A Century of Nuclear Theory: Severely Condensed

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Abstract. Defining the underlying structure of matter has been a holy grail for science. The discovery of the 'ultimate' nature of matter often has been seen as offering the ability to explain most everything else in the universe. During the twentieth century, nuclear physics became a science which promised to reveal many of these secrets. Through theory and experiment, nuclear scientists charted the underlying structure of matter and gave new insights into the fundamental nature of things. Over the last century, progress in nuclear physics has relied on a symbiotic relationship between theory and experiment. One cannot isolate developments in one from those in the other. However, as a full account of the developments in nuclear theory over the century would take multiple volumes, this article can be no more than a scant selection.

1. Introduction

Over the last hundred years nuclear physics has flourished. It is still an active and interesting field of study today. Through theory and experiment, nuclear scientists not only probed the underlying structure of matter but discovered a plethora of applications of the physics. Perhaps the most commonly known applications of nuclear physics are nuclear power generation and nuclear weapons technology. But the research has provided application in many other fields; magnetic resonance imaging, ion implantation in materials engineering, and radio-nuclide dating in geology and archaeology, to name a few.

Through the years, nuclear physics delved ever more deeply into the atom with larger and larger machines constituting what came to be known as 'Big Science' - a characteristic form of late twentieth-century science. But there were spin-offs. The ability to produce radioactive ion beams for experimentation has enabled the nuclear landscape to be vastly increased with new physics, especially with nuclear systems at and beyond the nucleon drip-lines.

As noted in the abstract, I have had to be super selective of what I cover. I begin with some history, though a very narrowly focussed history, of the first half-century of developments in nuclear theory. In the second half-century there was such a rapid growth of ideas, models of structure and of reactions (and of numbers of nuclear physicists), coupled with the effects of the computer revolution, that it is impossible herein to give even a very narrowly focussed history. So in Sec. 3, there is an overview of some questions posed, and of the diversity of methods used, in three prime areas of study. Then, in Sec. 4, the optical model is discussed and some results of its application to a modern form are given. Finally, in Sec. 5, some general comments are made on a few of the current and future studies in the field.

2. The first half-century

During the years from 1911 into the 1960's, nuclear theories established the content, and basic structures, of nuclei. Many of the mechanisms involved in their interactions also were defined. Later, with the computer revolution, evaluations became feasible so that very detailed studies of structure and reactions could be made.

A hundred years ago, Rutherford suggested to Geiger and Marsden to look in their experiment for back scattering of α particles from target foils. To their great surprise, they were observed. Using classical scattering in a Coulomb field to interpret this, Rutherford established a maximal size to the nucleus which fostered the new idea of the structure of atoms with a small but massive nucleus at its center. Later, in 1919, his group transmuted one element into another for the first time. In their experiment nitrogen was converted into oxygen through the nuclear reaction $\alpha + {}^{14}\text{N} \to {}^{17}\text{O} + p$. Thereby protons were identified as being a constituent of the nucleus. Radiation studies also established that nuclei could emit both β-rays and α-particles, so it was an obvious view that protons, electrons and α-particles were the building blocks of a nucleus. But there were problems even then with that view. In 1921 Rutherford theorised about the existence of neutrons to somehow compensate for the repelling effect of the positive charges of protons by causing an attractive nuclear force to keep nuclei from breaking apart. One had to wait until 1932 for the existence of neutrons to be demonstrated by Chadwick.

Prior to Chadwick's discovery, most studies on the nucleus related to ascertaining their gross properties, e.g. their size, mass, energy systematics, etc., and of the characteristics of spontaneous and induced radiations from them. Reaction data were limited to processes that could be obtained using nuclear projectiles (α's mostly) from radioactive sources. However, during preceding years, great developments were made in Quantum Theory (notably of atomic structure) that laid the foundation for much in nuclear theory as is know today. Notably, mindful of Heisenberg's uncertainty principle that it was impossible to determine simultaneously pairs of exact 'dynamical variables' of a system, Pauli found that an electron in an atom can be signified by four distinct quantum numbers. In 1926, Gamow was, perhaps, the first to apply quantum mechanics to deal with a nuclear physics question. He used the concept of tunnelling to explain α-emission from nuclei. Later (1928-29) he used quantum mechanics to re-analyse the Rutherford (α, p) data, ascertaining that nuclei had a critical radius given by $R = 1.21A^{1/3}$. Nuclear volumes then are proportional to A. This was a result of note. Nuclear volumes proportional to the atomic number meant that nuclei would 'saturate', i.e. have a constant central nucleon density. This also implied that the nucleon-nucleon force is short ranged.

