Linking nuclear masses with nucleon separation energies

S Karataglidis[1], K Amos[1,2], L Canton[3], P R Fraser[2,3], J P Svenne[4], and D van der Knijff[2]

[1] Department of Physics, University of Johannesburg, P.O. Box 524 Auckland Park, 2006, South Africa

[2] School of Physics, University of Melbourne, Victoria 3010, Australia

[3] Istituto Nazionale di Fisica Nucleare, Sezione di Padova, Padova I-35131, Italia

[4] Department of Physics and Astronomy, University of Manitoba, and Winnipeg Institute for Theoretical Physics, Winnipeg, Manitoba, Canada R3T 2N2

E-mail: [1]`stevenka@uj.ac.za`

Abstract. With the growing interest in masses of nuclei near the drip lines, and especially for those beyond the drip lines, we take a survey of mirror systems near the drip lines, where one of the mirror pair is unbound. Various methods are followed by which their masses may be determined. As an example, we consider the mass of ^{17}Na, and its energy relative to the $p+^{16}$Ne threshold.

1. Introduction

There is much interest in extending the knowledge of nuclear masses beyond the valley of stability. However, many of the masses of the exotic nuclei, especially those near or beyond the drip lines, are not known, and may never be known, given the limited availability of beams at Radioactive Ion Beam (RIB) Facilities, and even less information is available with regards to their spectra. So one looks to theory, such as the Multi-Channel Algebraic Scattering (MCAS) Theory [1] to fill in the gaps. With that theory, states in ^{15}F were predicted [2] from studies of low-energy $p+^{14}$O scattering. Those states were subsequently found in experiment [3].

Herein, we are interested in determining masses of exotic nuclei in three ways. The first method is to use mass formulae for nuclei within an isobar multiplet. That approach is (largely) model-independent. The second method relies on using the mirror system, which may have a well-measured spectrum, in determining the properties of the system in question. (Such was done to determine the spectrum of ^{15}F [2].) The third is to find trends among the known (measured) values from which to extrapolate to the drip lines. This last method serves as a guide to the more theoretic approaches.

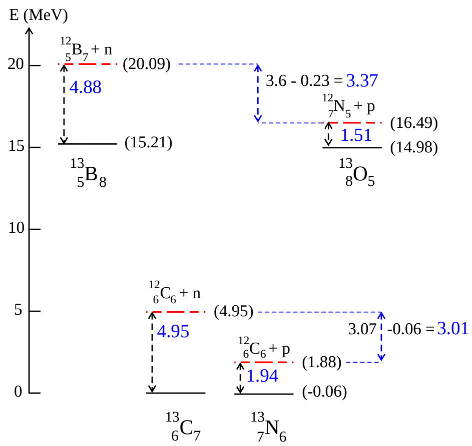

Figure 1. The mass-13 systems. All energies are relative to the ground state of ^{13}C.

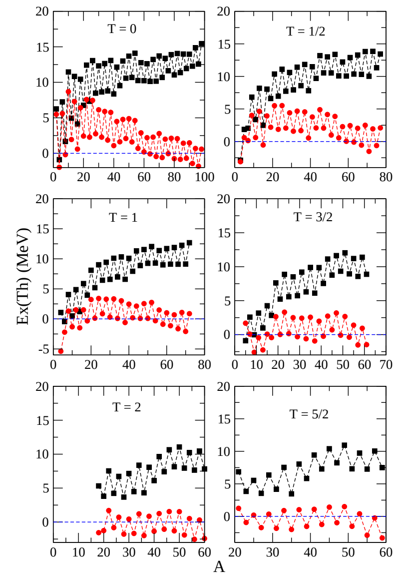

Figure 2. Excitation energies of particle-emission thresholds, $Th(nX)$ (squares) and $Th(pY)$ (circles). (The connecting lines are only a guide.)

