Nonplanar Integrability at Two Loops

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Introduction

We calculate the action of the two loop dilatation operator of $\mathcal{N} = 4$ super-Yang-Mills (SYM) theory on a class of operators known as restricted Schur polynomials. We perform these calculations in a large N nonplanar limit of the theory, on operators in the $su(2)$ sector.

We do this in order to determine the two-loop quantum corrections, known as anomalous dimensions, to the mass scaling dimension of these operators.

Introduction

It was possible to show that the spectrum of anomalous dimensions corresponded to a set of decoupled harmonic oscillators, indicating integrability in this sector of the theory. Another important result was that the anomalous dimensions found at two loops were additively corrected. The operators were not corrected.

Why study anomalous dimensions?

- Integrability can be shown for systems in $\mathcal{N} = 4$ SYM if the spectrum of anomalous dimensions is evenly spaced. This is because systems with evenly spaced spectra, resembling harmonic oscillators, are integrable.
- The AdS/CFT correspondence relates gauge theories to string theories. $\mathcal{N} = 4$ SYM is dual to type IIB string theory on an $AdS_5 \times S^5$ background. An evenly spaced spectrum of anomalous dimensions corresponds to a free string energy spectrum.
- We study AdS/CFT in order to study quantum gravity, since it allows us to do quantum gravity calculations in gauge theories and vice versa.

Anomalous dimensions are quantum corrections to the classical mass scaling dimension.

Note:

The anomalous dimension of an operator in a physical theory is something that can be measured through experiment. It describes how a system approaches critical points during phase change.

For example, all spins in a system will align at a certain temperature. The correlation length describing the distance between two aligned spins could be given by

$$< \mathcal{O}(x)\mathcal{O}(y) >= \frac{1}{|x-y|^{2(\Delta_0+\gamma)}}$$

This diverges as the system approaches the critical point.

$\mathcal{N} = 4$ SYM

This theory can be viewed as a toy model of QCD in that it is a renormalizable gauge theory with matrix valued fields. It is a simpler theory to study since is has additional super and conformal symmetries. The concept of mass as seen in QCD is replaced by a mass scaling dimension.

The Lagrangian of the theory has the following form:

$$\begin{aligned}\mathcal{L} \;=\; & -\frac{1}{4}Tr(F^{\mu\nu}F_{\mu\nu}) - \frac{1}{2}Tr(D^{\mu}\phi^{n}D_{\mu}\phi_{n}) \\ & -\frac{1}{4}g^{2}Tr\left[\phi^{m},\phi^{n}\right]\left[\phi_{m},\phi_{n}\right] + Tr(\dot{\psi}^{a}_{\dot{\alpha}}\sigma^{\dot{\alpha}\beta}_{\mu}D^{\mu}\psi_{\beta a}) \\ & -\frac{ig}{2}Tr(\psi_{\alpha a}\sigma^{ab}_{m}\epsilon^{\alpha\beta}\left[\phi^{m},\psi_{\beta b}\right]) - \frac{ig}{2}Tr(\dot{\psi}^{a}_{\dot{\alpha}}\sigma^{m}_{ab}\epsilon^{\dot{\alpha}\dot{\beta}}\left[\phi_{m},\dot{\psi}^{b}_{\dot{\beta}}\right])\end{aligned}$$

The operators that we used are built out of the six Higgs fields, ϕ_i.

The large N nonplanar limit

In quantum field theories of matrices, Feynman diagrams are known as ribbon graphs. Propagators are pairs of parallel lines, or ribbons, rather than single lines.

Planar Limit

Planar Feynman diagrams are those which can be drawn on a planar surface without any ribbons crossing. Nonplanar diagrams are diagrams which must be drawn on surfaces with holes, such as toruses, to avoid this. Summing only planar diagrams is a classical limit in quantum gravity.

- In QFT, loop corrections correspond to $\hbar$ corrections. $\hbar$ corrections in quantum gravity are dual to $\frac{1}{N^2}$ corrections in the gauge theory, where $N \times N$ is the size of matrices in the gauge group.
- A large N limit is a classical limit; sending $N \to \infty$ is like sending $\hbar \to 0$.
- It was thought to be the same limit as the planar limit, but it turns out that this is not the case. When operators are of dimension $\mathcal{O}(N)$, nonplanar diagrams must also be summed due to large combinatoric factors which arise when there are many fields being contracted.
- The fact that we are working in a large N nonplanar limit indicates that we are including stringy quantum corrections in the gravity theory.

The Dilatation Operator

- The action of the dilatation operator on an operator produces the anomalous dimensions of the operator as its eigenvalues.
- Anomalous dimensions are the quantum corrections to the conformal dimension, Δ. This is analogous to quantum corrections to the mass in other QFT's.
- We looked at the second loop correction to Δ in a large N nonplanar limit.

Integrability

An integrable system is one with as many conserved quantities as there are degrees of freedom. If a quantum mechanical system is integrable one can determine the spectrum of its Hamiltonian and its scattering matrix.

- Integrability of a quantum field theory is difficult to check for, since there are infinite degrees of freedom. On the gauge theory side of AdS/CFT, a way to do this is to allow the dilatation operator to act on operators, and to study the spectrum of anomalous dimensions that this produces.
- If this spectrum resembles the energy spectrum of an harmonic oscillator, the theory is integrable.

