Volume determination of irregular objects by hydrostatic weighing at NMISA

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Abstract. Accurate volume determination of irregular objects can be complex because there are no standard formulae to be applied. Various techniques based on Archimedes' principle where the buoyancy force exerted by a fluid on a submerged object is equated to the weight of the object may be applied. One technique is the water displacement method which is not very accurate. The other is by weighing a submerged object in a fluid of known density. At NMISA the volumes of the OIML-shaped stainless steel mass pieces (50 g to 1 kg) have been measured using the automated volume comparator VC1005 (VC) which weighs in the FC-40 fluid. The results were validated against the previous measurements that were done at KRISS. The volumes of the mass pieces were measured to an average agreement of greater than 99.998%. These measurements led to the determination and confirmation of densities of the mass pieces which is critical for high accuracy mass measurements. The density results show improved measurement uncertainties for NMISA. The current work demonstrates a new achievement of high accuracy volume and density measurements for South Africa by the hydrostatic weighing method. This paper presents the volume and density results found for 50 g to 1 kg mass pieces using the VC.

1. Introduction

The volume of a regular object can accurately be determined by measuring the object's dimensions to apply a formula. However, when it comes to irregular objects, measuring dimensions is not always possible. Various techniques that are based on Archimedes' principle can be used for volume and density determination of irregular objects. Archimedes' principle states that the buoyancy force on an object that is submerged in a fluid is equivalent to the weight of the displaced fluid [1]. The buoyancy force F_B exerted by a fluid of density ρ_f on a submerged volume V_0 is given by

$$F_B = V_0 \rho_f g \tag{1}$$

where g is the gravitational acceleration.

Figure 1 shows a summary of three volume determination techniques that are based on Archimedes' principle [2]. In technique (a) the object is submerged in water by a thin suspension wire and this creates a weight difference Δw on a balance which then enables the determination of the object's volume by $V_0 = \frac{\Delta w}{\rho_f}$. Technique (b) shows the water displacement technique where a rise in the water level is equivalent to the volume of the object. Technique (c) shows the liquid overflow technique in which the fluid displaced by the object is collected on a graduated container to read of the volume of the object directly.

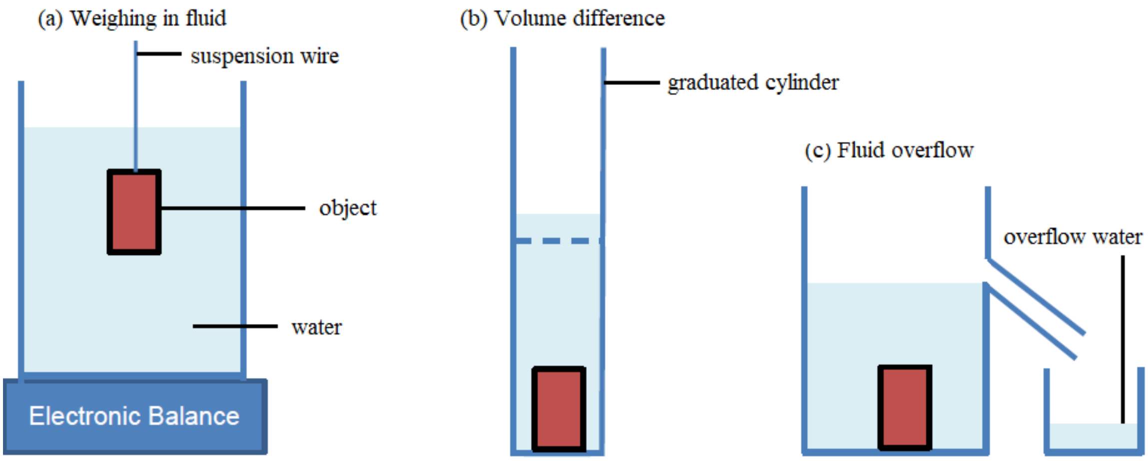

Figure 1: Schematic representation of (a) suspension, (b) liquid level and (c) overflow techniques for volume determination of solids [2].

This paper discusses the volume determination of irregular objects by hydrostatic weighing at the National Metrology Institute of South Africa (NMISA). The automated volume comparator VC1005 (VC) is used for volume and density determination of objects ranging from 1 g to 1 kg. The repeatability of weighing in liquid and temperature-stability on this system is critical in performing high accuracy measurements. The improved accuracy as indicated by smaller uncertainties at NMISA would mean improved accuracy for the South African volume and density measurements through dissemination from the national standards.

