Non-universality of a constrained period doubling route to chaos for Rössler’s system

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SOUTH AFRICAN INSTITUTE OF PHYSICS

SAIP CONFERENCE

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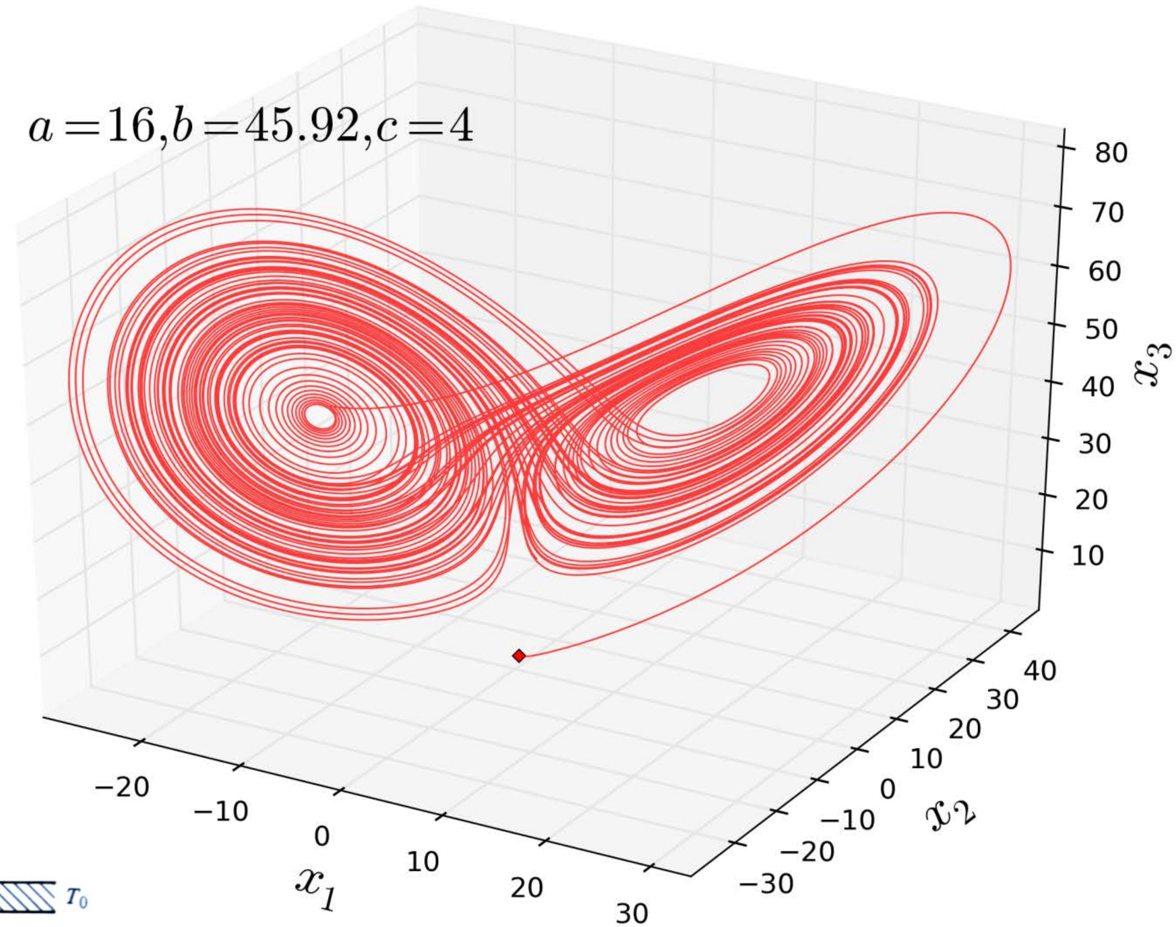
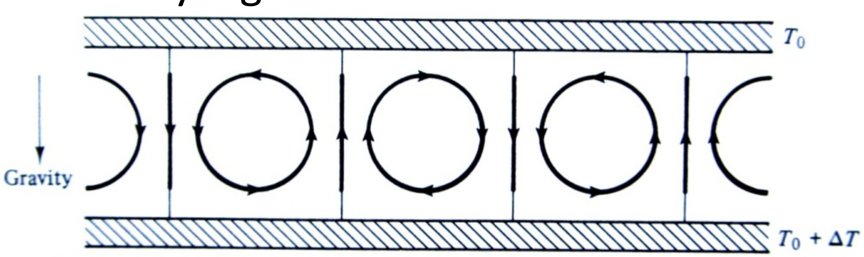
Edward Lorenz