2. Ground state energy and scattering threshold data brought together

For two mirror nuclei ${}^{A}_{(\pi=Z)}X_{(\nu=N)}$ and ${}^{A}_{(\pi=N)}Y_{(\nu=Z)}$, the energies (in MeV) of the nucleon plus nucleus thresholds are

$$\begin{aligned} Th(nX) &= E(n{+}X) - E_{\text{g.s.}}\left[{}^{(A+1)}_{(\pi=Z)}X_{(\nu=N+1)},\right] \\ Th(pY) &= E(p{+}Y) - E_{\text{g.s.}}\left[{}^{(A+1)}_{(\pi=Z+1)}W_{(\nu=N)}\right]; \\ \Delta(Th) &= Th(nX) - Th(pY). \end{aligned} \tag{1}$$

In Fig. 1 we show a plot of the various energies for the mass-13 systems, relative to the ground state of ^{13}C. The data were taken from the Ame2003 compilation [4]. We also evaluated the excitation energies of nucleon-emission thresholds in pairs of mirror systems, which are displayed in Fig. 2 for all nuclei with known mass and with core nucleus isospin $T < 3$. Negative values correspond to those nuclei which lie beyond the drip line, and are mostly proton emissive systems.

The associated $\Delta(Th)$ are plotted in Fig. 3. Therein the curves are results for $T = 0$ core nuclei ($N = Z = \frac{A}{2}$) with a proton, determined from

$$\Delta(Th) = \frac{\alpha Z \hbar c}{R} = \frac{197.3269602}{137.035999679}\; Z\; \frac{1}{R}. \tag{2}$$

where [5]

$$R = c_1\; A^{\frac{1}{3}} + c_2\; A^{-\frac{2}{3}}\; + r_p. \tag{3}$$

Ref. [5] gives $c_1 = 0.94$ and $c_2 = 2.81$ fm. The dashed curve in Fig. 3 was obtained with $r_p = 0.5$ fm, while the solid curve was obtained without any proton radius correction. For the latter, the resulting fitted curve gave values $c_1 = 1.07585$ fm and $c_2 = 1.95514$ fm.

The comparisons of the light mass results ($A \leq 20$) are shown on a larger scale in Fig 4. The two results give very good representations of data save one point: that of the mirror pair

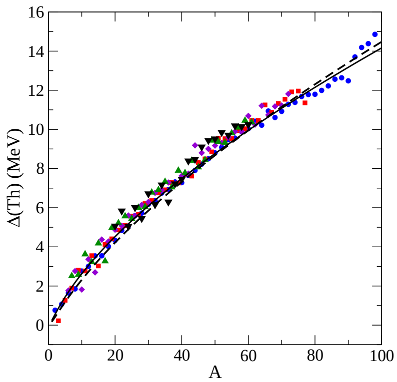

Figure 3. $\Delta(Th)$ plotted against the mass number of the core nuclei for isospins $T \leq 2$. The nuclear isospins are shown by circles ($T = 0$), squares ($T = \frac{1}{2}$), diamonds ($T = 1$), up-triangles ($T = \frac{3}{2}$), and down-triangles ($T = 2$).

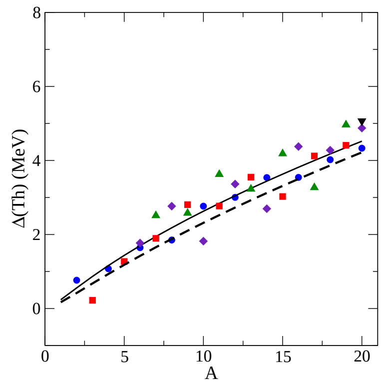

Figure 4. The Coulomb shifts $\Delta(Th)$ for the light mass core nuclei. Notation is as used with Fig. 3

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^{19}N–^{19}Mg. Both those nuclei are represented by a neutron coupled to ^{18}N and a proton coupled to ^{18}Na, respectively, and there is very little information available for both ^{18}Na and ^{19}Mg, both lying beyond the proton drip line.