2. The volume determination

The VC works by performing comparisons between two objects (reference and test) to detect the difference in volume. When an object of mass m and volume V balances a reference mass piece of mass m_c and volume V_c in a fluid of density ρ_f, the net force equation in the vertical direction which considers the downward weight due to the object and the upwards buoyancy due to the fluid can be written as

$$mg - \rho_f V g = m_c g - \rho_f V_c g\,. \tag{2}$$

For a weight taken at a temperature of 20 °C, the conventional mass m_c is defined as the mass of a reference of density 8000 kg/m^3 that the weight would balance in air density of 1.2 kg/m^3 [3]. Setting $V_c = \frac{m_c}{\rho_c} = \frac{m_c}{8000}$ and $\rho_f = \rho_{air} = 1.2$ in Equation (2), this definition may be expressed mathematically as

$$m = m_c\left(1 - \frac{1.2}{8000}\right) + V(1.2). \tag{3}$$

The weighing equation that is used for calculating the volume difference ΔV between the reference and test mass pieces, in terms of the conventional masses, is given by

$$\Delta V = \frac{(m_{ct} - m_{cr})\left(1 - \frac{1.2}{8000}\right)}{\rho_f - 1.2} - \frac{B\Delta m_w}{\rho_f - 1.2} \tag{4}$$

where: m_{ct} and m_{cr} are the conventional masses for the test and reference respectively,

$\Delta m_w = m_{wt} - \frac{m_{wr1} + m_{wr2}}{2}$ is the drift-eliminated weighing difference,

$B = \left(1 - \frac{\rho_{air}}{\rho_{bw}}\right)$ is the balance correction factor,

ρ_{air} is the air density and ρ_{bw} is the density of the balance's internal weights.

The volume $V_t(20)$ of the test object at 20 °C is then determined using the volume difference by

$$V_t(20) = \frac{V_r\left[1+\alpha_r\left(T_{av}-20\right)\right]+\Delta V}{1+\alpha_t\left(T_{av}-20\right)} \tag{5}$$

where: α_r and α_t are the thermal expansion coefficients of the reference and test since the measurements are performed at temperature T_{av},

V_r is the volume of the reference mass [4]. In the next section we discuss the experimental details and set-up of the VC.

3. Experimental details

The volumes of the OIML-shaped stainless-steel mass pieces (50 g, 100 g, 200 g, 500 g and 1 kg), shown in Figure 2, were determined using the VC. The front view of the NMISA's VC with the side view of the fluid bath is shown in Figure 3.

Figure 2: OIML-shaped weights.

The VC system is mounted on top of a stable granite table to minimize vibrations. The VC consists of the outer and inner fluid baths. The outer bath is filled with sterilized distilled water to prevent the build-up of micro-organisms and contaminants inside. This water provides some sort of fluid gradient from the dense FC-40 in the inner bath to the outside air which minimizes the risk of the inner glass cracking due to the heavy FC-40. The mass pieces are submerged in the FC-40 fluid for weighing. The FC-40 electronic fluid is a fluorinert colourless liquid that is thermally stable and single-phase liquid over a temperature range of -57 °C to 165 °C which makes it suitable for relatively constant temperature measurements as in this current work [5]. Prior to measurements the fluid density has to be determined to ensure the correct density which is about 1.87 g/cm^3 is recorded. Fluid calibration for density may be performed using internal weights of the VC or using the calibrated external mass pieces, with the latter being more accurate.

The turntable inside the fluid bath has four loading positions. The mass pieces are loaded using the loading arm which can move up or down. Figure 4 shows a schematic representation of the experimental set-up of the apparatus. The weighing pan inside the fluid bath hangs from the comparator. Close to this weighing pan is a temperature sensor connected outside to a Thermometer Readout. This ensures that the fluid temperature is measured close to where the weighing occurs. A

ClimaLog was used to measure the air temperature (T), pressure (P) and relative humidity (RH) for calculating the air density. The measured data is then collected onto the computer software ComVol for processing where the data for the reference and test objects to be measured are also stored.