$$\dot{x} = a(y - x)$$
$$\dot{y} = x(b - z) - y$$
$$\dot{z} = xz - bz$$



Rayleigh-Bernard Convection



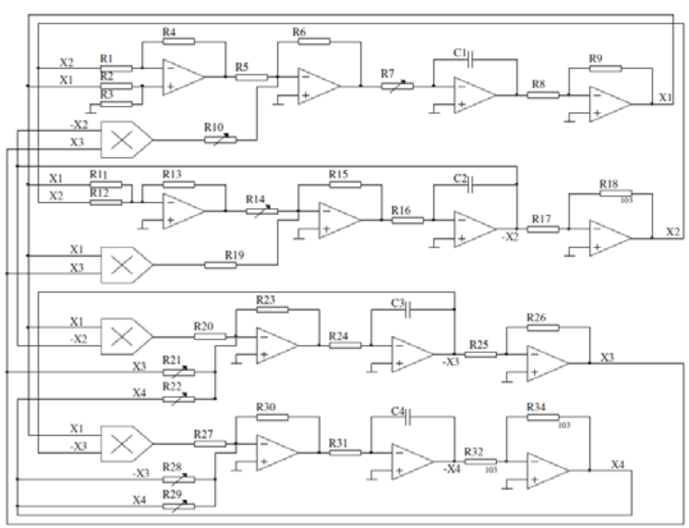
E.N. Lorenz, J. Atmos. Sci. **20**, 130 (1963)