A most important development in Quantum Theory was the work of Dirac in formulating a relativistic equation to describe fermions, and concomitantly, antiparticles. Fermi was to use this later to explain β-decay. Those two theories, coupled with Segre's later discovery of the anti-proton, arguably may be the genesis of the modern topic of particle physics.

The real start to theories of the nucleus began only with Chadwick's discovery of the neutron. For even growth of the liquid drop model, a theory that should have been attributed to Gamow, awaited the idea of two types and two spin values of fluids in the drop. Heisenberg, in 1932, was the first to state categorically that nuclei consisted of protons and neutrons without any constituent electrons. He introduced the concept of isospin, deeming the neutron to be a fermion. Of great importance, the discovery of the neutron meant that one could finally calculate the binding energy fractions of each nucleus by comparing the nuclear mass with the sum of those of the constituent protons and neutrons. Differences between nuclear masses were calculated in this way and, when nuclear reactions were measured, they were found to agree to high accuracy with Einstein's mass-energy equivalence.

In 1935, Yukawa proposed the first significant theory of the strong force to explain how the nucleus is held together. In the Yukawa interaction a virtual particle, later called a meson, mediated a force between all nucleons, including protons and neutrons. This force explained why nuclei did not disintegrate under the influence of proton repulsion and it also gave an explanation of why the attractive strong force had a very short range of interaction.

Electrons do not exist in their own right within the nucleus, though the process of internal conversion revealed that the atomic K-shell electron could be captured. The energy systematics were established to show that not all nuclei would be stable. Unstable nuclei may decay by hadronic-particle emission (α, p, n) or it may β-decay with emission of an electron or a positron. After one of these decays the resultant nucleus may be left in an excited state, from which it may cascade by γ-decay to its ground state. Understanding β-decay of nuclei revealed the existence of the weak force of nature. Electrons, protons, and neutrons are fermions and β-decay (electron) was supposed to be the disintegration of a neutron to a proton and an electron. If that were all, then spin would not be conserved. The neutrino is required. Postulated by Pauli (1931) and used by Fermi (1933), the neutrino allowed preservation of conservation laws in β-decay and explained the quite distinctive structure of observed energy distribution of electrons. Eventually, β-decay would lead to the concept of parity violation and a re-examination of the symmetries of physical laws which often have been taken for-granted.

The period 1936-60 was the golden age of nuclear theory with the specification of diverse models of nuclear structure and of nuclear reaction theories that are the bases of study to this day. Nuclear theory, along with the numbers of its practitioners, burgeoned. Many textbooks were written during this period with that of Blatt and Weisskopf being a 'bible'. A look at the contents of this 850 page tome shows clearly just how intense had been the research activity into nuclear theory to the date of its publication (1952). Even so, there is little in that book about the shell model; a concept that underpins a vast range of studies now.

By 1960, the computer revolution was in full swing. One only has to consider what exists today to see this as a revolution. As examples, consider what size of evaluations can be done and how quickly, and consider the development the Internet and social networking systems and how they have affected societies, and, finally, consider the almost prime necessity of a computer in every house and on every desk.

3. The second half-century

From the discovery of the neutron, basic aims of nuclear theory were to form a picture of the structure of the nucleus, to understand how nuclei interact, and to ascertain what results from those interactions. Answers were sought to three basic questions.

