Student difficulties in vectors: foothold ideas

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Abstract. This investigation was conducted with the aim of developing a research based teaching strategy in vectors for non-major physics students. The study reports the explanations used by 132 first year university students to answer five questions without context in two dimensions. The two angled analysis of the methods and explanations used for answering the five questions show lack of understanding of basic vector concepts and inappropriateness in the methods used to solve the problems. The findings indicate that the majority of students used one method and reasoning in all questions irrespective of its suitability and thus answered a few questions correctly and others incorrectly. This paper describes the methods used by students and the productive foothold ideas identified.

1. Introduction

To understand many physics concepts, at any level, it is vital to have a good understanding of vectors. The vectors are introduced, in first year university courses and high schools, by the definition of vectors and scalars. After that the distance and displacement followed by speed and velocity are discussed as examples. This is followed by describing the motion of an object to a distance in one direction and then to another direction, thereafter to find the distance between the initial position and final position to define the resultant displacement. This is easily done by the method of head to tail method rather than parallelogram method. Thus the first step in the introduction of the resultant is done using the method of head to tail followed by parallelogram (without much emphasis in the latter).

Nguyen and Meltzer [1] investigated the understanding of vector addition presented in graphical format, in terms of magnitudes and directions. The seven question set used in the study, a few of them were presented with grid and others without grid, requested explanations for the answers provided. They found that majority of the students were unable to perform vector additions correctly. Hawkins, Thompson and Wittmann [2] investigated the vector addition skills of students using interviews. They used two dimensional graphical vector questions to obtain different solutions and found that most of the students used only one method to answer all questions irrespective of the suitability. Flores, Kanim and Kautz [3] used two dimensional questions and interviews in their studies and found that students lack the ability to reason vectors after traditional instructions. They suggested that modifications in the instruction could improve the student understanding of vector additions. Shaffer and McDermott [4] found that students were able to solve vector problems better without the real life situations. Barniol and Zavala [5] investigated the effect of context and position of vectors in two dimensions. They found that the student responses are contextually dependent. In a similar note, Southey and Allie [6] investigated the student responses in different vector contexts: force, displacement and momentum. They found that additions of different physical vectors are not perceived as the same by majority of students. From these studies it is evident that students have difficulties, however, it is not clear that the reason for these difficulties at a fundamental level is addressed in the instruction. The present study tries to identify the methods and reasoning used by students while solving simple one and two dimensional problems without any physical context. The absence of physical context in the study enabled us to focus on the fundamental aspects in vectors to identify the “ideas” that helped the “correct” students to come to the acceptable answer. The result can be interpreted using the “knowledge in pieces” perspective.

2. Methodology

The present study is trying to probe the aspects in vectors by analysing the student responses to a single situation in which no physical contextual changes were made but only the direction. A questionnaire consisted of five questions was used in the study. All questions were presented in graphical representation and each question consisted of two vectors of different magnitudes and directions. Of the five, four were in two dimensions and the last was in one dimension orientated in the same direction. Thus the study is trying to answer the following questions:

1. What are the methods used by students in answering the questions, namely the parallelogram method, head to tail method or resolution of components method or any other method?
2. While answering the questions, did they use graphical methods in the process or used some other familiar “concepts” from their prior knowledge?
3. What are the ideas used by the students to answer each question irrespective of its correctness?

2.1. Instrument

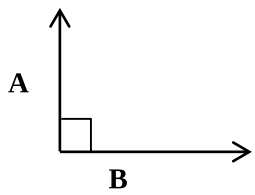
A five question instrument was used in the probe. Each question consisted of two vectors of different magnitudes. While keeping one of the vectors with the same magnitude and horizontal direction in all five questions, the direction of the second vector was changed in each question i.e. the angle between the vectors changed between $0^{!}$ and $180^{!}$. The two vectors were connected tail to tail in all questions, except the last one. All questions were presented graphically without a grid and the possible answers to the resultant vector were given as options. The students were requested to choose one of the given options and explain in detail the reason for choosing the particular option. The answer to the resultant was always related to the previous question and answer, except the first. The first question asked for the magnitude of the resultant vector of two vectors acting perpendicular to each other. The following two questions had the angle between the vectors was changing between $0^{!}$ and $90^{!}$, and the fourth question formed an angle greater than $90^{!}$. In the last question both vectors were parallel to each other. The format of a question is shown in Figure 1. The full questionnaire is presented in Figure 2.

For the five questions below, choose the resultant of the two vectors. Circle the BEST ANSWER of the given options and explain your reasons: $\vec{A} = 3\ units\ and\ \vec{B} = 4\ units$

1. The resultant $\overrightarrow{R_1} = \ldots$

A. 1 unit B. 3 units C. 4 units

D. 5 units E. 7 units F. none of the above

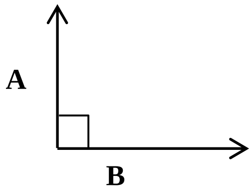


Explain your answer………………………………………………………………………………………….
………

Figure 1: Format of a question used in the study.