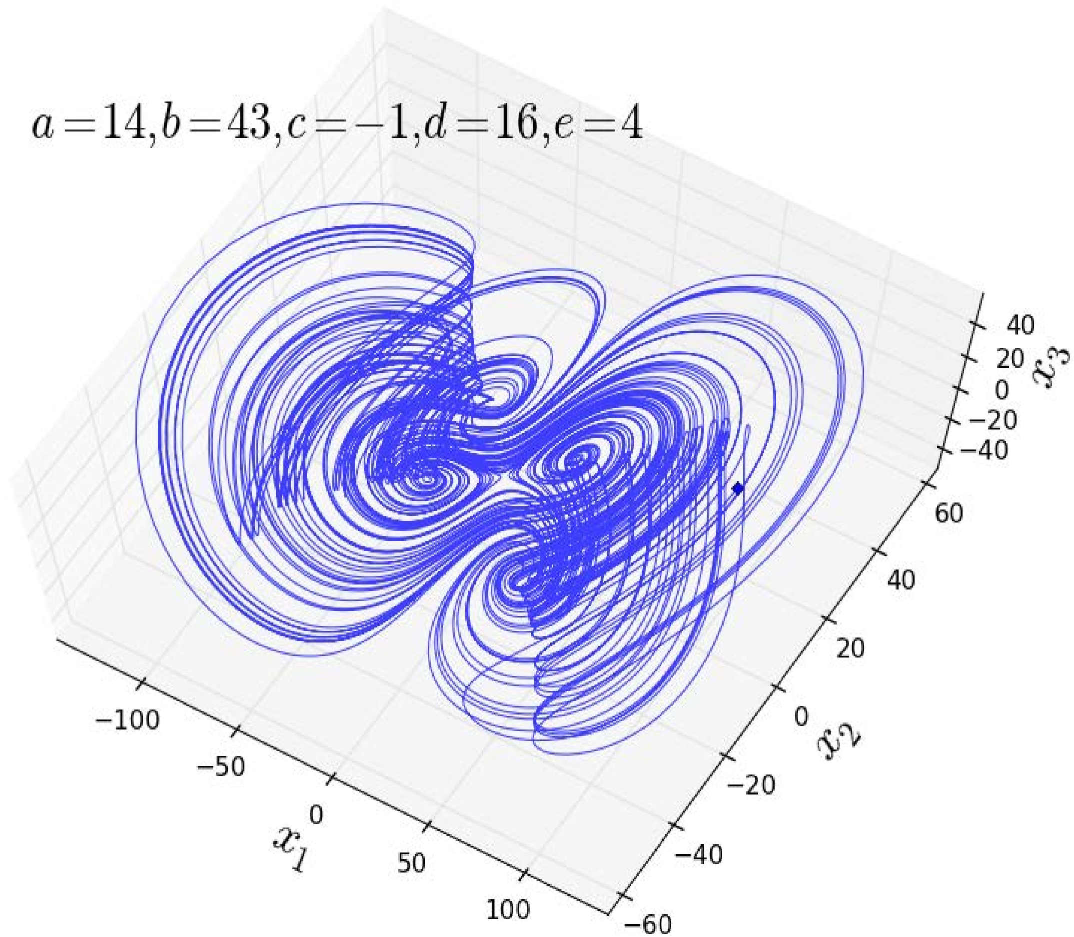
Guoyuan Qi

$$\dot{x} = a(y - x) + eyz$$

$$\dot{y} = cx + dy - xz$$

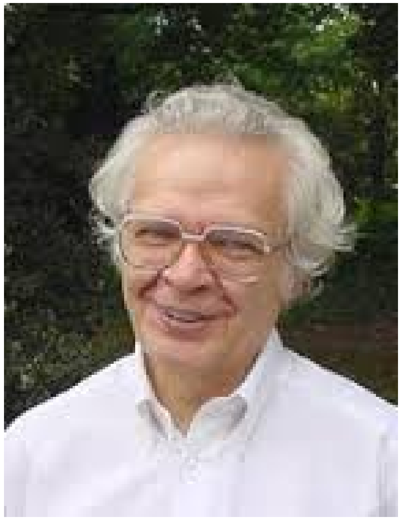
