Quantitative Predictions of Heavy Flavor Photon Bremsstrahlung in Heavy Ion Collisions from AdS/CFT

W A Horowitz

Department of Physics, University of Cape Town, Private Bag X3, Rondebosch 7701, South Africa

E-mail: `wa.horowitz@uct.ac.za`

Abstract. We present quantitative predictions for the spectra of photon bremsstrahlung from heavy quarks propagating through a strongly-coupled quark-gluon plasma using the techniques of AdS/CFT. The spectra are the result of including both drag and diffusion terms in the heavy flavor propagation in the plasma. The predictions show that future experimental upgrades and high-luminosity runs at the Large Hadron Collider at CERN will allow for a quantitative comparison between data and our calculations, providing novel insight into the non-trivial, emergent many-body dynamics of a non-Abelian gauge theory.

1. Introduction

Experimental indications give a mixed picture of the relevant dynamics of the quark-gluon plasma (QGP) created at the Relativistic Heavy Ion Collider (RHIC) and the Large Hadron Collider (LHC). In particular, measurements of the distribution of low transverse momentum ($p_T \lesssim 2$ GeV/c) particles can be understood through near perfect viscous hydrodynamics with a rapid hydrodynamization time [1–7] or from parton cascades [8–13]. Similarly, observables related to single partons at high transverse momentum ($p_T \gtrsim 15$ GeV/c) can also be qualitatively described using energy loss models based on either strong-coupling AdS/CFT [14–22] or weak-coupling pQCD [23–39].

We therefore seek novel experimental handles to provide insight into the relevant dynamics of the quark-gluon plasma (QGP) produced at the temperatures $T_{QGP} \sim 400$ MeV accessible at RHIC and LHC. In particular with the massive increase in luminosity and detector sensitivity that will come to the LHC after upgrades, the study of rare observables becomes possible.

A natural observable of potential interest is the measurement of the *photon* bremsstrahlung associated with open heavy flavor propagation in QGP. Presumably the radiation pattern for photons will differ depending on whether the heavy quark undergoes rare hard scattering events as one expects from pQCD or if it rather is plowing through a strongly-coupled soup as described by AdS/CFT. (Note that photon tagged heavy flavor production is different and refers to the $2 \rightarrow 2$ or $2 \rightarrow 3$ prompt hard photon production at heavy ion collision time $t = 0$ that is clearly well described by standard perturbative field theory methods [40, 41].)

We provide here quantitative predictions for the photon bremsstrahlung produced by an open heavy quark strongly-coupled to a strongly-coupled plasma. In this exploratory study we use

leading order heavy quark drag as derived from a steady state string setup in AdS/CFT [42, 43]. The influence of fluctuations [44–46] in the momentum loss experienced by the heavy quark may be an important contribution, but we leave it to future work.

2. Setup

We treat the heavy quark–anti-quark pair produced at the initial nuclear overlap at $t = 0$ and moving in opposite directions as a classical current [47]

$$j^\mu(x) = Q\, e\, \theta(t) \left[v^\mu(t) \delta^{(3)}(\vec{x} - \vec{x}(t)) - \tilde{v}^\mu(t) \delta^{(3)}(\vec{x} + \vec{x}(t)) \right]. \tag{1}$$

coupled to the electromagnetic field

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + j_\mu A^\mu. \tag{2}$$

In Eq. 1, Q is the fractional electric charge carried by the heavy quark, $\theta(t)$ is the usual Heaviside step function, and

$$\begin{aligned} v^\mu(t) &= (1, \vec{v}(t))^\mu \\ \tilde{v}^\mu(t) &= (1, -\vec{v}(t))^\mu \\ \vec{x}(t) &= \vec{x}_0 + \int_0^t dt'\, \vec{v}(t'). \end{aligned} \tag{3}$$

