Solitary Waves in Magnetized Plasma with Two Temperature Electrons

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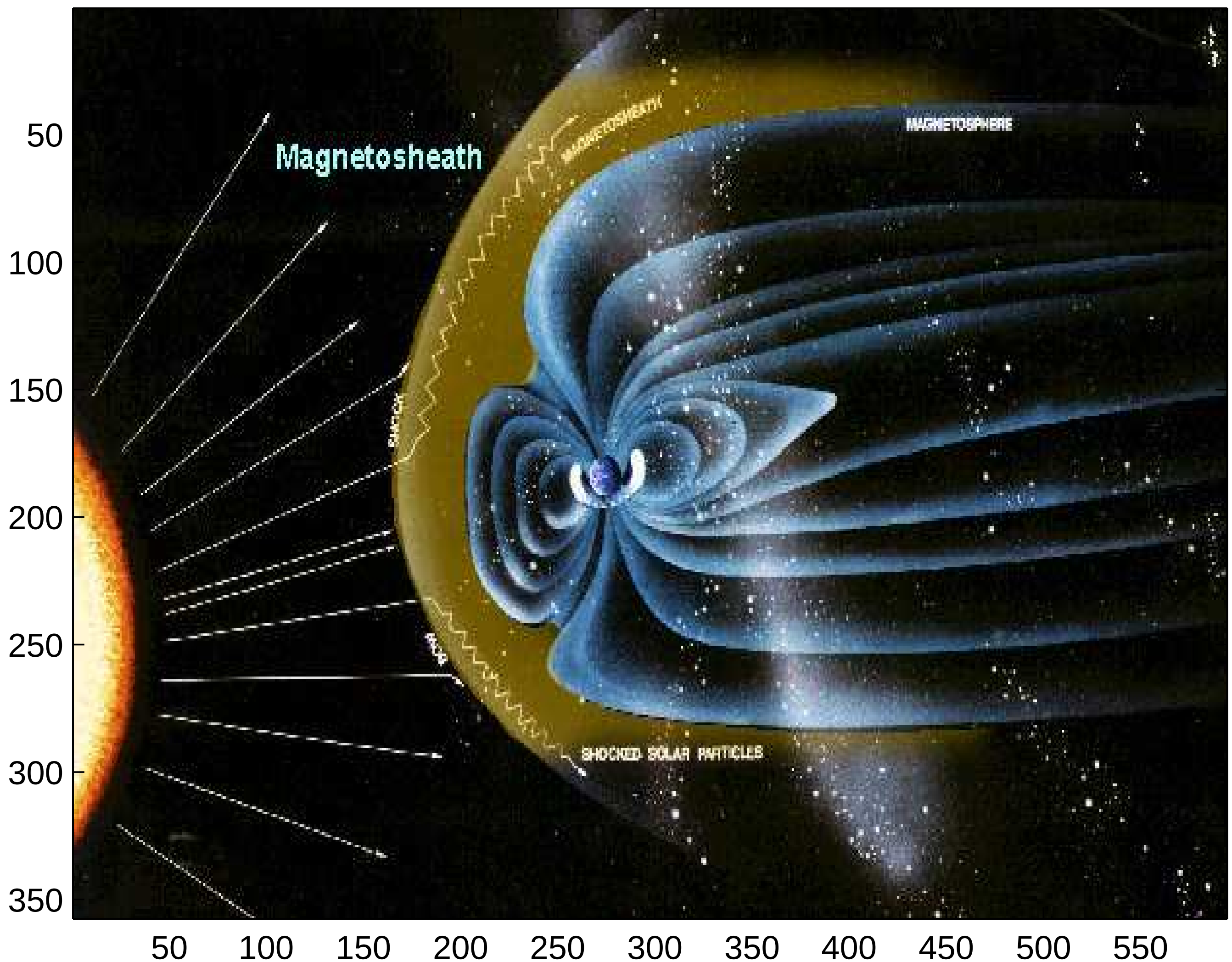
Introduction

S3-3 and Viking satellities have clearly indicated that the solitary waves stuctures are frequently observed along auroral magnetic field lines (Temerin et.al., 1982; Boström et.al., 1988; Berthomier et. al., 1998). Ergun et.al., (1998) reported the observation of large-amplitude electromagnetic structures from Fast Auroral Snapshot (FAST) satellite called “fast solitary waves”. Further evidence of nonlinear solitary stuctures in the presence of two electron components in the auroral plasma has been provided by Berthomier et.al., 1998.
Various theoretical models have been developed to describe the observed solitary wave structures at different regions of Earth’s magnetosphere (for example, Bharuthram et.al., 1986; Baluku et al., 2010).

