Coupled-channel studies of nucleon scattering from oxygen isotopes

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Abstract. The Multi-Channel Algebraic Scattering (MCAS) method for the description of nucleon-nucleus scattering has been used with a (collective) rotational model of structure describing the target. The success of that model, when incorporating the Pauli Principle in the interactions describing the scattering and the formation of the compound systems, has been quite good. We extend that method to include the vibrational model in describing the target states, and apply the method to the scattering of low-energy nucleons from oxygen isotopes. Preliminary results for neutron scattering from \textsuperscript{16}O, leading to states in \textsuperscript{17}O will be reported.

1. Introduction

The structures of \textsuperscript{17}O and \textsuperscript{17}F play an important role in the synthesis, in the stellar environment, of elements beyond carbon. Once carbon burning begins, with successive proton capture reactions \cite{1} and $\beta$-decays, elements to nitrogen and oxygen are formed, as well as fluorine. Of particular note within the CNOF cycle are the reactions \cite{2}

\[
\begin{align*}
\textsuperscript{16}O + p &\rightarrow \textsuperscript{17}F \\
\textsuperscript{17}F &\rightarrow \textsuperscript{17}O + e^+ + \nu \\
\textsuperscript{17}O + p &\rightarrow \textsuperscript{14}N + \alpha \\
\textsuperscript{17}O + p &\rightarrow \textsuperscript{18}F + \gamma,
\end{align*}
\]

the last reaction being the breakout from the CNO cycle. Proton capture on \textsuperscript{16}O can proceed via either the ground state of \textsuperscript{17}F, capture to the $d_{5/2}$ level, which has a $Q$ value of 600 keV, or via the first excited state, capture to the $s_{1/2}$ level, with a $Q$ value of 105 keV. It is likely to proceed via the first excited state, as that state is a proton halo \cite{3}.

Yet the two nuclei are of interest for their own sake. They are each a single nucleon outside an \textsuperscript{16}O core, and are mirror nuclei, with the first three positive-parity states reflecting the single particle energies of the $0d_{5/2}$, $1s_{1/2}$, and $0d_{3/2}$ levels in the $sd$-shell model. But the model for each nucleus is not so simple: in a $(0 + 2)\hbar\omega$ prescription, there is significant admixing of $2\hbar\omega$
components, \( \sim 25\% \), in the ground states. This largely stems from 2p-2h components giving rise to additional nucleons in the \( sd \) shell. With this in mind, it is instructive to compare the extreme shell model picture, with one particle in the \( sd \) shell, or the more general \((0 + 2)h\omega\) model, to the collective model description contained in the Multi-Channel Algebraic Scattering theory (MCAS) [4], which describes low-energy nucleon-nucleus scattering, and the resonances (both bound-state and scattering) in the compound nucleus. For the mass-17 nuclei, the model would entail coupling a single nucleon to the \( ^{16}\text{O} \) core. This is, however, not a trivial exercise, as it is well-known that the description of the spectrum of \(^{16}\text{O}\) requires a \( 4h\omega \) shell model at the minimum [5, 6, 7].

2. MCAS
MCAS has been described in detail elsewhere [4], and so we present a brief description for the purposes of highlighting those aspects which are of relevance to the present calculation. The method is a means of solving the coupled-channel Lippmann-Schwinger (LS) equations, describing a coupled two-cluster system, in momentum space. For the most part, it is assumed that the two clusters are a nucleon and an even-even nucleus. It is also assumed that the target (core) is described by a collective model, which allows for the specification of the matrix of interaction potentials defining the coupled-channel problem. Nothing is assumed of the spectrum of the target states. The matrix of potentials is expanded in terms of sturmian functions, and a finite set of \( \sim 30 \) are used to ensure convergence. Pauli-exclusion is handled by the use of orthogonalising pseudo-potentials (OPP) with weights of \( 10^6 \text{ MeV} \) to guarantee that the sturmians are orthogonal to any states corresponding to a nucleon coupling to a filled orbit in the target [8, 9].

Once the sturmians and OPPs are set, the matrix of potentials is re-expressed as a sum of separable potentials in momentum space as input to the LS equations. As the equations are expressed in momentum space, solutions of the LS equations may be found for both bound (negative energy) and scattering (positive energy) states in the compound system. Energies and widths are found for the states in the compound system, assuming that the channels specified are only those of a nucleon coupled to a target nucleus, in which case the widths are partial widths corresponding only to nucleon emission from the compound nuclear states.