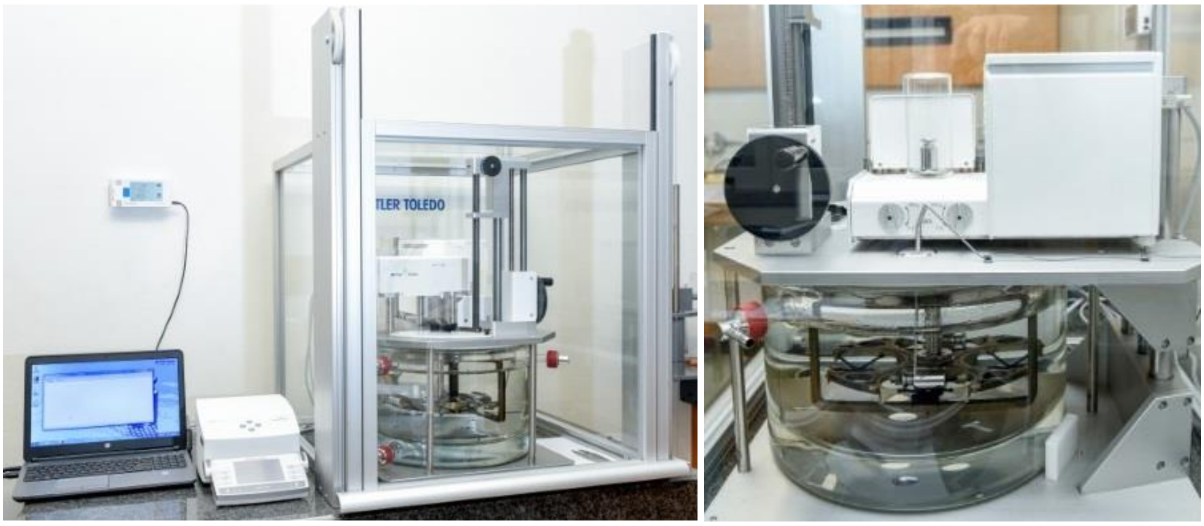

Figure 3: The front view (left) of the VC1005 and the side view (right) of the fluid bath on the right.

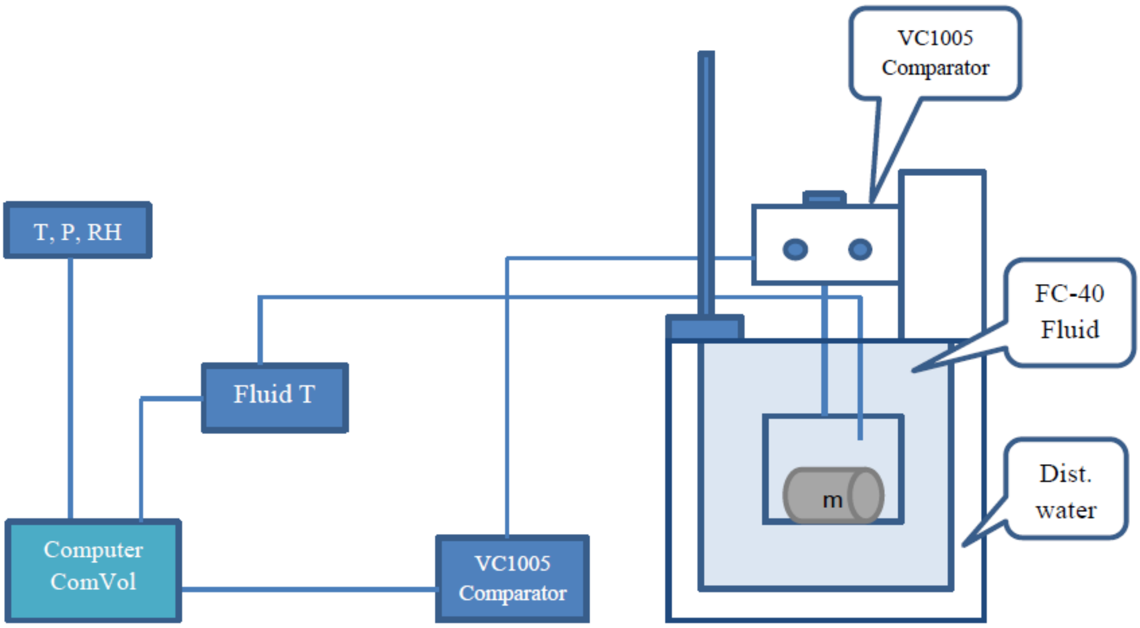

Figure 4: Schematic representation of the VC1005 experimental set-up.

The reference and test weights of the same nominal mass and known conventional masses were loaded across each other on the turntable positions. Weights were thermally stabilized in the fluid for a minimum of two hours before performing measurements.

4. Results and discussion

The mass pieces used in this current work were previously calibrated for mass and density at Korea Research Institute of Standards and Science (KRISS) in the year 2010. The volumes V of the weights by KRISS were calculated from the calibration certificates using the measurement results of mass m and density ρ, as per the formula $V = m/\rho$. The calculated volumes from KRISS measurements were used to validate the results found. The associated volume uncertainties $u(V)$ for KRISS were determined by the relation $\left(\frac{u(V)}{V}\right)^2 = \left(\frac{u(m)}{m}\right)^2 + \left(\frac{u(\rho)}{\rho}\right)^2$ where $u(m)$ and $u(\rho)$ are the uncertainties in mass and density certificates respectively [7]. The measured volumes with the calculated uncertainties (K=2) that were found by NMISA and KRISS are shown in Table 1. The accredited calibration and measurement capabilities (CMCs) for NMISA solid weights density calibrations [6] are shown in Table 2 which also shows the density results of NMISA and KRISS. The agreement between the results was calculated using the relation $y = \left(1 - \frac{|x_1 - x_2|}{(x_1 + x_2)/2}\right) \times 100\%$ [8].