Sun Earth connection solar wind plasma



Objectives

- To formulate a model for satellite observations of nonlinear structures observed along auroral magnetic field lines of Earth's magnetosphere.
- To analytically and numerically solve the nonlinear model for ion-acoustic solitary waves in a magnetized two-temperature electron population.
- To obtain the solitary wave structures using the Sadgeev pseudo-potential technique .
- To study the soliton structures as a function of plasma parameters.
- To relate the theoretical results to the actual satellite measurements.

Formulation

We consider a collisonless, magnetized plasma consisting of cold ions fluid and two distinct groups of electrons, a cool component electrons (temperature T_c) and a hot component electrons (temperature T_h). The density of the two electrons species is given by the Boltzmann distribution.

Model Formulation

- Cool Electrons

(1) $$N_c = N_{c0} \exp\left(\frac{e\phi}{T_c}\right)$$

- Hot Electrons

(2) $$N_h = N_{h0} \exp\left(\frac{e\phi}{T_h}\right)$$

Model Formulation cont.,

- Magnetized Cold ions

Continuty Equation

(3) $$\frac{\partial N_i}{\partial t} + \nabla(N_i V_i) = 0.$$

Momentum equation

(4) $$\frac{\partial V_i}{\partial t} + V_i \nabla V_i = -\frac{e\nabla\phi}{m_i} + e\frac{\mathbf{V}_i \times \mathbf{B}_o}{m_i c}.$$

Normalization Terms

Densities n_c , n_h are normalized with respect to the total electron density $N_{c0} + N_{h0} = N_0$, velocities V by $C_s = (T_{eff}/m_i)^{1/2}$ the acoustic speed, distance x by $\rho_i = c_s/\Omega$, time t by Ω^{-1}, where Ω is the ion gyrofrequency and potential ϕ by T_{eff}/e. Here temperature ratio $\tau = T_c/T_h$, cool density ratio $f = N_{c0}/N_0$, where $N_{j0} = (j = c, h, i)$ are the equilibrium densities, an effective temperature $T_{eff} = T_c/(f + (1 - f)\tau)$, $\alpha_c = T_{eff}/T_c$, $\alpha_h = T_{eff}/T_h$, $\psi = e\phi/T_{eff}$ and $\alpha = k_x/k = \sin\theta$, $\gamma = k_z/k = \cos\theta$ (θ is the propagating angle between $\mathbf{k} = (k_x, 0, k_z)$ and $\mathbf{B}_0$, in which $k = \sqrt{k_x^2 + k_z^2}$) and $n_i = n_c + n_h$.

Normalization Set of Equations

The equations in (1) - (4) are presented in normalized form

(5) $$n_c = f\exp(\alpha_c\psi).$$

(6) $$n_h = (1-f)\exp(\alpha_h\psi).$$

(7) $$\frac{\partial n_i}{\partial t} + \nabla.(n_i v_i).$$

(8) $$\frac{\partial v_i}{\partial t} + v_i\nabla v_i = -\nabla\psi + \mathbf{v}_i \times \hat{\mathbf{z}}.$$

Normalization Set., cont.,

For ion motion in the (x, z) plane in (7)-(8) yield

(9) $$\frac{\partial n_i}{\partial t} + \frac{\partial (n_i v_x)}{\partial x} + \frac{\partial (n_i v_z)}{\partial z} = 0$$

(10) $$\frac{\partial v_x}{\partial t} + \left(v_x \frac{\partial}{\partial x} + v_z \frac{\partial}{\partial z} \right) v_x = -\frac{\partial \psi}{\partial x} + v_y.$$

(11) $$\frac{\partial v_y}{\partial t} + \left(v_x \frac{\partial}{\partial x} + v_z \frac{\partial}{\partial z} \right) v_y = -\frac{\partial \psi}{\partial y} - v_x$$

(12) $$\frac{\partial v_z}{\partial t} + \left(v_x \frac{\partial}{\partial x} + v_z \frac{\partial}{\partial z} \right) v_z = -\frac{\partial \psi}{\partial z}$$

Localized Solution

Using transformation $\xi = (\alpha x + \gamma z - Mt)/M$, where M is the Mach number, to the stationary frame, equations (9)-(12) becomes