For the five questions below, choose the resultant of the two vectors. Circle the BEST ANSWER of the given options and explain your reasons: $\vec{A} = 3\ units\ and\ \vec{B} = 4\ units$

1.



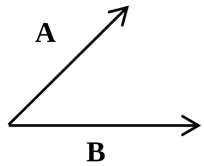
The resultant $\overrightarrow{R_1}$ = …

A. 1 unit B. 3 units C. 4 units
D. 5 units E. 7 units F. none of the above

Explain your answer……

………

2.

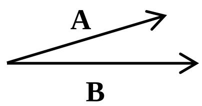


The resultant $\vec{R}_2$ = …

A. smaller than $\overrightarrow{R_1}$ B. bigger than $\overrightarrow{R_1}$
C. equal to $\overrightarrow{R_1}$ D. none of the above

Explain your answer……

3.

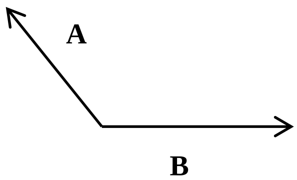


The resultant $\vec{R}_3$ = …

A. smaller than $\vec{R}_2$ B. bigger than $\vec{R}_2$
C. equal to $\vec{R}_2$ D. None of the above

Explain your answer……

4.

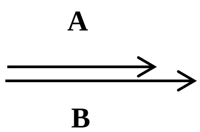


The resultant $\vec{R}_4$ = …

A. smaller than $\overrightarrow{R_1}$ B. bigger than $\overrightarrow{R_1}$
C. smaller than $\overrightarrow{R_2}$ D. bigger than $\overrightarrow{R_2}$
E. none of the above

Explain your answer……

5.



The resultant $\vec{R}_5$ = …

A. smaller than $\vec{R}_4$ B. equal to $\vec{R}_4$
C. bigger than $\vec{R}_4$ D. bigger than $\vec{R}_3$
E. smaller than $\vec{R}_1$ F. bigger than $\vec{R}_1$
G. None of the above

Explain your answer……

Figure 2: The questionnaire used in the study.

The test was conducted during a physics lecture period, as a weekly formative assessment. Students were told that they may use any type of explanation, in terms of either textual or graphical or a combination of both. A few students completed the test in fifteen minutes and others took more than 20 minutes.

2.2. Sample
The cohort consisted of 132 first year university students registered for various courses in the Science Faculty (non-physics major). All these students passed Physical Science (a combination of Physics and Chemistry) in Grade 12 and vectors were part of their syllabus in high school. The majority of the students were from rural schools, aged about 18 years, and for whom English is second or third language. The students received instruction on vectors and kinematics at the university before the test.

3. Result
The data was captured in a spread sheet that consisted of ten main columns (5 x 2). The answer to each question was recorded as two parts in each column: (i) the forced choice responses and (ii) the written responses.

3.1 Forced Choice Responses
Majority of the students answered questions 1 and 5 correctly. 94% chose the answer 5 units in Q1 and 84% chose the answer 7 units in Q5. The reason given by all students for Question 1 was the same with the Pythagoras theorem being stated as the reason. The two reasons for question 5 were: “sum” of two vectors and vectors are in the “same direction”. Since the majority of them answered these questions correctly, with acceptable reasoning, no further analysis was done on these two responses. Thus the questions 3, 4 and 5 was analysed in detail. Figure 3 shows the full responses of the cohort. The first set of three bars shows that each of the questions were answered correctly by almost half of the students. However, only 10% of the students answered all questions correctly.

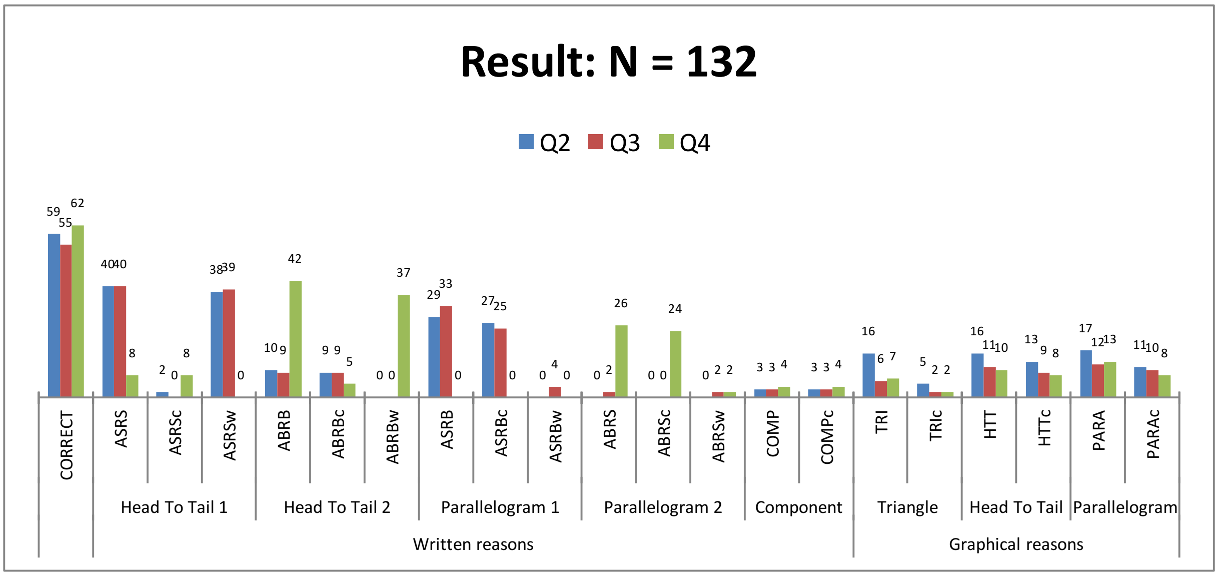


Figure 3: Student reasoning responses for the three questions: the two versions of head to tail, parallelogram and component method are presented including graphical reasons.

