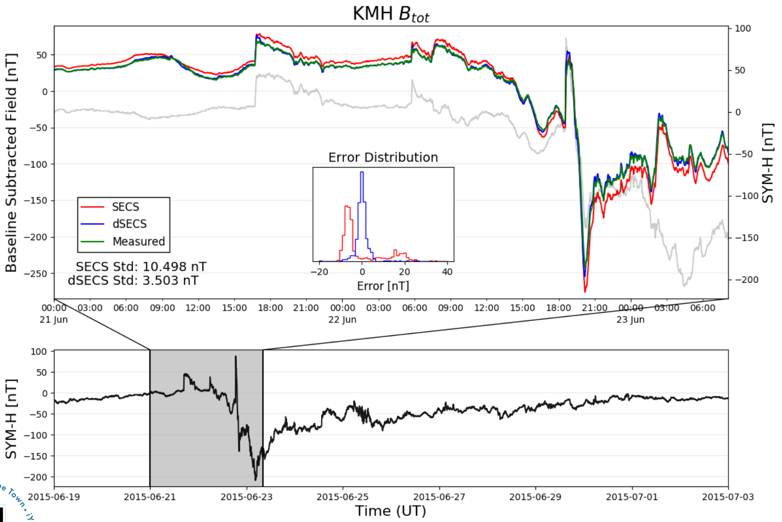
2015-06-23
2015-06-25
2015-06-27
2015-06-29
2015-07-01
2015-07-03
Time (UT)

KMH B_{tot}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dSECS
Measured
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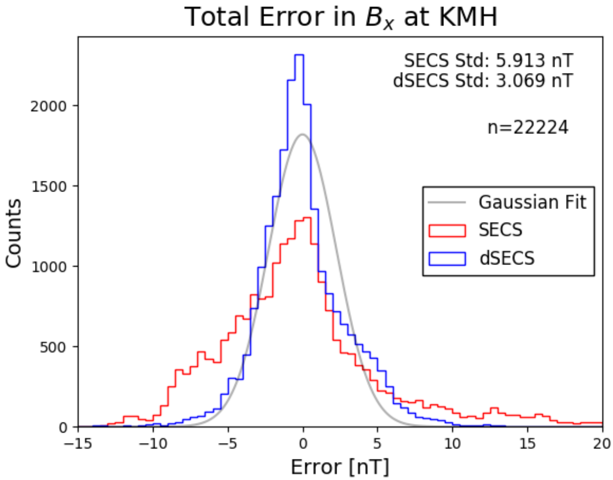
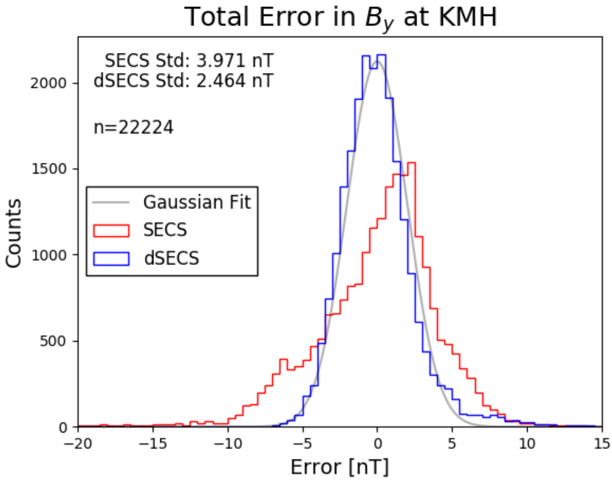
Error Distribution
Error [nT]
21 Jun
22 Jun
23 Jun
SYM-H [nT]
Time (UT)
2015-06-19
2015-06-21
2015-06-23
2015-06-25
2015-06-27
2015-06-29
2015-07-01
2015-07-03

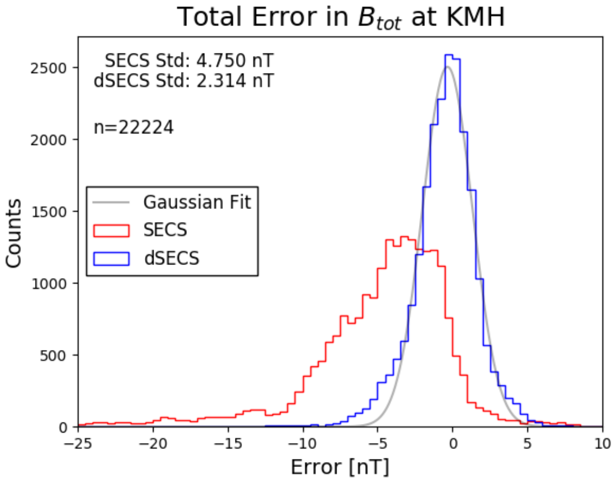
KMH B_x - All Storms



KMH B_y - All Storms



KMH B_{tot} - All Storms



Take Home Points...

1. Integrating variometers into magnetometer networks improve geomagnetic interpolation, especially during geomagnetic storms.
   - This improvement comes at a much reduced cost!
2. Variometers can be used alone in geoelectric field studies.
3. Systematic error is reduced, particularly in B_y component (which relates to induction effects).

'The more the merrier.'