3. Shell model considerations
While the spectrum of \(^{16}\text{O}\) requires a full \( 4h\omega \) shell model for description [5], the ground state found by Brown and Green is dominated by \( 0h\omega \) and \( 2h\omega \) components which correspond approximately to those found from a pure \((0 + 2)h\omega \) shell model [7]. In that respect, we may calculate the spectra of \(^{17}\text{O}\) and \(^{17}\text{F}\) in a \((0 + 2)h\omega \) model space, for the positive parity states, and a \((1 + 3)h\omega \) model space for the negative parity states. In both sets of calculations all shells from the \( 0s \) to the \( 0f1p \) are used, with all particles active. We calculate the spectrum using OXBASH [10] with the WBP interaction of Warburton and Brown [11]. Any calculations of transition rates between states in \(^{16}\text{O}\) to indicate the strength of the couplings, however, will require a full \( 4h\omega \) model space calculation to ensure a proper description of the \(^{16}\text{O}\) spectrum; that is work in progress. The resultant spectrum, together with the known spectra for \(^{17}\text{O}\) and \(^{17}\text{F}\) [12], is shown in Fig. 1.

It is clear that the spectrum obtained from the shell model compares well with both spectra. Discrepancies between the model spectrum and the known spectra may be due to limitations in the model space and/or the underlying limitation on the ground state of \(^{16}\text{O}\). Nevertheless, this result serves to illustrate that the extreme single-particle picture of the mass-17 system is too simplistic. It points to the need for a coupled-channel description of the nuclei, with as many states in the target \((^{16}\text{O}\)) spectrum included in the coupling as possible.
4. MCAS considerations

The MCAS has been applied to $n + ^{16}O$, leading to compound states in $^{17}O$, using the vibrational model and 5 target states in $^{16}O$, namely the ground state, the $0_{2}^{+}$ state (6.049 MeV), the $3_{1}^{-}$ state (6.130 MeV), the $2_{1}^{+}$ state (6.917 MeV), and the $1_{1}^{+}$ state (7.117 MeV). The parameters for the Woods-Saxon functions used in the calculation are $R_0 = 3.15$ fm, and $a = 0.65$ fm. The potential parameters (positive and negative parity) are, in units of MeV,

$$
V_{0}^{-} = -47.50; V_{0}^{+} = -50.50 \\
V_{ll}^{-} = 2.55; V_{ll}^{+} = 0.00 \\
V_{ls}^{-} = 6.90; V_{ls}^{+} = 7.20 \\
V_{ss}^{-} = 2.50; V_{ss}^{+} = -2.0 ,
$$

with two deformation parameters, $\beta_2 = 0.21$ and $\beta_3 = 0.42$. To obtain the spectrum of $^{17}F$, we add a Coulomb potential. Also, while Pauli blocking of the 0$s$ and 0$p_{\frac{3}{2}}$ orbits have been incorporated by the addition of the OPP, Paul hindrance [9] has also been included for the higher orbits. In particular, this includes the 0$p_{\frac{1}{2}}$ orbit, to account for the ground state of $^{16}O$ being $4h\omega$ in character. It should be noted that the results presented herein from these calculations are preliminary.

Fig. 2 shows the spectra of $^{17}O$ and $^{17}F$ as compared to the result obtained from MCAS. Agreement with the known spectra is quite good, with the low-lying states well-reproduced. Comparison to Fig 1 shows the results from MCAS agree well also with the results from the shell model. Above the nucleon thresholds, the density of states make identification of states difficult, though the trends in the groupings of states in the known spectra are reproduced. Changes to the Coulomb potential parameters have little influence on the spectrum of $^{17}F$.

The low-energy neutron scattering cross section from $^{16}O$ is shown in Fig. 3. It is clear from Fig. 3 that the cross section to 300 keV obtained from MCAS is in agreement with the...
Figure 2. Spectra of $^{17}$O and $^{17}$F obtained from MCAS, as compared to the known spectra. Notation is as for Fig. 1, and the zero energy corresponds to the nucleon scattering threshold.

Figure 3. Low-energy neutron scattering cross section from $^{16}$O, showing states in $^{17}$O. Note that the energy is on a log-scale.
data. However, the resonance observed at 450 keV is not in the correct position; the MCAS result places it at 1 MeV. That resonance corresponds to the $3/2^-$ state in $^{17}\text{O}$. Further work is continuing in order to understand this discrepancy. Work is also continuing on the low-energy proton scattering cross sections with regards to states in $^{17}\text{F}$.

5. Conclusions

We have calculated the spectra of $^{17}\text{O}$ and $^{17}\text{F}$ from the shell model and also from MCAS. Results of both models agree well with the known spectra. However, preliminary results for the neutron scattering cross section from $^{16}\text{O}$, leading to states in the compound $^{17}\text{O}$ does not reproduce the resonances observed. Further work is being pursued to understand this discrepancy. This has to be resolved before investigating the proton scattering cross sections, leading to states in $^{17}\text{F}$.

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