$$\dot{z} = -bz + xy$$





G. Qi et al., Chaos Solit. Fract. **38**, 705 (2008)

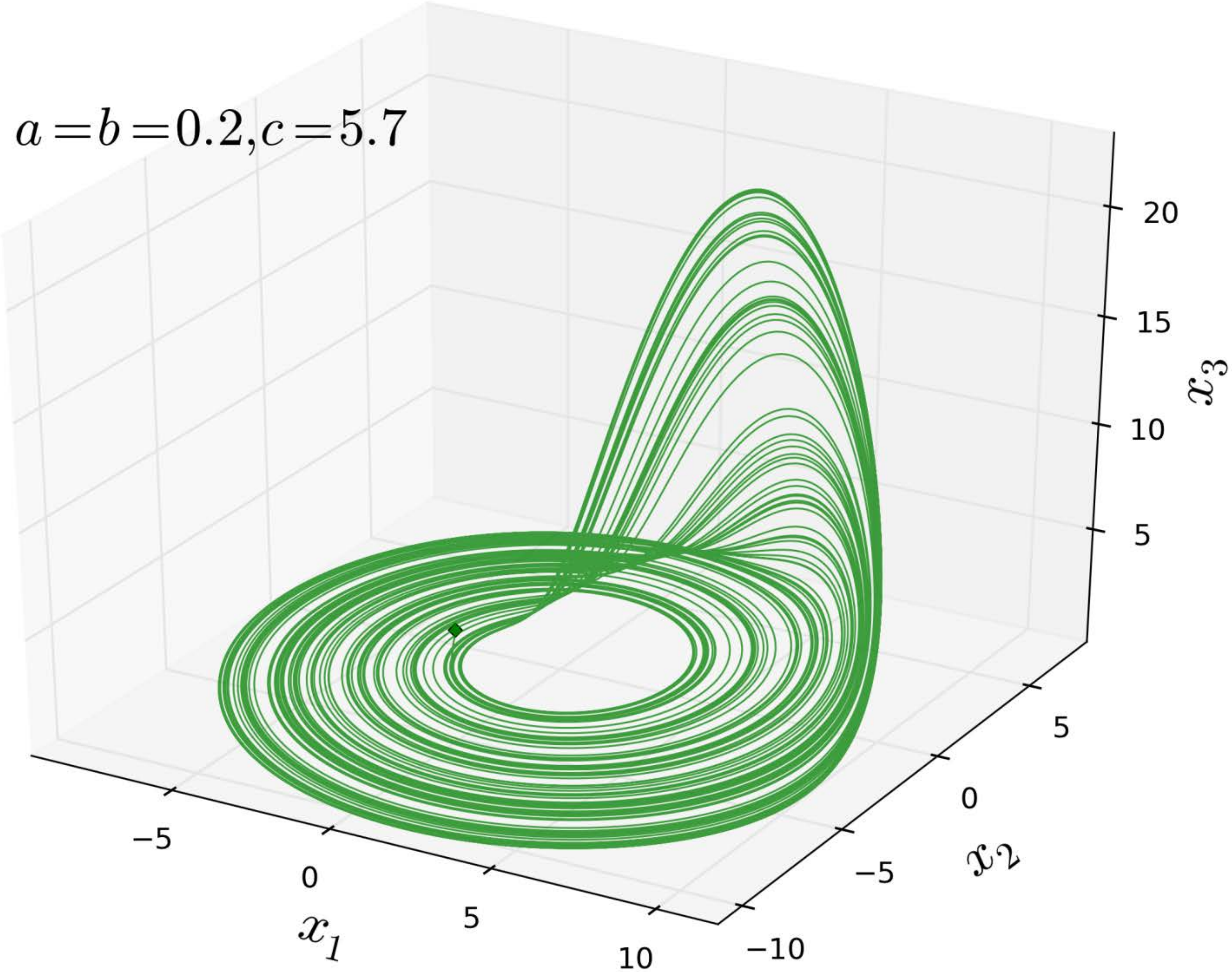
Otto Rössler