Bibliography

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Separation of the geomagnetic variation field on the ground into external and internal parts using the spherical elementary current system method.
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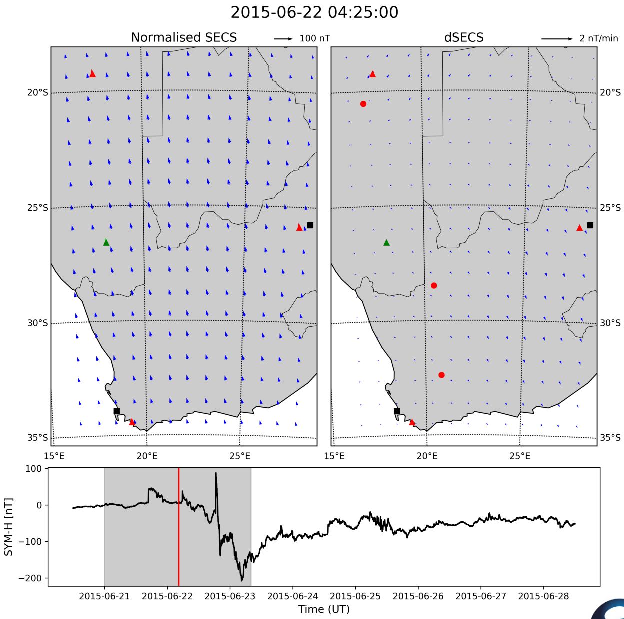
Questions (Suggestions)?


2015-06-22 04:25:00
Normalised SECS
100 nT
dSECS
2 nT/min
20°S
25°S
30°S
35°S
15°E
20°E
25°E
100
0
−100
−200
SYM-H [nT]
2015-06-21
2015-06-22
2015-06-23
2015-06-24
2015-06-25
2015-06-26
2015-06-27
2015-06-28
Time (UT)

SECS - Background Theory

From Helmholtz's theorem, any current flowing on a surface can be broken into curl-free (allows current flow in and out of sheet) and divergence-free (allows current flow on sheet) parts. The divergence-free part is what is typically measured by ground based magnetometers.

In Earth-centred spherical coordinates (r, θ, ϕ), the divergence-free current at a point $\vec{r}$ on the surface R_{surf}, θ away from a pole at $\vec{r}'$ is,

$$\vec{J}_{df,i}(\vec{r}) = \frac{I_i}{4\pi R_{surf}} \cot\left(\frac{\theta}{2}\right) \vec{e}_\phi = \frac{\int\int_S \tilde{r} \cdot \vec{\nabla} \times \vec{J}(\vec{r}')dS}{4\pi R_{surf}} \cot\left(\frac{\theta}{2}\right) \vec{e}_\phi .$$

SECS - Planar Approximation

Given a current element of amplitude of I at height h in cylindrical coordinates ($r = \sqrt{x^2 + y^2}, \phi, z$), the surface current density would be,

$$\vec{J_{df}} = I/(2\pi r)\vec{e_\phi},$$

Assuming z is downwards and a harmonic time dependence (i.e. $e^{i\omega t}$), the electric field resulting from the element would be,

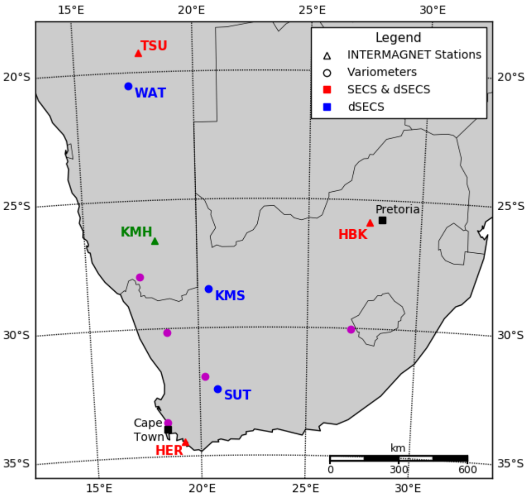
$$\vec{E} = -\frac{i\omega\mu_0 I}{4\pi}\frac{\sqrt{r^2 + h^2} - h}{r}\vec{e_\phi}.$$

The magnetic field would then be,

$$\vec{B} = \frac{\mu_0 I}{4\pi r}\left(\left(1 - \frac{h}{\sqrt{r^2 + h^2}}\right)\vec{e_r} + \left(\frac{r}{\sqrt{r^2 + h^2}}\right)\vec{e_z}\right).$$

Relevant Stations and Interpolation Grid

- Latitude range: 34.5-18.5°S
- Longitude range: 16.5-28.0°E
- Grid dimensions: 13 x 18
- Grid span: 1 214km EW and 1 781km NS
- Grid spacing: 101km and 104km
- Internal field model: Enhanced Magnetic Model (EMM2017)



Analytical Pseudo Inverse

$$A^{-1} = \begin{bmatrix} -1 & 1 & 0 & 0 & \cdots & 0 \\ 0 & -1 & 1 & 0 & \cdots & 0 \\ 0 & 0 & -1 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & -1 & 1 \end{bmatrix}^{-1}$$

$$= \overset{N \longrightarrow}{\begin{bmatrix} -\frac{N}{N+1} & -\frac{N-1}{N+1} & -\frac{N-2}{N+1} & \cdots & -\frac{1}{N+1} \\ \frac{1}{N+1} & -\frac{N-1}{N+1} & -\frac{N-2}{N+1} & \cdots & -\frac{1}{N+1} \\ \frac{1}{N+1} & \frac{2}{N+1} & -\frac{N-2}{N+1} & \cdots & -\frac{1}{N+1} \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ \frac{1}{N+1} & \frac{2}{N+1} & \cdots & -\frac{2}{N+1} & -\frac{1}{N+1} \\ \frac{1}{N+1} & \frac{2}{N+1} & \cdots & \frac{N-1}{N+1} & -\frac{1}{N+1} \\ \frac{1}{N+1} & \frac{2}{N+1} & \cdots & \frac{N-1}{N+1} & \frac{N}{N+1} \end{bmatrix}} \Big\downarrow N+1$$