3. Mass Equations

Kelson and Garvey [8] gave a formula to relate masses of isobars, namely

$$M(A, T_z = -T) - M(A, T_z = T) = \sum_x \left[M(A + x, T_z = -1/2) - M(Z + x, T_z = 1/2) \right], \quad (4)$$

where $-(2T - 1) \leq x \leq (2T - 1)$. Using experimentally known masses, they predicted results for nuclei with atomic numbers 4 to 22. Reasonable results were obtained for nuclei that are not weakly bound using this prescription.

Antony *et al.* [9] proposed another mass formula to specify isobaric mass multiplet energies for $A < 40$, for which they considered multiplets with $T \leq 2$. The energy (in MeV) of a generic (less-stable) ground-state nucleus is given with respect to that of a more stable one, taken as a 'base', *viz.*

$$\epsilon = \epsilon(Z, A) = E(Z, A) - E(Z_s, A)$$
$$E(Z, A) = M_{Z,A} - ZM_{\rm H} - (A - Z)M_{\rm n} - 0.6Z(Z - 1)A^{\frac{1}{3}}, \quad (5)$$

where Z_s is the charge of the base nucleus, $M_{\rm n}$ is the mass of the neutron, and $M_{\rm H}$ is the mass energy of the hydrogen atom. The last term is a correction for the Coulomb energy.

Using data from Ame2003 [4], we have used Eq. (5) to estimate gap energies between the ground-states within isobars with members of mass 6 to 19. Values of those estimates are shown in Table 1. There is a close pairing of the ground-state energies of nuclei within each set of isobars according to their isospin - an effect that has been noted before [9]. We plot the energy differences between ground states of isospin pairs, $D_T = \epsilon(Z, A) - \epsilon(A - Z, A)$, with $Z < A - Z$ in Fig. 5. For the three separate isospin values, the trend is for the energy differences of the pairs to decrease as mass increases. However, strong deviations do occur for some masses within each isospin set.

Table 1. Ground state gap energies (in MeV) of light mass isobars determined using Eq. (5).

Base	A	Z	$\epsilon(Z,A)$	Base	A	Z	$\epsilon(Z,A)$
	6	1	28.19		7	2	11.66
^{6}Li	6	2	4.05	^{7}Li	7	4	−0.24
	6	4	3.09		7	5	10.13
	8	2	28.09		9	2	30.91
^{8}Be	8	3	17.02		9	3	14.55
	8	5	16.36	^{9}Be	9	5	−0.46
	8	6	26.32		9	6	13.93
	10	2	39.42				
	10	3	23.33		11	3	34.34
^{10}B	10	4	2.00		11	4	12.88
	10	6	1.64	^{11}B	11	6	0.07
	10	7	22.18		11	7	11.26
	12	4	28.23				
^{12}C	12	5	15.21		13	5	15.21
	12	7	14.97	^{13}C	13	7	−0.06
	12	8	26.80		13	8	14.92
	14	5	24.72		15	5	32.66
^{14}N	14	6	2.36		15	6	11.91
	14	8	2.40	^{15}N	15	8	0.13
					15	9	10.94
	16	6	23.06		17	6	26.35
^{16}O	16	7	12.97		17	7	11.16
	16	9	12.39	^{17}O	17	9	−0.19
	16	10	22.20		17	10	10.90
	18	6	31.32		19	6	41.00
	18	7	17.54		19	7	22.53
^{18}F	18	8	1.23		19	8	7.64
	18	10	1.10	^{19}F	19	10	−0.03
					19	11	7.43

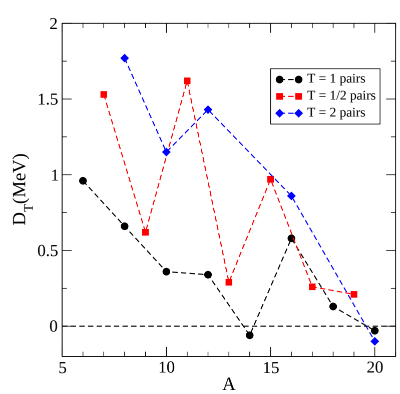

Figure 5. The energy differences between mirror ground-state energies calculated using Eq. (5).