3.1. What is the force between two nucleons?

General properties of nuclei indicate that the internucleon (NN) force has to be

- strong: The binding energies of nuclei are large and the nucleus is quite small and compact.
- short ranged: Rutherford's analysis (of α scattering) showed that nuclear forces would have no noticeable effect at ranges of a few times the nuclear diameter.
- basically attractive: Since nuclei exist, the components had to 'stick'
- repulsive at short range: If it were otherwise, heavy nuclei would collapse catastrophically.

The combination of repulsive character inside an attractive one for the short ranged NN force meant that nuclei saturate. Nuclei then increase in size with mass (A), nuclear radii vary as $A^{1/3}$, and nuclear binding energies per nucleon tend to a constant value of ~ 8 MeV/A. Note also that the exchange nature of the force specified by Heisenberg contributes to forming saturation. Thus more than just the exchange of a single pion is required in defining the NN force.

3.2. How can nuclear structures be described?

Understanding the structure of a nucleus is one of the central challenges in nuclear physics. Essentially it is a quantum mechanical problem of a few to fairly many strongly interacting fermions, with the precise knowledge of the interactions between the constituent nucleons, to this day, not totally settled. Given that a nucleus has protons and neutrons confined within quite a small volume, does the two-nucleon interaction vary from the free space form when the two are embedded in nuclear matter? Are there three and more nucleon forces at play? Those vagaries, as well as the diversity of the properties of nuclear systems, and the serious difficulties of finding 'exact' solutions to the many-fermion Schrödinger equations, has lead to development of many disparate models for nuclear structure. Attempts to unify the structure models have had with limited success. Some nuclear structure models that have been developed are:

- Collective models: The liquid drop model, quantised rotors and vibrators, vibrating potential model, and other harmonic vibration models.
- The shell model: Standard shell model, deformed (Nilsson) shell model, Random phase approximation (RPA), quasi RPA, and the continuum shell model.
- Mean field theories (nonrelativistic and relativistic): Hartree-Fock (HF), Hartree-Fock-Bogoliubov (HFB), Time dependent HF methods.
- Cluster models: α-cluster models, cluster-orbital model, generator coordinate methods.
- Few body models: Fadde'ev three-body, four-body (AGS), quantum Monte Carlo models.
- Group theoretic models: Wigner supermultiplet theory, SU(3) and other group classifications of states, interacting boson models.

3.3. How can one describe nuclear reactions?

The number of possible nuclear reactions which these theories seek to describe is immense. However, there are several common, or otherwise notable, types. Some examples include:

- Fusion and capture reactions: Two light nuclei join to form a heavier one either as a final system or as a temporary 'compound' that breaks up with one or more particles ejected.
- Fission reactions: A very heavy nucleus spontaneously, or after absorbing additional light particles (often neutrons), splits into two or more pieces.
- Spallation: A nucleus hit by another with sufficient energy and momentum to knock out several small fragments, or to be smashed into many.
- Direct reactions: A projectile transfers energy or picks up or loses nucleons to the nucleus in a single quick (10^{-21} sec.) event.
- Compound nuclear reactions: A low-energy projectile undergoes many collisions and is absorbed forming a nucleus with too much energy to be fully bound. On a time scale of about 10^{-19} sec., particles, usually neutrons, are 'boiled' off. The compound exists until enough energy concentrates on a neutron to escape the mutual attraction.

There were almost as many reaction theories developed through these years. Some are

- Hauser-Feshbach: Used in analyses of low-energy compound nuclear reactions.
- $\mathcal{R}$-matrix theory: Used to classify resonance structures; a central feature of large modern data-analysis codes.
- Transport theories: Used to analyse spallation, heavy ion fragmentation data especially with relativistic heavy ion collisions. They include evaporation and transport models, *viz.* the Boltzmann-Uehling-Uhlenbeck (BUU) model and its relativistic form.
- Direct reactions: The optical model, distorted wave approximation (DWA), and the multi-step Feshbach-Kerman-Koonin (FKK) theory

A key ingredient for many analyses is the optical model, which is now considered in more detail.