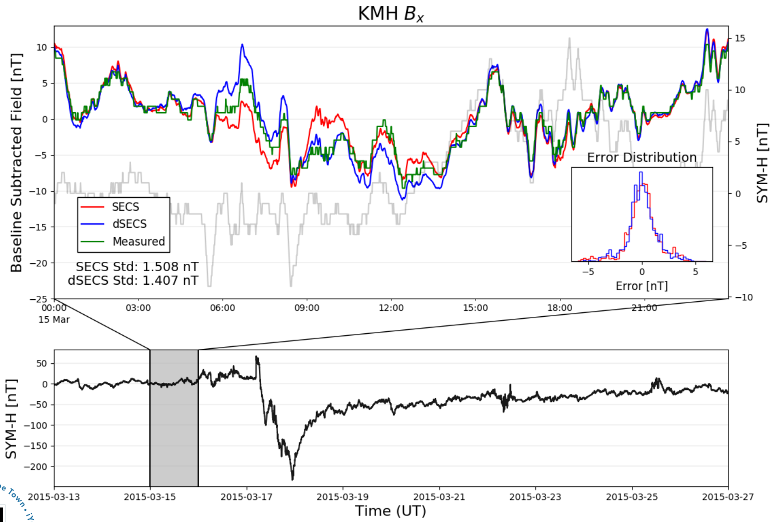
Analytical Pseudo Inverse

$$AA^{-1} = \mathbb{1}(N \times N)$$

$$A^{-1}A = \overbrace{\begin{bmatrix} \frac{N}{N+1} & -\frac{1}{N+1} & -\frac{1}{N+1} & \cdots & -\frac{1}{N+1} \\ -\frac{1}{N+1} & \frac{N}{N+1} & -\frac{1}{N+1} & \cdots & -\frac{1}{N+1} \\ -\frac{1}{N+1} & -\frac{1}{N+1} & \frac{N}{N+1} & \cdots & -\frac{1}{N+1} \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ -\frac{1}{N+1} & -\frac{1}{N+1} & \cdots & \frac{N}{N+1} & -\frac{1}{N+1} \\ -\frac{1}{N+1} & -\frac{1}{N+1} & \cdots & -\frac{1}{N+1} & -\frac{1}{N+1} \\ -\frac{1}{N+1} & -\frac{1}{N+1} & \cdots & -\frac{1}{N+1} & \frac{N}{N+1} \end{bmatrix}}^{N+1} \Big\downarrow N+1$$

$$\approx \mathbb{1}(N+1 \times N+1) \text{ for } N \gg 1$$

KMH B_x



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SYM-H [nT]
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dSECS
Measured
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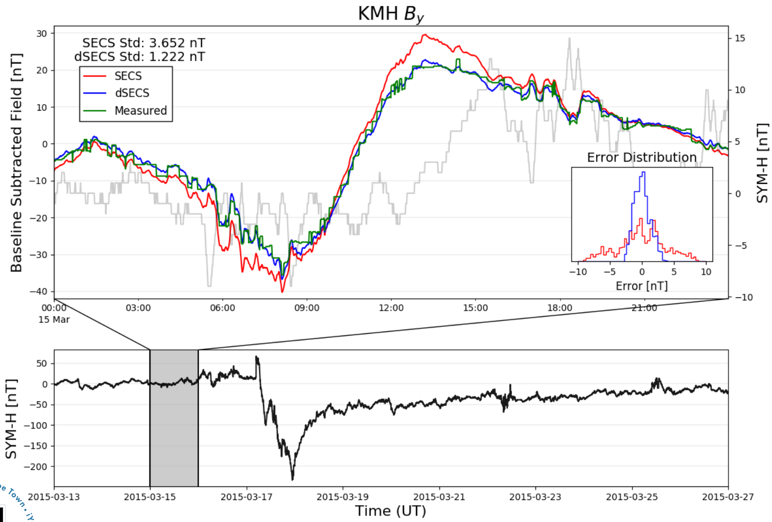
dSECS Std: 1.407 nT
Error Distribution
Error [nT]
00:00
15 Mar
03:00
06:00
09:00
12:00
15:00
18:00
21:00
2015-03-13
2015-03-15
2015-03-17
2015-03-19
2015-03-21
2015-03-23
2015-03-25
2015-03-27
Time (UT)

KMH B_y



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dSECS
Measured
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SYM-H [nT]
Error Distribution
Error [nT]
00:00
15 Mar
03:00
06:00
09:00
12:00
15:00
18:00
21:00
SYM-H [nT]
2015-03-13
2015-03-15
2015-03-17
2015-03-19
2015-03-21
2015-03-23
2015-03-25
2015-03-27
Time (UT)