3.2 Free Writing Response
The majority of students used different written reasons to explain their answers. However, they did not use any method consistently or did not apply any method appropriately in all three questions. Instead

they used different reasoning without properly analysing the situation at hand. Some used only words to explain and others used words and drawings; the drawings they used were also different from question to question. Thus the free writing responses provided by students were divided into two categories: Written reasons (WR) and Graphical reasons (GR), as shown in Figure 3. The blue, red and green bars represent the number of responses for questions 2, 3 and 4 respectively.

3.2.1. Reasoning in Words (WR)

The analysis of free writing responses revealed that most of the students were familiar with the head to tail method. The different explanations in words they used were categorised into five reasoning categories: (i) when the angle is small, the resultant is small (ASRS) (ii) when the angle is big, the resultant is big (ABRB) (iii) when the angle is small, the resultant is big (ASRB) (iv) when the angle is big, the resultant is small (ABRS) and (v) resolution of components (COMP).

The first two reasoning can be related to head to tail method and the third and fourth can be related to parallelogram method. Of the 40 students who used the category ASRS, almost all of them answered Q2 and Q3 incorrectly (ASRSw); however, the few who used this category in Q4 answered it correctly (ASRSc). Of the 42 students who used the category ABRB in Q4, most of them answered incorrectly (ABRBw), while a few who used this category in Q2 and Q3 answered all correctly (ABRBc). The subscripts c and w represent the correct and wrong answers respectively.

A fifth of the cohort used the category ASRB in Q2 and Q3. Of this, more than 80% answered these questions correctly (ASRBc). However, none of them used this category in Q4. The category ABRS used by a few students in Q4; all answered correctly.

All students who used the components reasoning (COMP) answered all questions correctly although this group was very small in number.

3.2.2. Graphical Reasoning (GR)

An average of 10% of students used written reasons and graphical reasons in combination for their reasoning. The graphical reasons are subdivided to three categories: closed Triangle (TRI), head to tail (HTT) and parallelogram (PARA). More than 70% of the students who used correct graphical methods, HTT and PARA, answered all questions correctly and those who used TRI explanation (which itself is an incorrect method) answered most of the questions incorrectly.

4. Discussion

The general features of all questions used in this probe were the same. All questions were presented in graphical form, in which the vectors were connected tail to tail in two dimensions. The only variation in each question was the angle between the two vectors. Since the vectors were connected tail to tail, it would have been easier to answer these questions by employing the parallelogram method in all cases. However, it was interesting to note that most of the students used head to tail method rather than parallelogram or components method. Even though, most of them were familiar with head to tail method, they were unable to use this method correctly in all situations. In general, students who used only the written explanation in all three questions did not answer all of them correctly. For example, the idea of "angle small resultant small" and "angle big resultant big" are correct if used in the head to tail method and incorrect if used in the parallelogram method. Similarly, the idea "angle small resultant big" and "angle big resultant small" are correct if used in the parallelogram method and incorrect if used in the head to tail method. In summary the students who used the head to tail method (ASRS, ABRB) answered incorrectly and the students who used the parallelogram method (ASRB, ABRS) answered correctly and all students who used the components method answered all questions correctly. The reason for this phenomenon may be attributed to the majority of college physics (and high school) textbooks introducing scalars and vectors using the concepts of distance and displacement. While doing so, it is reasonable to start with two displacements, one after the other and ask for the distance between the initial position and final position to determine the resultant displacement. Thereafter, the parallelogram method (may be followed by components method) is introduced but not emphasis. This initial process may be strongly embedded in the minds of the students as the best method, although, this method may not work efficiently in the context of forces. These variations in context and misunderstanding or partial understanding of different methods used in different contexts and the confusion between them can be explained using the framework of the 'knowledge in pieces' perspective.

Thus from the study we can argue that the students who used the parallelogram method and the component method performed significantly better than the students who used the head to tail method. This was evident both in terms of the written explanations and graphical explanations. Thus the parallelogram method and component method seems to be the productive foothold ideas in understanding vectors. These results have huge teaching implications. At this stage, it can be suggested that introducing vectors with forces in two dimensions using the parallelogram method and component methods may be a better strategy to introduce vector addition. Furthermore, the context in which students frame these questions needs to be ascertained by investigating if they use a mathematical or a physical context.

Future research is considered to include a post test after the feedback and with more than two vectors.

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