For any generic current coupling to the electromagnetic field given by Eq. 2, one finds [16, 48] that the momentum differential energy distribution of the emitted electromagnetic radiation is given by

$$\frac{dE}{d^3 k} = -\frac{1}{2} \frac{1}{(2\pi)^3} \tilde{j}^\mu(k) \tilde{j}_\mu(-k), \tag{4}$$

where

$$\tilde{j}^\mu(k) \equiv \int d^4 x\, e^{-ik \cdot x} j^\mu(x). \tag{5}$$

2.1. Hard Production Electromagnetic Radiation

Since we will ultimately wish to compare to pQCD-based energy loss calculations for which there are usually non-trivial UV and IR catastrophes associated with the factorization of the hard production and subsequent dynamics, we wish to compute the *difference* in the radiated energy in medium as compared to the vacuum. For the vacuum case, we have that, in the soft radiation approximation, the heavy quark pair does not lose momentum after it is produced. In this case we then take $\vec{v}(t) \equiv \vec{v}_0$ and $\vec{x}(t) = \vec{v}_0 t$ in Eq. 1. To perform the Fourier transform in Eq. 5 one must insert a small convergence factor η. Evaluation of Eq. 5 then yields

$$\begin{aligned} \tilde{j}^\mu_{vac}(k) &= \lim_{\eta \to 0} \int d^4 x\, e^{-i[(\omega - i\,\eta)t + \vec{k} \cdot \vec{x}(t)]}\, j^\mu(x) \\ &= \lim_{\eta \to 0} Q\, e \left[\frac{v_0^\mu}{i\,\omega - i\,\vec{k} \cdot \vec{v}_0 + \eta} - \frac{\tilde{v}_0^\mu}{i\,\omega + i\,\vec{k} \cdot \vec{v}_0 + \eta} \right] \\ &= -i\, Q\, e \left[\frac{v_0^\mu}{\omega - \vec{k} \cdot \vec{v}_0} - \frac{\tilde{v}_0^\mu}{\omega + \vec{k} \cdot \vec{v}_0} \right]. \end{aligned} \tag{6}$$

After some manipulation and taking the motion of the quarks to be along the z direction, with $\vec{k} \cdot \vec{v}_0 = \omega\, v_0 \cos\theta$, $\omega \equiv k^0$, and $v_0 \equiv |\vec{v}_0|$, we find Eq. 4 yields for the differential energy radiated in vacuum

$$\frac{dE_{vac}}{d^3k} = \frac{2}{(2\pi)^3}(Q\,e)^2 \frac{1}{\omega^2} \frac{v_0^2 \sin^2\theta}{(1 - v_0^2 \cos^2\theta)^2}. \tag{7}$$

As is usual, we see that the total integrated energy radiated by the vacuum current grows linearly with an artificially imposed UV cutoff ω_{max}. Interestingly, the inclusion of the *pair* of quarks has tamed the usual IR divergence: the long wavelength physics knows that the total charge remains 0 as the $q\bar{q}$ pair separates.

2.2. AdS/CFT Induced Electromagnetic Radiation

The leading order energy loss of a heavy quark in a strongly-coupled $\mathcal{N} = 4$ SYM plasma is given by [42, 43]

$$\frac{d\vec{p}}{dt} = -\mu\,\vec{p}, \quad \mu = \frac{\pi\sqrt{\lambda}}{2}\frac{T^2}{M_Q}, \tag{8}$$

where $\lambda = g^2 N_c$ is the 't Hooft coupling, T is the temperature of the plasma, and M_Q is the mass of the heavy quark.

From Eq. 8, and assuming motion is only in one dimension, we may solve for

$$
\begin{aligned}
p(t) &\equiv |\vec{p}(t)| = p_0 e^{-\mu\,t} \\
E(t) &= \sqrt{m^2 + p^2(t)} \\
v(t) &= p(t)/E(t) \\
x(t) &= \frac{1}{2\mu}\ln\left[\frac{E_0 + p_0}{E_0 - p_)}\times\frac{E(t) - p(t)}{E(t) + p(t)}\right].
\end{aligned}
\tag{9}
$$

We may plug in our results from Eq. 9 into our equation for the Fourier transform of the current, Eq. 5, to find the Fourier transform of the current when the heavy quarks are subject to the AdS/CFT heavy quark drag. Unfortunately, the result cannot be evaluated analytically.

3. Results

Once we have the Fourier transform of the current associated with a heavy quark pair separating in a strongly-coupled AdS/CFT plasma, we may compute the difference in energy radiated by the heavy quark in medium minus the energy radiated in vacuum. We show in Fig. 1 (left) the result for $m_c = 1.5$ GeV/c^2 charm quarks and (right) $m_b = 4.75$ GeV/c^2 bottom quarks. Not surprisingly, the biggest medium modification to $\omega^2 dE/d^3k$ is centered at $\theta_{max} \sim m_Q/p_T$; one can show numerically that the depth of the difference at θ_{max} is a function of $(m_Q/p_T)^2$.