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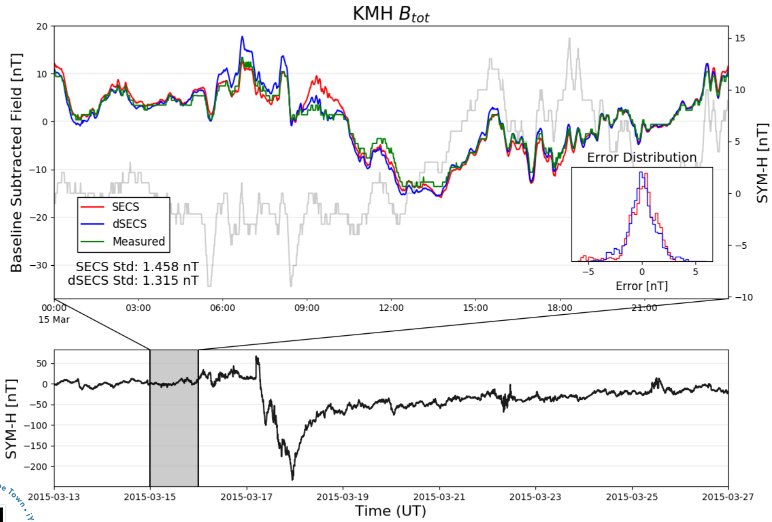
SPES BONA
SOUTH AFRICAN NATIONAL SPACE AGENCY

KMH B_{tot}



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dSECS
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15 Mar
03:00
06:00
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15:00
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21:00
SYM-H [nT]
2015-03-13
2015-03-15
2015-03-17
2015-03-19
2015-03-21
2015-03-23
2015-03-25
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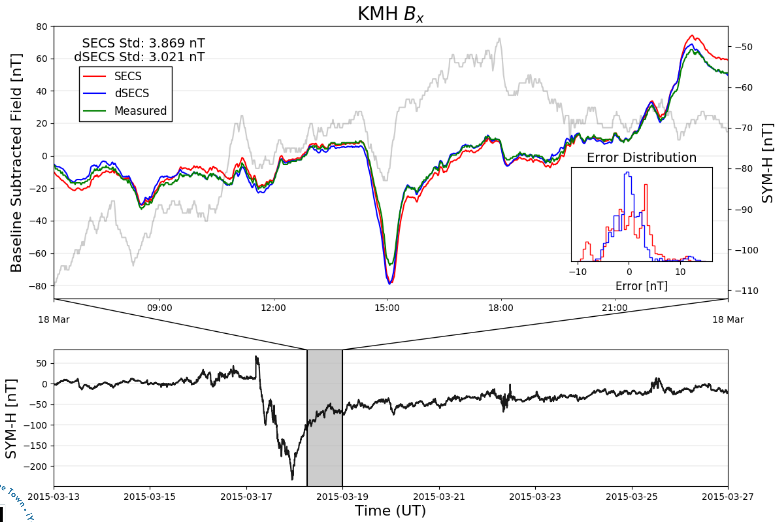
Time (UT)

KMH B_x



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SECS
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SYM-H [nT]
Error Distribution
Error [nT]
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12:00
15:00
18:00
21:00
18 Mar
18 Mar
2015-03-13
2015-03-15
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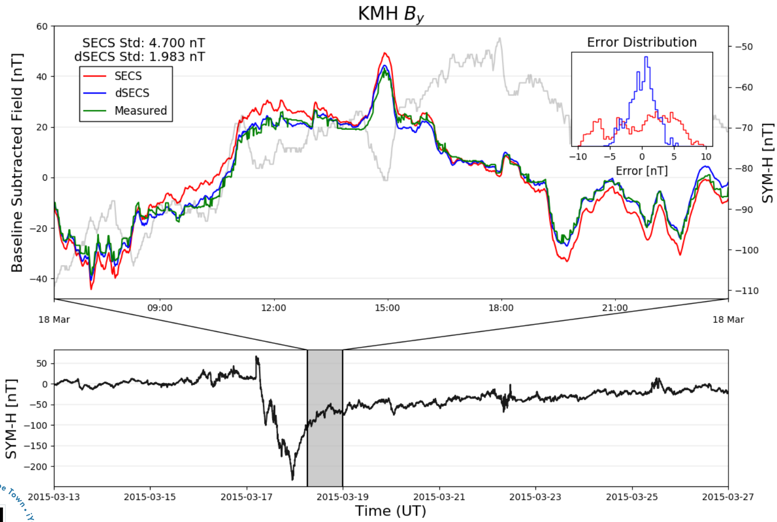
2015-03-23
2015-03-25
2015-03-27
Time (UT)

KMH B_y



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dSECS
Measured
Error Distribution
Error [nT]
Baseline Subtracted Field [nT]
SYM-H [nT]
18 Mar
09:00
12:00
15:00
18:00
21:00
18 Mar
SYM-H [nT]
2015-03-13
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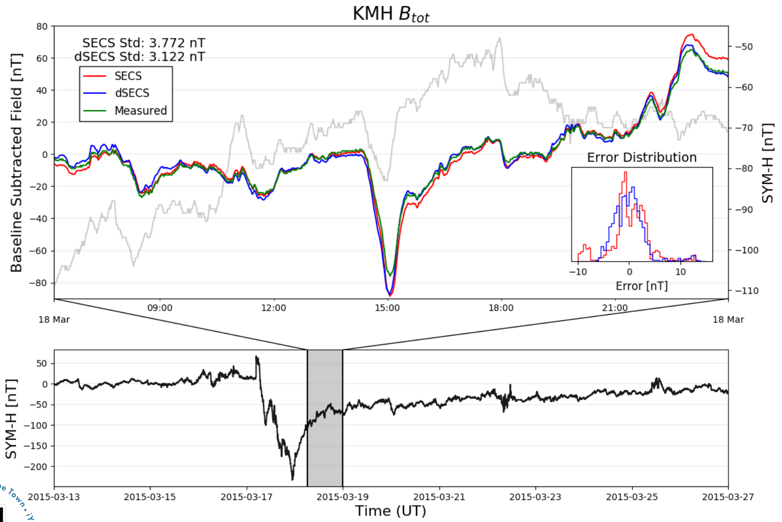
2015-03-17
2015-03-19
2015-03-21
2015-03-23
2015-03-25
2015-03-27
Time (UT)