4. The optical model

The optical potential plays a vital role in the history of nuclear reaction theories. The concept of a complex optical potential as a single particle representation of the interaction between two nuclei dates at least to the study by Bethe in 1940 of neutron-nucleus cross sections. All early optical potentials were phenomenological. Studies of that phenomenology proceeded apace thereafter, culminating in attempts to prescribe global forms for all target masses and for projectile energies up to ~ 1 GeV. Phenomenological and semi-phenomenological optical potentials are still used to interpret elastic scattering data as well as to define the distorted waves required in DWA analyses of non-elastic processes. Likewise the semi-phenomenological approach has reached a very sophisticated stage.

4.1. A formal derivation of the optical potetial

Whatever approach is used to specify the optical potential acting between two nuclei, its formal derivation has the requirement to project the equations for the state vector onto the 'elastic channel subspace'. An elegant way of doing this was given by Feshbach in 1958. His formalism divides the Hilbert space using two projection operators P and Q. P projects onto the elastic channel and Q on to all others. Thus there is an algebra: $P^2 = P, Q^2 = Q, P + Q = 1, PQ = QP = 0$ and $Q\,|\Psi_{gs}\rangle = 0$. With these projection operators, the Schrödinger equation segments

$$(E - H_{PP})P\,|\Psi^+\rangle = H_{PQ}Q\,|\Psi^+\rangle \quad ; \quad (E - H_{QQ})Q\,|\Psi^+\rangle = H_{QP}Q\,|\Psi^+\rangle\,,$$

where $H_{XY} = XHY$. Using the second equation to eliminate $Q\,|\Psi^+\rangle$ from the first gives

$$\left(E - H_{PP} - H_{PQ}[E - H_{QQ} + i\epsilon]^{-1}H_{QP}\right) P\,|\Psi^+\rangle = 0\,,$$

where outgoing wave boundary conditions are assumed. The Feshbach formalism reduces the many nucleon problem to an effective one body one by invoking explicitly the ground state and transitions to other channels from the ground state. With $H = H_0 + V$, $H_{PQ}(\equiv H_{QP}) = V_{PQ}$,

$$\left(E - H_0 - \langle\Phi_{gs}\,|V|\,\Phi_{gs}\rangle - \left\langle\Phi_{gs}\left|VG^{(+)}_{QQ}V\right|\Phi_{gs}\right\rangle\right)\;|\chi^+\rangle = 0 \quad \text{where} \quad G^{(+)}_{QQ} = [E - H_{QQ} + i\epsilon]^{-1}.$$

Thereby the single nucleon *distorted wave* $|\chi^+\rangle$ for elastic scattering is defined. The intermediate state propagator is complex due to pole contributions. It contains the whole complex spectrum of many body excitations of the projectile and target nucleons in bound as well as in continuum states through $G^{(+)}_{QQ}$. Thus the optical potential is identified by

$$U = U_{OM}(E) = \langle\Phi_{gs}\,|V|\,\Phi_{gs}\rangle + \left\langle\Phi_{gs}\left|VG^{(+)}_{QQ}V\right|\Phi_{gs}\right\rangle.$$

In general it is non-local, energy dependent, and if the incident energy is larger than the first excited state threshold, complex due to the second term. That is so even if the leading term is real, local, and energy independent. The second term is the dynamic polarisation potential.

For incident energies in the range of the discrete excited states of a target (usually less than a few MeV), specific coupled-channel effects are paramount. For higher energies, ones coinciding with a dense continuum in the spectrum of the target, the leading interaction can be taken as a sum of pairwise interactions between the projectile and each and every bound nucleon.

With regard to the latter condition, initially it was supposed that, for sufficiently high incident energies, those NA interactions would be ascertained from free NN scattering. Bethe in 1958 showed that the cross section and polarization from the scattering of 310 MeV protons from ^{12}C at forward scattering angles are consistent with that conjecture. Then, in 1959, Kerman, McManus, and Thaler (KMT) developed the Watson multiple scattering approach expressing the NA optical potential by a series expansion in terms of free NN scattering amplitudes. Those formulations define an effective interaction between projectile and the target nucleons.