Integrability

- It is conjectured that $\mathcal{N} = 4$ SYM is integrable in the planar limit.
- In more recent work, it was also shown that certain classes of operators of $\mathcal{N} = 4$ SYM are integrable to one loop, so our goal was to test this to higher loops.
- AdS/CFT predicts that we can perform calculations in gauge theories to obtain quantum gravity results. Checking for integrability is a useful way to test this prediction, since it displays the relation between anomalous dimensions and free string energies.

Operators

The operators that we studied are known as restricted Schur polynomials. Their general form is as follows:

$$\chi_{R,(r,s)\alpha\beta}(Z,Y)$$

- $Y = \phi_3 + i\phi_4$ and $Z = \phi_5 + i\phi_6$, where these ϕ fields are scalar matrix fields from the action of $\mathcal{N} = 4$ SYM.
- These operators are labelled by Young diagrams, R. These are representations of the symmetric group S_{n+m}.

For example:

$R =$ 

Group Representation Theory

- Elements of the symmetric group permute numbers. For example, the element $(1,2,3)$ means

$$1 \rightarrow 2 \rightarrow 3 \rightarrow 1$$

- A group representation is a mapping from an element of a group, $g \in G$ to some matrix ρ. ρ is an element of the general linear group, which is a group of $n \times n$ matrices on an n dimensional vector space that is closed by matrix multiplication.
- A representation of a group is not unique, but it must satisfy

$$\rho(g_1 g_2) = \rho(g_1)\rho(g_2) \quad g_1, g_2 \in G$$

$R \to (r, s)$

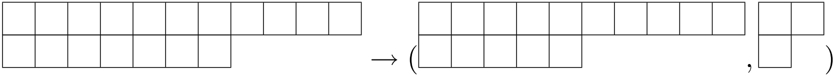
- The label $R \to (r, s)$ indicates that we are projecting from a representation of S_{n+m} to a representation of $S_n \times S_m$. The label $\alpha\beta$ indicates which copy of the representation (r, s) is being used.

If

$R =$ 

the label (r, s) indicates the decomposition of R into two Young diagrams:

$$R \to (r, s)$$



Giant Gravitons

- Schur polynomials labelled by a Young diagram R which has $\mathcal{O}(1)$ rows or columns of lengths $\mathcal{O}(N)$ are dual to giant gravitons. These are $D3$ branes with energy of order N.
- In a Young diagram, each block can be labelled with a factor $N + i - j$, where i is the column number and j is the row number.
- If the factors in the corner boxes have a difference that goes to ∞ in the large N limit, we call this the displaced corners approximation. We concentrated on operators in this approximation since it allowed us to use some simplifying properties of the large N limit.

Each row of the Young diagram R labeling the Schur polynomial corresponds to a giant graviton. The length of the row is related to the size of the graviton.

The fact that we are studying Young diagrams in the displaced corners approximation means that the gravitons have large differences in size, and are thus well separated, or non-interacting.

Since they are non-interacting, one expects that these branes would have evenly spaced energy spectra, like harmonic oscillators.

- We concentrated on operators in the $su(2)$ sector (only Y and Z fields present), in which there are Schur polynomials containing a large number n of Z fields, known as the background, and a small number m of Y fields, known as impurities.
- This configuration can also be thought of as a giant graviton with strings attached.
- It has been shown to one loop that there are evenly spaced spectra of anomalous dimensions corresponding to giant graviton energy spectra. We wished to find out whether this was the case at two loops.

Calculation

Dilatation Operator

The $\mathcal{N} = 4$ SYM two loop dilatation operator has the following general form:

$$\begin{aligned} D_4 &= -g_{YM}^4 Tr\left(\left[\left[Y, Z\right], \partial_Z\right]\left[\left[\partial_Y, \partial_Z\right], Z\right]\right) \\ &\quad -g_{YM}^4 Tr\left(\left[\left[Y, Z\right], \partial_Y\right]\left[\left[\partial_Y, \partial_Z\right], Y\right]\right) \\ &\quad -g_{YM}^4 Tr\left(\left[\left[Y, Z\right], T^a\right]\left[\left[\partial_Y, \partial_Z\right], T^a\right]\right) \end{aligned}$$

This operator multiplies out to sixteen terms per piece, and the calculation of the action of each was done on a restricted Schur polynomial.

When a term from the dilatation operator acts on a Schur polynomial labelled by a Young diagram R, a box is being pulled off of R. In the case of the one loop operator, terms arise where R has one box removed to become R', whereas in the two loop case terms arise where $R \to R'$ and $R \to R''$.

The fact that we obtain terms with both one and two boxes removed would lead one to expect that the two loop operator would yield corrections to the one loop result. The two loop eigenvalue problem that was solved contained more eigenvalues than the one loop problem.

Result and Outlook

The two loop dilatation operator of $\mathcal{N} = 4$ SYM is integrable in the large N displaced corners approximation.

From this result, we could try to find a general way to show that the dilatation operator is integrable to higher loop corrections.

The anomalous dimensions have additive corrections to what was found at one loop. The operators are not corrected at two loops.

This leads to the hope that the operators are not corrected at any loop and that it would be possible to calculate exact anomalous dimensions.