KMH B_{tot}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Error Distribution
Error [nT]
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09:00
12:00
15:00
18:00
21:00
18 Mar
SYM-H [nT]
2015-03-13
2015-03-15
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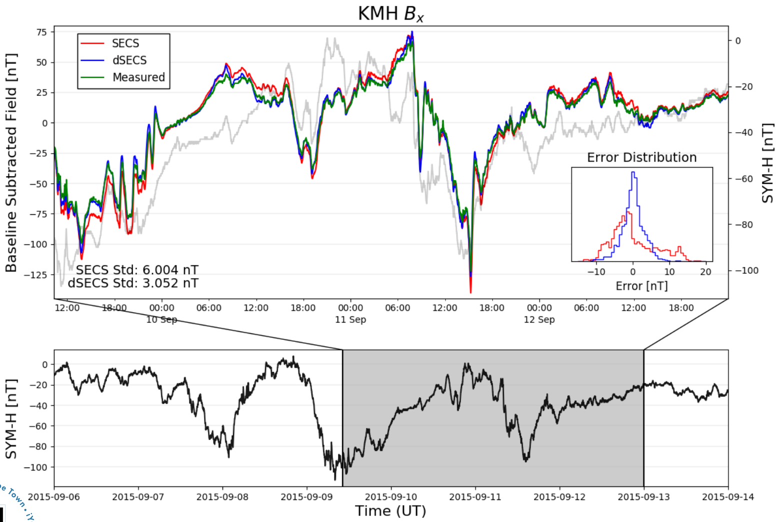
Time (UT)

KMH B_x



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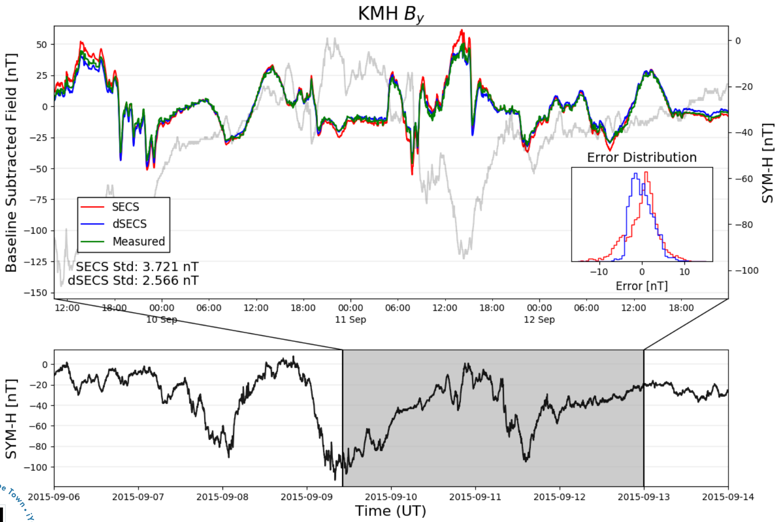
KMH B_y



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Measured
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dSECS Std: 2.566 nT
Error Distribution
Error [nT]
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00:00
10 Sep
06:00
11 Sep
12 Sep
2015-09-06
2015-09-07
2015-09-08
2015-09-09
2015-09-10
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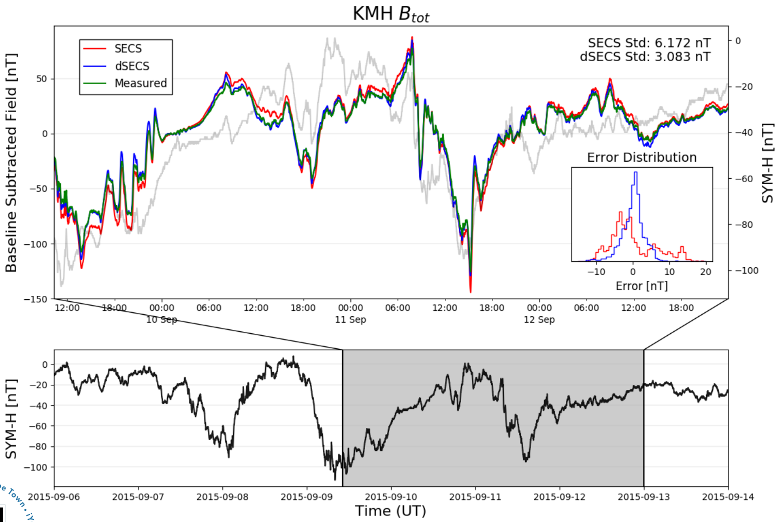
2015-09-13
2015-09-14
Time (UT)