Table 1: Volumes measured at NMISA with VC1005 and calculated volumes from KRISS results.

Nominal mass (g)	NMISA 2016		KRISS 2010		Agreement of results (%)
	Measured Volume (cm^3)	Calculated Uncertainty (cm^3)	Calculated Volume (cm^3)	Calculated uncertainty (cm^3)	
50	6.244 41	0.000 50	6.244 69	0.000 78	99.996
100	12.490 02	0.000 50	12.489 85	0.000 84	99.999
200	24.980 12	0.000 50	24.979 52	0.001 28	99.998
500	62.444 93	0.000 71	62.444 49	0.002 89	99.999
1 000	125.843 33	0.001 12	125.842 07	0.005 54	99.999

Table 2: Densities measured at NMISA with VC1005 and at KRISS.

Nominal mass (g)	NMISA 2016			KRISS 2010		Agreement of results (%)
	Measured Density (kg/m^3)	Calculated Uncertainty (kg/m^3)	Accredited CMC (kg/m^3)	Density (kg/m^3)	Uncertainty (kg/m^3)	
50	8007.16	0.32	8.00	8006.80	1.00	99.996
100	8006.38	0.32	5.00	8006.49	0.54	99.999
200	8006.37	0.16	5.00	8006.56	0.41	99.998
500	8007.05	0.09	5.00	8007.11	0.37	99.999
1 000	7946.40	0.07	5.00	7946.48	0.35	99.999

It can be seen in Table 1 and Table 2 that the volume and density determined by NMISA in year 2016 are in agreement with those determined by KRISS in year 2010 with an average percentage agreement of greater than 99.998%. There is significant improvement in the density uncertainties compared to the currently accredited CMCs for NMISA. Graphical representation of densities in Figure 5 shows good overlap between NMISA's and KRISS' results.

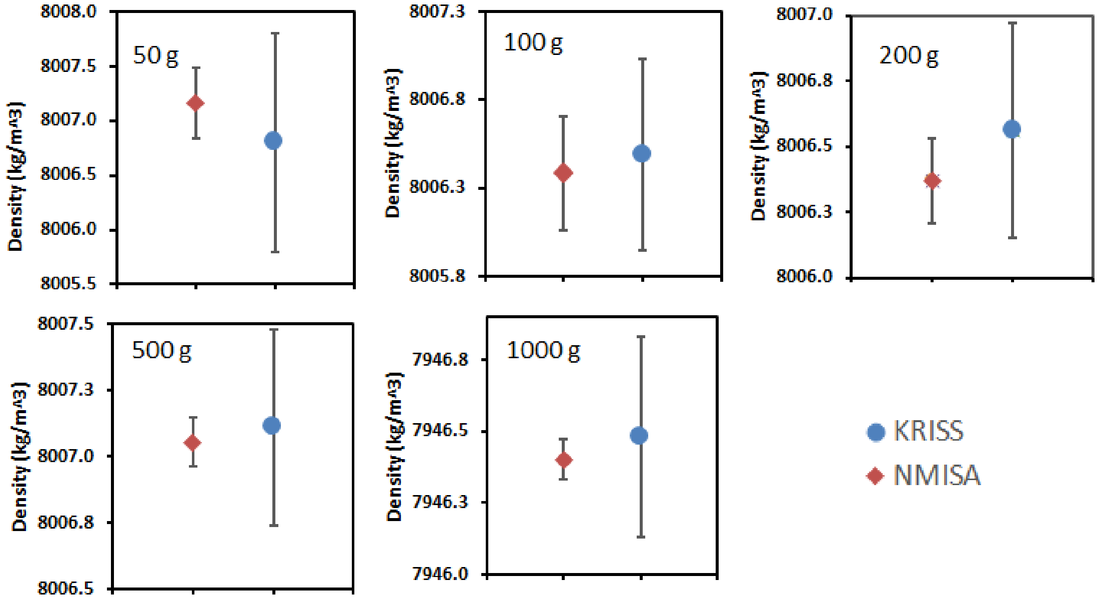

Figure 5: Graphical representation of the measured densities.

5. Conclusions

We have accurately measured the volumes and densities of weights ranging from 50 g to 1000 g using the automated volume comparator VC1005. The results for density show a significant improvement on the accredited measurement uncertainties for NMISA. Through this work, we have demonstrated a new achievement of high accuracy volume and density measurements for South Africa by the hydrostatic weighing method. Moreover, NMISA's results for volume and density are in agreement with the results that were found by KRISS.

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