One can understand the reduction in emitted photon radiation shown in the plot as follows. In vacuum, the quarks are accelerated from rest to some non-zero velocity v_0 associated with the initial p_T of the particles. The quarks in medium, on the other hand, immediately begin decelerating due to the presence of the plasma. The produced photons will always have some non-zero formation time. For the photons produced in-medium, the effective acceleration of the quark is smaller than that for the quark in vacuum because of the immediate deceleration. Therefore there is less radiation emitted from the in-medium quarks, and thus the suppression observed in the subtracted spectrum.
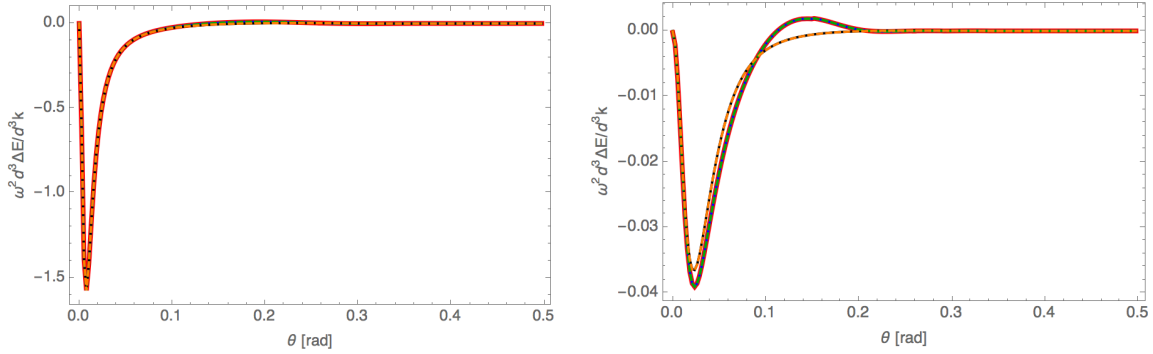
Figure 1: (Colour online) $dE/d^3k|_{med} - dE/d^3k|_{vac}$ for $T = 400$ MeV strongly-coupled $\mathcal{N} = 4$ SYM plasma for (left) $m_c = 1.5$ GeV/c^2 charm quarks and (right) $m_b = 4.75$ GeV/c^2 bottom quarks. For both plots the quarks have $p_T = 200$ GeV/c, the photon has energy 20 GeV, and $\lambda = 12$. In both plots, the thicker red curve corresponds to both quarks traversing an infinite length plasma, the thick blue curve to both quarks traversing up to 5 fm of plasma, and the thick black curve to both quarks traversing up to 1 fm of plasma. For the dashed green curve, the quark moving in the z direction traverses a distance of up to 5 fm while the away side quark traverses a 1 fm thick plasma; for the dashed orange curve, the distances are reversed.

4. Conclusions and Outlook

We presented the first prediction of photon bremsstrahlung for heavy quarks produced in a strongly-coupled $\mathcal{N} = 4$ SYM plasma. We derived the spectrum of emitted QED radiation dE/d^3k for a classical point current of given (potentially) time-dependent velocity in Eq. 4. We then focused on the trajectory of a heavy quark strongly coupled to a strongly coupled plasma as predicted by leading order AdS/CFT, in which case the momentum of the heavy quark is modified according to the usual drag result $dp/dt = -\mu p$, where $\mu = \pi\sqrt{\lambda}T^2/m_Q$. We plotted the difference in the spectra of the photons produced in medium compared to those produced in vacuum, $\omega^2 dE/d^3k|_{med} - \omega^2 dE/d^3k|_{vac}$, in Fig. 1. The presence of the medium suppresses the radiation emitted by heavy quark compared to that in the vacuum. Given the very small angle at which the modification is significant, $\theta_{max} = m_Q/p_T \ll 1$, it is currently unclear whether the current experiments at the LHC could distinguish between a modification to the production bremsstrahlung as shown here or a potential modification to the photons generated in the hadronization process in which the heavy quarks become heavy mesons.

Acknowledgments

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