(13) $$\frac{d}{d\xi}(L_v n_i) = 0.$$

(14) $$L_v \frac{dv_x}{d\xi} = -\alpha \frac{d\psi}{d\xi} + M v_y.$$

(15) $$L_v \frac{dv_y}{d\xi} = -M v_x.$$

(16) $$L_v \frac{dv_z}{d\xi} = -\gamma \frac{d\psi}{d\xi}.$$

where $L_v = -M + \alpha v_x + \gamma v_z$

Localized Solution Cont.,

the system is closed with quasi-neutrality condition,

(17) $$n_i = f\exp(\alpha_c\psi) + (1-f)\exp(\alpha_h\psi)$$

solving equations (13)-(17) with the appropriate boundary conditions for solitary waves structures (namely, $n_i \to 1$, $\psi \to 0$, and $d\psi/d\xi \to 0$ at $\xi \to \pm\infty$), and eliminating v_x, v_y, and v_z, we can reduce (13)-(17) to

(18) $$\frac{1}{2}\left(\frac{d\psi}{d\xi}\right)^2 + V(\psi, M) = 0$$

The "Energy integral" of an oscillating particle of unit mass, with the velocity $\frac{d\psi}{d\xi}$ and the position ψ in a potential $V(\psi, M)$.

Sagdeev potential

$$V(\psi, M) = -\frac{1}{\left[1-M^2\left(\frac{\alpha_c f\exp(\alpha_c\psi)+\alpha_h(1-f)\exp(\alpha_h\psi)}{(f\exp(\alpha_c\psi)+(1-f)\exp(\alpha_h\psi))^3}\right)\right]^2} \times .$$

$$\left(\frac{M^4}{2n_i^2}(1-n_i)^2 - M^2(1-\gamma^2)\psi\right.$$

$$+M^2\left(\frac{f}{\alpha_c}(\exp(\alpha_c\psi)-1)+\frac{(1-f)}{\alpha_h}(\exp(\alpha_h\psi)-1)\right)$$

$$-\frac{\gamma^2}{2}\left(\frac{f}{\alpha_c}(\exp(\alpha_c\psi)-1)+\frac{(1-f)}{\alpha_h}(\exp(\alpha_h\psi)-1)\right)^2$$

$$\left.+M^2\gamma^2\left(\frac{\frac{f}{\alpha_c}(\exp(\alpha_c\psi)-1)+\frac{(1-f)}{\alpha_h}(\exp(\alpha_h\psi)-1)}{f\exp(\alpha_c\psi)+(1-f)\exp(\alpha_h\psi)}\right)\right).$$

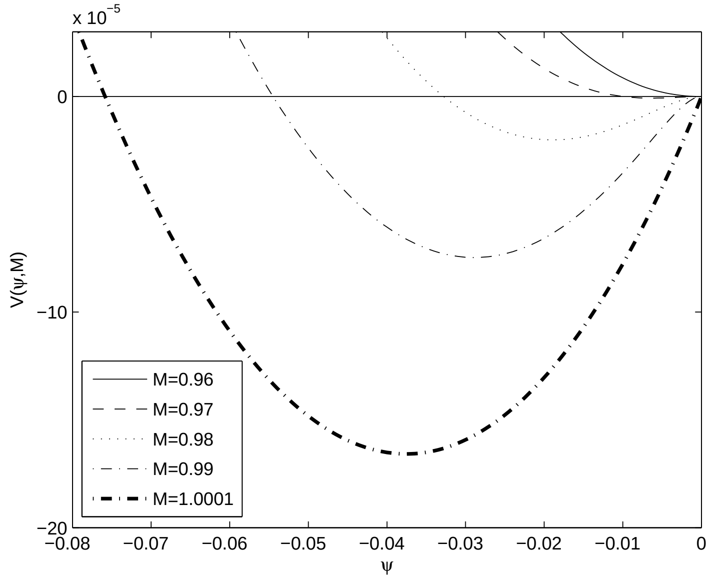
(19)

Soliton Characteristics

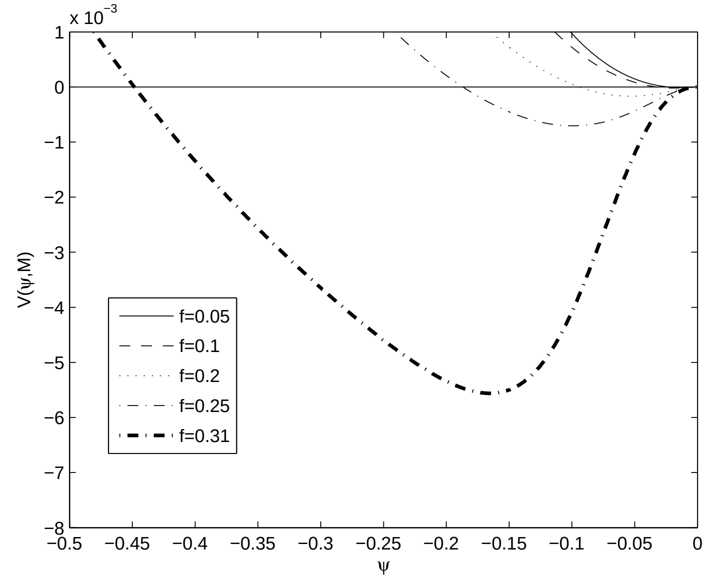
The origin at $\psi = 0$ defines the equilibrium state, which represents a local maximun of the Sagdeev potential $V(\psi, M)$. For solitary wave solutions of equation (18) to exist, the following requirements must be satisfied:

- $V(\psi, M) = \frac{dV(\psi,M)}{d\psi} = 0$ at $\psi = 0$.
- $\frac{d^2V(\psi,M)}{d\psi^2}|_{\psi=0} < 0$, the function $V(\psi, M)$ has a maximum at the origin, and
- $V(\psi, M) < 0$ for $0 < |\psi| < \psi_m$, where ψ_m is the maximum.

Results

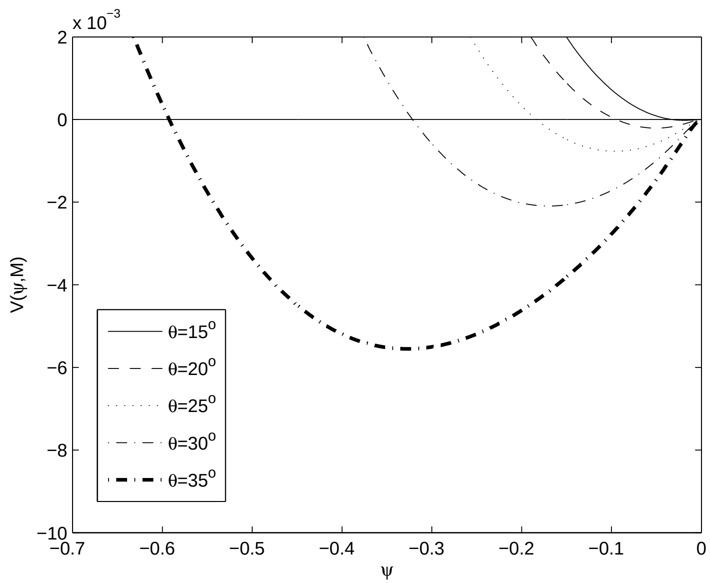


(a) τ=0.04,f=0.1,θ=15^o

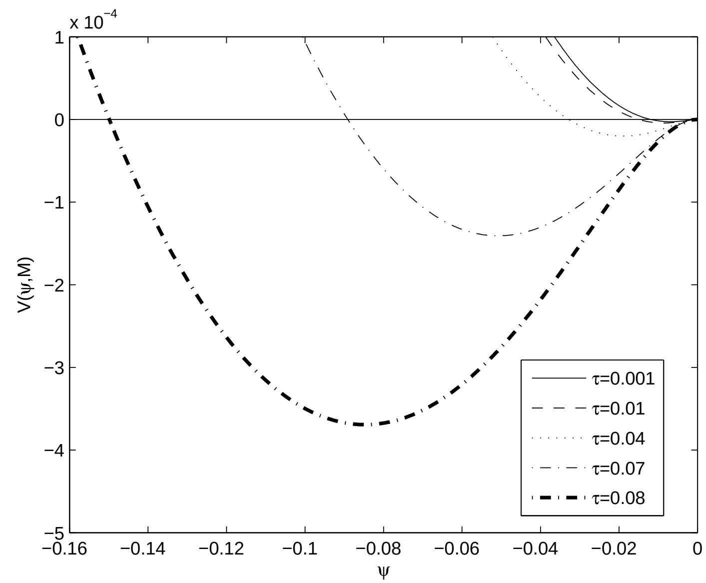


(b) τ=0.04,θ=15^o,M=0.98

Results Cont.,



(c) τ=0.04,f=0.1,M=0.98



(d) $f = 0.1$,$\theta = 15^o$,$M = 0.98$

Figure 1: The behavior of the Sagdeev potential $V(\psi, M)$.

Conclusions

- A finite amplitude theory for nonlinear ion-acoustic solitary waves in magnetized plasma is studied with cold ions and two Boltzmann distributed electrons species using the Sadgeev pseudpotential technique.
- The existence of Solitary waves with only a negative potential have been shown.
- As the Mach number M, density ratio $f = n_{c0}/n_{h0}$, temperature ratio $\tau = T_c/T_h$ and the propagating angle θ increases the amplitude increases.
- Our calculation provide good agreement with Viking satellite observations. (The input parameter values were taken from satellite data).

References

References

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Thank you