Table 2. Mass-17 system properties deduced from inversion of Eq. (5). The base system defining $E(Z_s, A)$ is ^{17}O. All energies are in units of MeV and masses are in atomic mass units.

	$\epsilon(Z,A)$	$M_{Z,A}$	$Th(nX)$	$Th(pY)$
^{17}C	26.35	17.02259	0.73	23.33
^{17}N	11.16	17.00845	5.88	13.11
^{17}O	0.0	16.99914	4.14	13.78
^{17}F	−0.19	17.00209	16.8	0.60
^{17}Ne	10.92	17.01778	15.6	1.49
^{17}Na	25.5(1.0)	17.03752(107)	26.8(1.0)	−3.66(1.0)

Table 3. Predicted ground state energies of ^{17}Na relative to the p+^{16}Ne threshold (in MeV).

System.	KG$_{\text{old}}$	KG$_{\text{new}}$	Antony	cluster	MCAS
3.3(8)	3.65	4.28	3.66(1.0)	2.4	1.03

4. The case of ^{17}Na

^{17}C and ^{17}Na are a mirror pair of current interest [12, 13] but there is very little actually known about ^{17}Na. An early tabulation of nuclear masses [14] estimates the mass excess for ^{17}Na at values of 35.61, 35.81, and 35.84 MeV.

From Figs. 3 and 4 we estimate the ground state of ^{17}Na, relative to the p+^{17}Ne threshold, to be 3.3 ± 0.8 MeV. Kelson and Garvey [8] find mass excesses of 35.61 MeV and 24.67 MeV for ^{17}Na and ^{16}Ne, respectively, with the proton threshold to be 3.65 MeV. However, using the masses of Ame2003 in Eq. (4), we obtain a mass excess of 35.56 MeV for ^{17}Na. The measured mass excess for ^{16}Ne is 23.996 MeV and the proton mass excess is 7.288 MeV, giving a value for the proton threshold of 4.28 MeV.

By inverting Eq. (5), we can find masses for exotic nuclei. For mass-17 nuclei, those are listed in Table 2, with data taken from Refs. [4] and [10, 11]. The agreement is excellent. Given the close pairing of the ground state energies for nuclei of the same T within an isobar multiplet, and noting from Fig. 5 that as T increases, so does the gap between these ground states, we estimate the gap between the $T = \frac{5}{2}$ ^{17}C and ^{17}Na ground states to be around 1 MeV. Then, since the gap energy between ^{17}C and ^{17}O ground states is 26.54 MeV, we can assume a gap energy for ^{17}Na above ^{17}O to be 25.5±1.0 MeV. Thus, the last unknown in the equation, the atomic mass of ^{17}Na, is estimated as 17.03752 ±0.00107, which is 3.66 ± 1.0 MeV above the proton-^{16}Ne threshold. These values are slightly higher than those obtained using a microscopic cluster model [12]. And the results from MCAS [16] suggest a much lower value. All results are summarized in Table 3, and the differences may be due to the treatment of Coulomb effects.

5. Conclusions

A systematic study to predict the ground state energies of exotic nuclei has been presented. It is based upon examining the nucleon removal thresholds for mirror systems across the existing tabulated data for light-mass nuclei. A strong correlation in the data is found and does well for a wide range of masses, with variations from observed values usually being a few hundred keV and largest for the highest T-values. However, there are exceptional points with differences of up to an MeV.

We used the result from the systematic method and results from two mass formulae to estimate the mass of the unbound ^{17}Na. The systematics suggest a ground state of 3.3 ± 0.8 MeV above the proton-^{16}Ne threshold, in accord with predictions from mass formulae. And considering the differences in model predictions for the ground state energy, there is a need for a direct measurement of ^{17}Na.

SK acknowledges support from the National Research Foundation of South Africa. JPS acknowledges support from the National Sciences and Engineering Research Council (NSERC) of Canada. PRF acknowledges funds from the Dipartimento di Fisica e Astronomia dell'Università di Padova and PRIN research project 2009TWL3MX.

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