4.2. The g-folding model

This model of the optical potential is that which my colleagues and I use today. It is based upon a non-relativistic multiple scattering theory but with the NN scattering amplitudes modified from the free NN values. Those modifications are caused by the two nucleons interacting within the nuclear medium and are due to *Pauli blocking* and *mean field effects* for both projectile and bound state nucleons. Those effects are taken into account in the Brueckner-Bethe-Goldstone (BBG) equations for infinite nuclear matter systems, solutions of which are termed NN g-matrices. Also of importance is the antisymmetrization of the incident nucleon with each of the A nucleons in the target. That leads to direct and knock out exchange components to scattering. The effect of the exchange terms is not small at any energy and they are a source of non-locality. With non-local NA optical potentials formed using the Melbourne (effective) NN g-matrices,

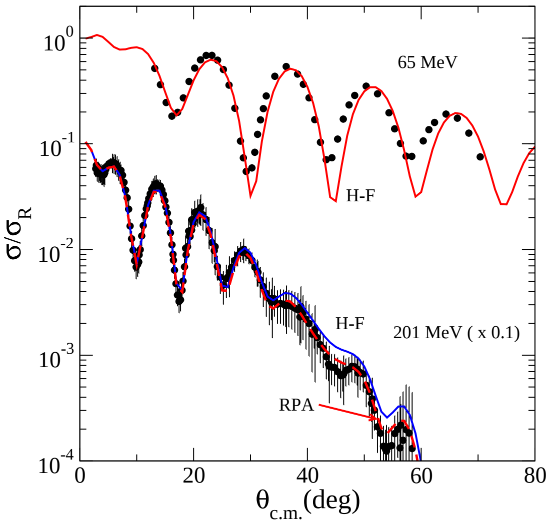

Figure 1. p-^{208}Pb elastic cross sections

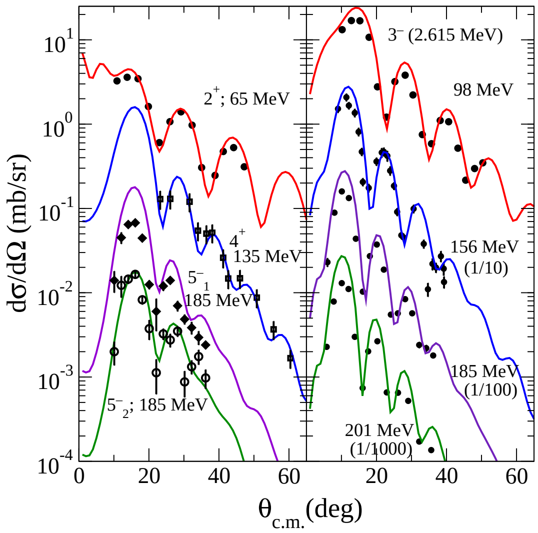

Figure 2. p-^{208}Pb inelastic cross sections

and two structure models for the ground state of ^{208}Pb, Dupuis *et al.* (Phys. Rev. C **73**, 014605 (2006)) obtained the results given in Fig. 1. The structure models from which the required one-body density matrix elements (OBDME) were obtained are as indicated. Clearly the matches with data are very good and can be used to discriminate between the model structures. Results of similar quality were found for spin observables. In particular, such data analyses lead to the belief that, in its ground state, ^{208}Pb has a neutron skin thickness of ~ 0.16 fm; a value consistent with that found from a recent parity violating electron scattering experiment.

The relative motion wave functions (the distorted waves), whatever was chosen for the optical model potential, are considered the appropriate ones to use in other reaction evaluations provided elastic scattering data are well fit. While that is an essential condition, it is not a sufficient one because elastic scattering cross section evaluations require knowledge only of a set of phase shifts. Those are quantities determined from the asymptotic properties of the wave functions. The veracity of those wave functions throughout the nuclear volume is not guaranteed. For a sufficient condition, more information about an optical potential is needed. Use of the relative motion wave functions in a successful fit to other scattering data, e.g. of inelastic scattering data, could suffice. This may be so, but a predictive application, i.e. one without additional parameterisations or scale adjustments, is needed since, often, the DWA is used to analyze data.