KMH B_{tot}



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dSECS
Measured
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dSECS Std: 3.083 nT
Error Distribution
Error [nT]
Baseline Subtracted Field [nT]
SYM-H [nT]
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18:00
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10 Sep
06:00
12:00
18:00
00:00
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18:00
2015-09-06
2015-09-07
2015-09-08
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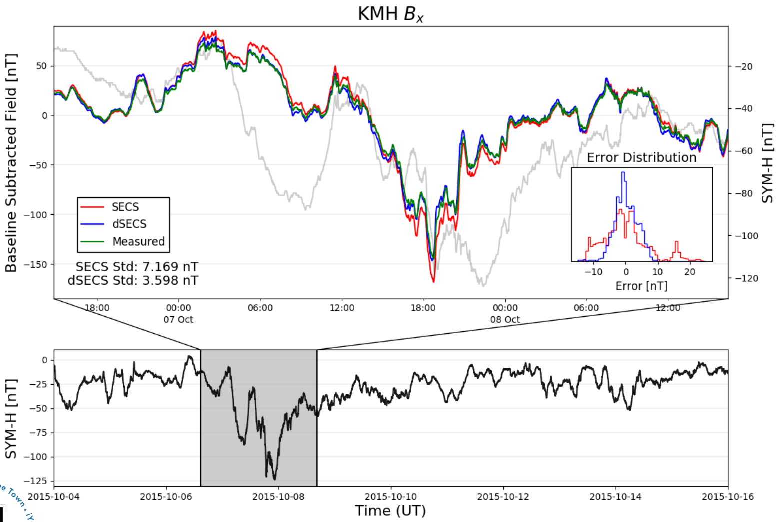
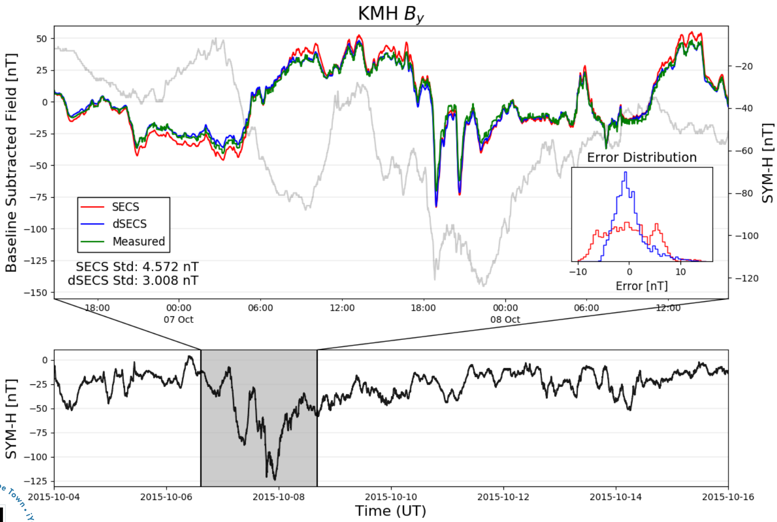
2015-09-12
2015-09-13
2015-09-14
Time (UT)

KMH B_x



KMH B_x
Baseline Subtracted Field [nT]
50
0
−50
−100
−150
SYM-H [nT]
−20
−40
−60
−80
−100
−120
SECS
dSECS
Measured
SECS Std: 7.169 nT
dSECS Std: 3.598 nT
Error Distribution
−10
0
10
20
Error [nT]
18:00
00:00
07 Oct
06:00
12:00
18:00
00:00
08 Oct
06:00
12:00
0
−25
−50
−75
−100
−125
2015-10-04
2015-10-06
2015-10-08
2015-10-10
2015-10-12
2015-10-14
2015-10-16
Time (UT)

KMH B_y



KMH B_y
Baseline Subtracted Field [nT]
SYM-H [nT]
SECS
dSECS
Measured
SECS Std: 4.572 nT
dSECS Std: 3.008 nT
Error Distribution
Error [nT]
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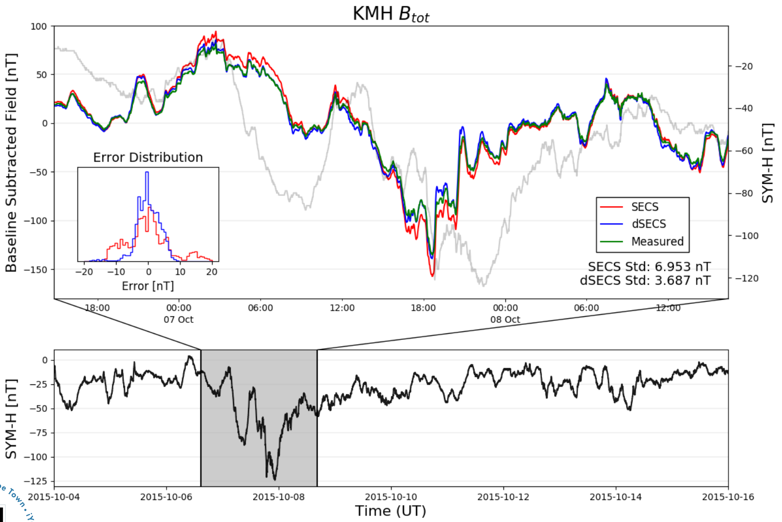
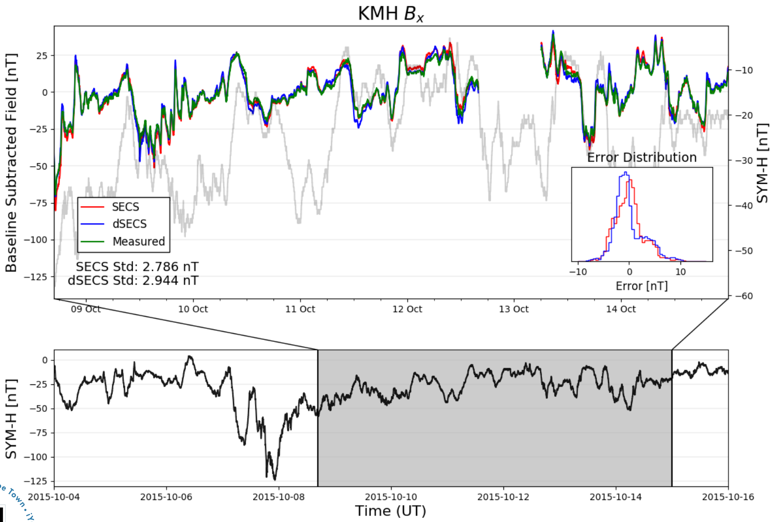
00:00
07 Oct
06:00
12:00
18:00
00:00
08 Oct
06:00
12:00
2015-10-04
2015-10-06
2015-10-08
2015-10-10
2015-10-12
2015-10-14
2015-10-16
Time (UT)