The best one can do at present is to use a formulation of the optical potential that accounts as best possible for physical laws. Notably one should take into account the Pauli principle and treat the ensuing non-localities exactly. Only then might analyses of elastic and inelastic proton scattering data be used as good tests of structure. The results of M. Dupuis *et al.* (Phys. Lett. **B665**, 152 (2008)), are shown in Fig. 2. They were found using the DWA with distorted waves of the g-folding optical model potentials, with the OBDME for a transition as given by the same model of structure. The Melbourne effective NN interaction was used as the transition operator. In the left-hand panel, cross-section data are compared with DWA results for excitations of different spin states in ^{208}Pb, while in the right-hand panel, results of calculations for excitation of the 3^- state at 2.615 MeV are compared with data.

The OBDME required in the scattering calculations (elastic and inelastic) were obtained from self-consistent RPA calculations where self-consistency means that the same interaction (the D1S interaction) was used to determine the mean field single particle states, as well as being the residual interaction in RPA calculations. As Dupuis *et al.* noted in the conclusions, 'excitations of high spin states up to the 12_1^+ state have also been well explained with the model and so, proton scattering has been shown to be a means to precisely investigate the structure description of heavy spherical nuclei. The goal has been achieved because no phenomenological input or arbitrary renormalization process enters the microscopic model analyses."

5. The future very briefly

The development of nuclear theories of structure and reaction has been enormous over the last few decades. Research continues in the physics of relativistic heavy-ion collisions, in nuclear astrophysics processes, in the structure of new and exotic nuclear systems and their interactions, and in the new aspects of strangeness with hypernuclei, to name a few.

There are many nuclear structure theories that are in use, no-core shell models, Green's function Monte Carlo models with three-nucleon forces included, diverse mean field models some being relativistic, and various cluster models like the antisymmetrized molecular cluster model, are examples. Nuclear reaction theories also have progressed. These, in particular, result from the development in the last few decades of heavy ion facilities. Some produce relativistic heavy ion beams whose collision with targets, in particular it is hoped, will give evidence of a quark-gluon plasma, while others produce radioactive ion beams (RIBs) for use in delineating the structure and reaction properties of nuclei off the line of stability, and indeed beyond the nucleon drip-lines. RIB scattering from stable nuclei enables one to study exotic nuclei; ones that lie well off the line of stabilty. There are two methods of making them. The first, the isotopic separation on-line (ISOL), is appropriate to make low energy RIBs, e.g. ≤ 10 MeV/A.

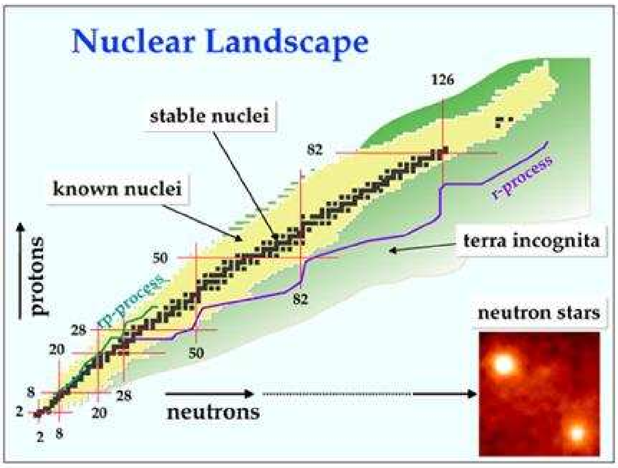

Figure 3. A stylised modern chart of nuclei

For higher energies, e.g. > 10 MeV/A, RIBs are made by in-flight fragmentation. The scope is seen in the stylised modern chart of the nuclides shown in Fig. 3.

- *'terra incognita'* edges are the nucleon drip-lines
- Most nuclei $\beta^{\pm}$ decay
- Some are particle emissive (p, n, α)
- In nuclei near to the drip lines, nucleons are weakly bound
- The general lines of the astrophysics r- and rp- processes are shown
- Not shown is a third axis, for strangeness and hypernuclei.