KMH B_{tot}



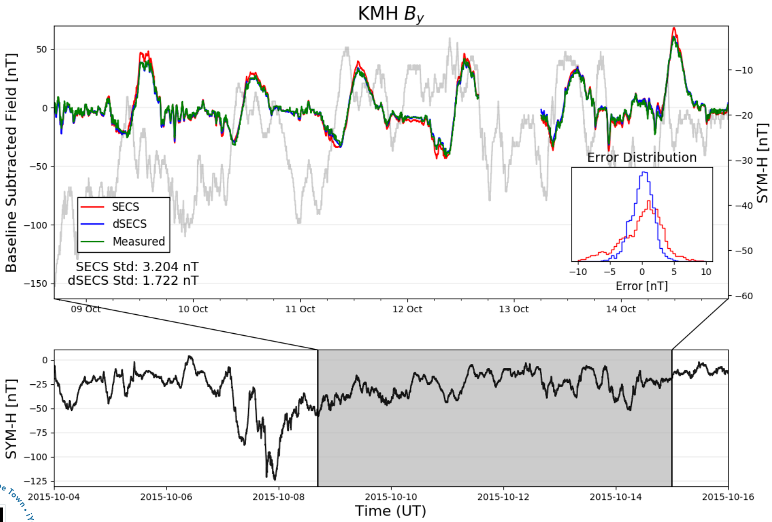
KMH B_{tot}
Baseline Subtracted Field [nT]
SYM-H [nT]
Error Distribution
Error [nT]
SECS
dSECS
Measured
SECS Std: 6.953 nT
dSECS Std: 3.687 nT
18:00
00:00
07 Oct
06:00
12:00
18:00
00:00
08 Oct
06:00
12:00
SYM-H [nT]
2015-10-04
2015-10-06
2015-10-08
2015-10-10
2015-10-12
2015-10-14
2015-10-16
Time (UT)

KMH B_x



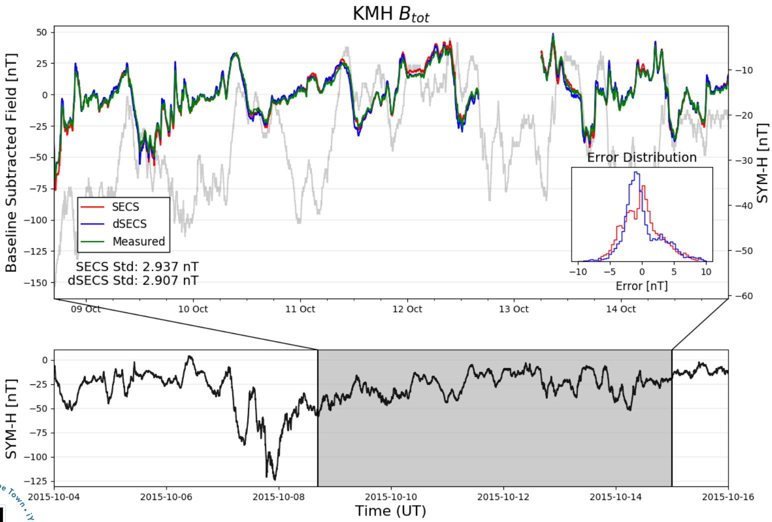
KMH B_x
Baseline Subtracted Field [nT]
SYM-H [nT]
SECS
dSECS
Measured
SECS Std: 2.786 nT
dSECS Std: 2.944 nT
Error Distribution
Error [nT]
09 Oct
10 Oct
11 Oct
12 Oct
13 Oct
14 Oct
2015-10-04
2015-10-06
2015-10-08
2015-10-10
2015-10-12
2015-10-14
2015-10-16
Time (UT)

KMH B_y



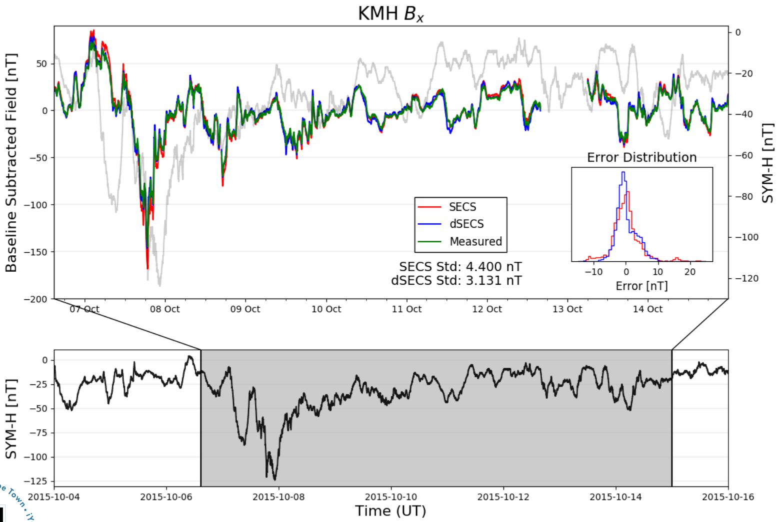
KMH B_y
Baseline Subtracted Field [nT]
SYM-H [nT]
SECS
dSECS
Measured
SECS Std: 3.204 nT
dSECS Std: 1.722 nT
Error Distribution
Error [nT]
09 Oct
10 Oct
11 Oct
12 Oct
13 Oct
14 Oct
2015-10-04
2015-10-06
2015-10-08
2015-10-10
2015-10-12
2015-10-14
2015-10-16
Time (UT)