As an example, consider the radioactive nucleus, ^{6}He, which has two weakly bound neutrons. The β-decaying system is known to be Borromean in that, under stimulus, it will break readily into an α plus those two neutrons. RIBs of ^{6}He have been formed and the scattering from hydrogen targets measured. Those data are compared with g-folding and DWA results in Figs 4 and 5. The elastic scattering results shown in Fig. 4 compare well (magnitude and shape)

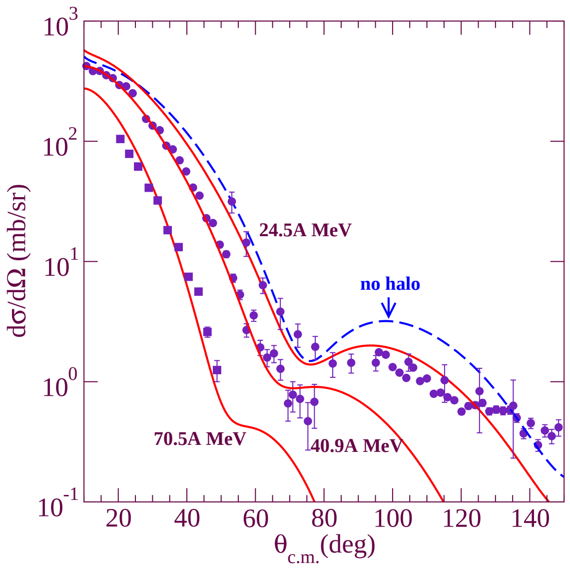

Figure 4. ^{6}He-p elastic scattering for three incident energies and a neutron halo density

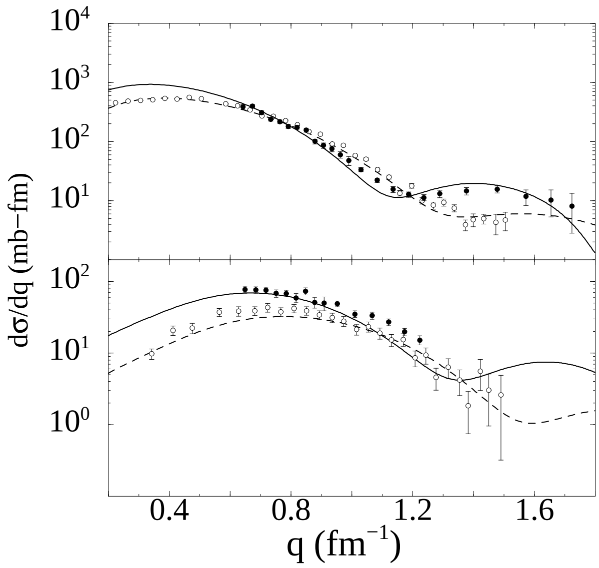

Figure 5. ^{6}He-p elastic (top) and inelastic scattering to the 2_1^+ state (bottom)

with calculations made using large space no-core shell model wave functions with single particle states chosen to give ^{6}He an extended neutron halo. When wave functions are used that give the nucleus only a neutron skin (no halo), the match is quite imperfect.

With elastic scattering, the halo character affects evaluations for high momentum (large angle scattering) as is shown by the results that are compared with data taken at 24.5A MeV (filled circles) and at 40.9A MeV (open circles) in the top segment of Fig. 5. In the bottom segment DWA results at those incident energies are compared with data from the inelastic excitation of the 2_1^+ (1.8 MeV) state in ^{6}He. In this case it is the smaller momentum transfer results that are most affected by the halo prescription. This reflects the reduction in strength of wave functions in the surface region needed to have the extended neutron distribution that is the halo. These momentum transfer diagrams also show that the energy dependence of the effective NN interaction used in the folding and the DWA calculations is important since the structure used in all calculations was fixed.

Acknowledgments

Material in this presentation has been taken from many books, papers, and articles; so many that I have not given a list of references. I reference in text the two recent papers, on which I am not a co-author, and from which I have taken figures. The nuclear landscape figure was taken from a Google search and is one of many in the literature.