KMH B_{tot}



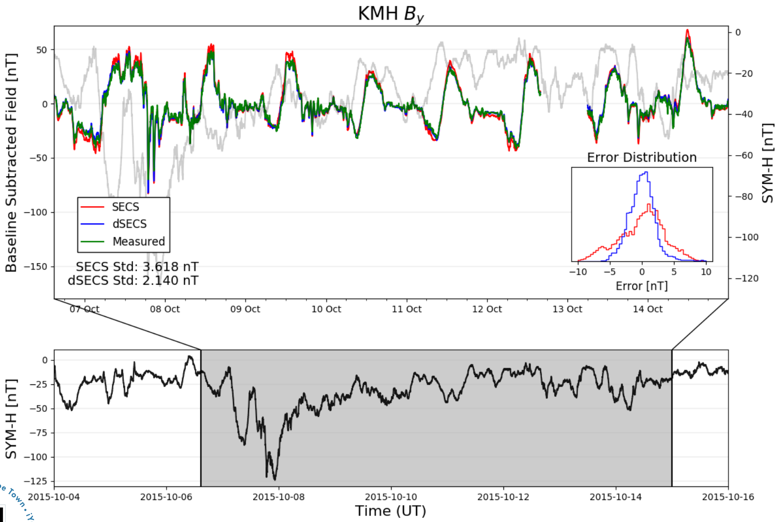
KMH B_{tot}
Baseline Subtracted Field [nT]
SYM-H [nT]
SECS
dSECS
Measured
SECS Std: 2.937 nT
dSECS Std: 2.907 nT
Error Distribution
Error [nT]
09 Oct
10 Oct
11 Oct
12 Oct
13 Oct
14 Oct
2015-10-04
2015-10-06
2015-10-08
2015-10-10
2015-10-12
2015-10-14
2015-10-16
Time (UT)

KMH B_x

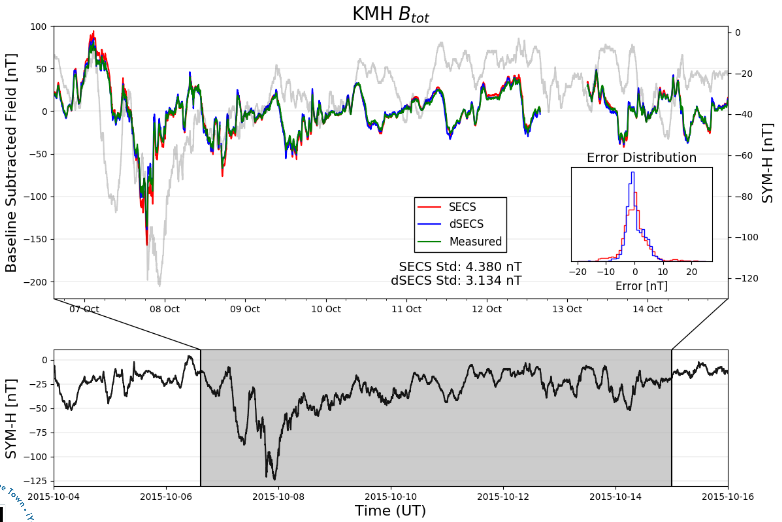


KMH B_x
Baseline Subtracted Field [nT]
SYM-H [nT]
SECS
dSECS
Measured
SECS Std: 4.400 nT
dSECS Std: 3.131 nT
Error Distribution
Error [nT]
07 Oct
08 Oct
09 Oct
10 Oct
11 Oct
12 Oct
13 Oct
14 Oct
2015-10-04
2015-10-06
2015-10-08
2015-10-10
2015-10-12
2015-10-14
2015-10-16
Time (UT)

KMH B_y


KMH B_y
Baseline Subtracted Field [nT]
SYM-H [nT]
SECS
dSECS
Measured
SECS Std: 3.618 nT
dSECS Std: 2.140 nT
Error Distribution
Error [nT]
07 Oct
08 Oct
09 Oct
10 Oct
11 Oct
12 Oct
13 Oct
14 Oct
2015-10-04
2015-10-06
2015-10-08
2015-10-10
2015-10-12
2015-10-14
2015-10-16
Time (UT)

KMH B_{tot}



KMH B_{tot}
Baseline Subtracted Field [nT]
SYM-H [nT]
Error Distribution
Error [nT]
SECS
dSECS
Measured
SECS Std: 4.380 nT
dSECS Std: 3.134 nT
07 Oct
08 Oct
09 Oct
10 Oct
11 Oct
12 Oct
13 Oct
14 Oct
2015-10-04
2015-10-06
2015-10-08
2015-10-10
2015-10-12
2015-10-14
2015-10-16
Time (UT)