$$\dot{x} = -y - z$$

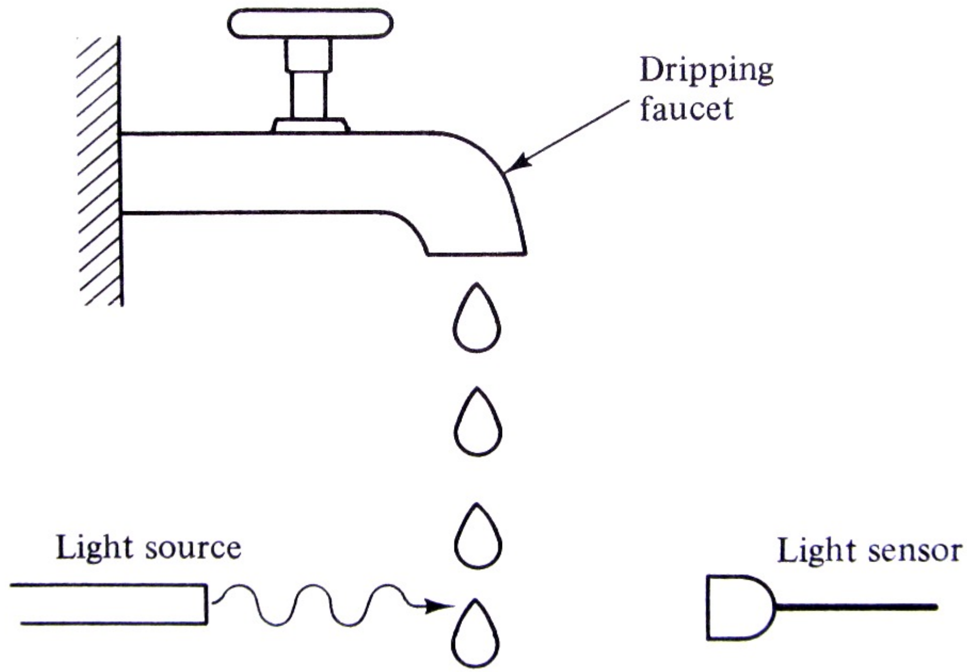
$$\dot{y} = x + ay$$

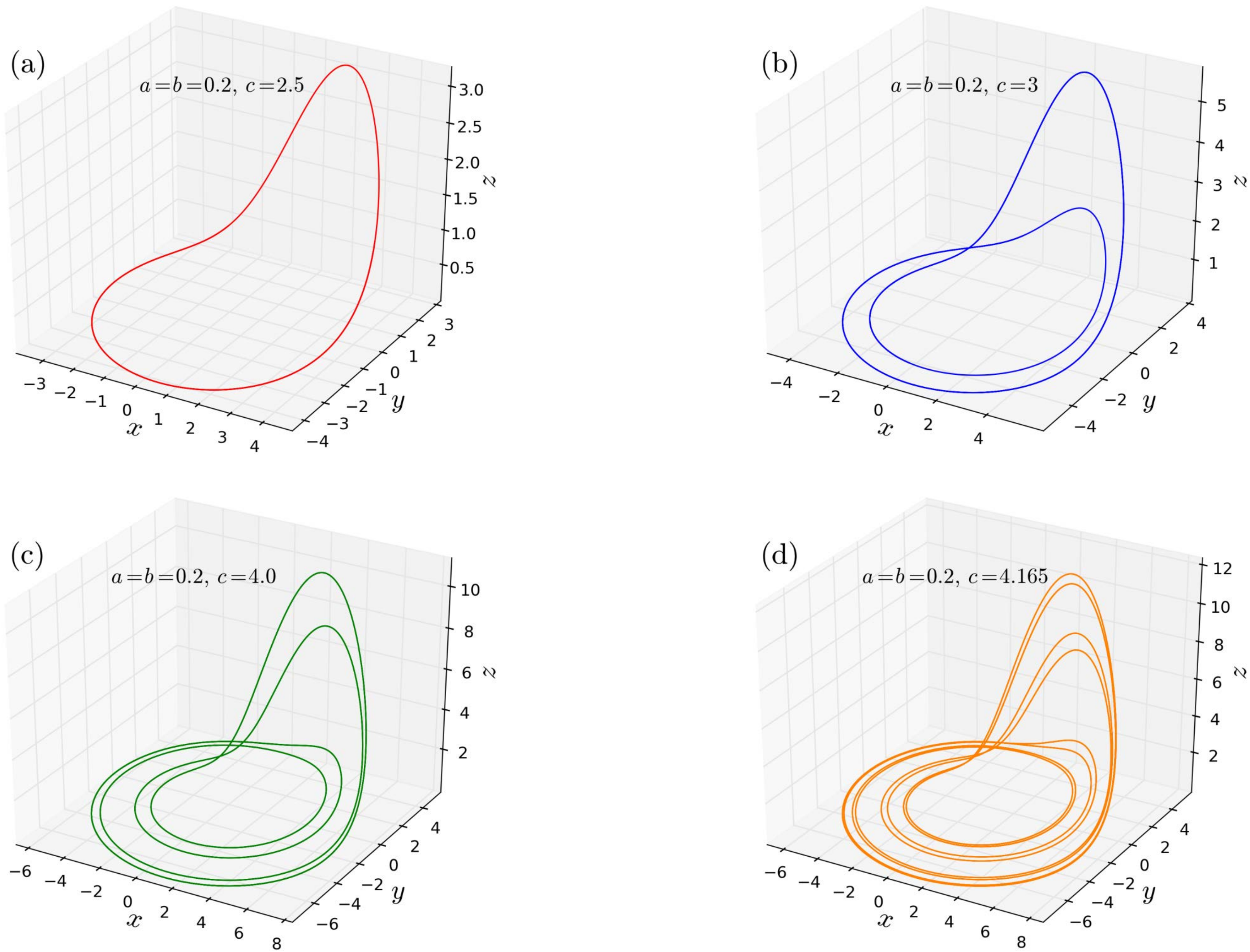
$$\dot{z} = b + z(x - c)$$



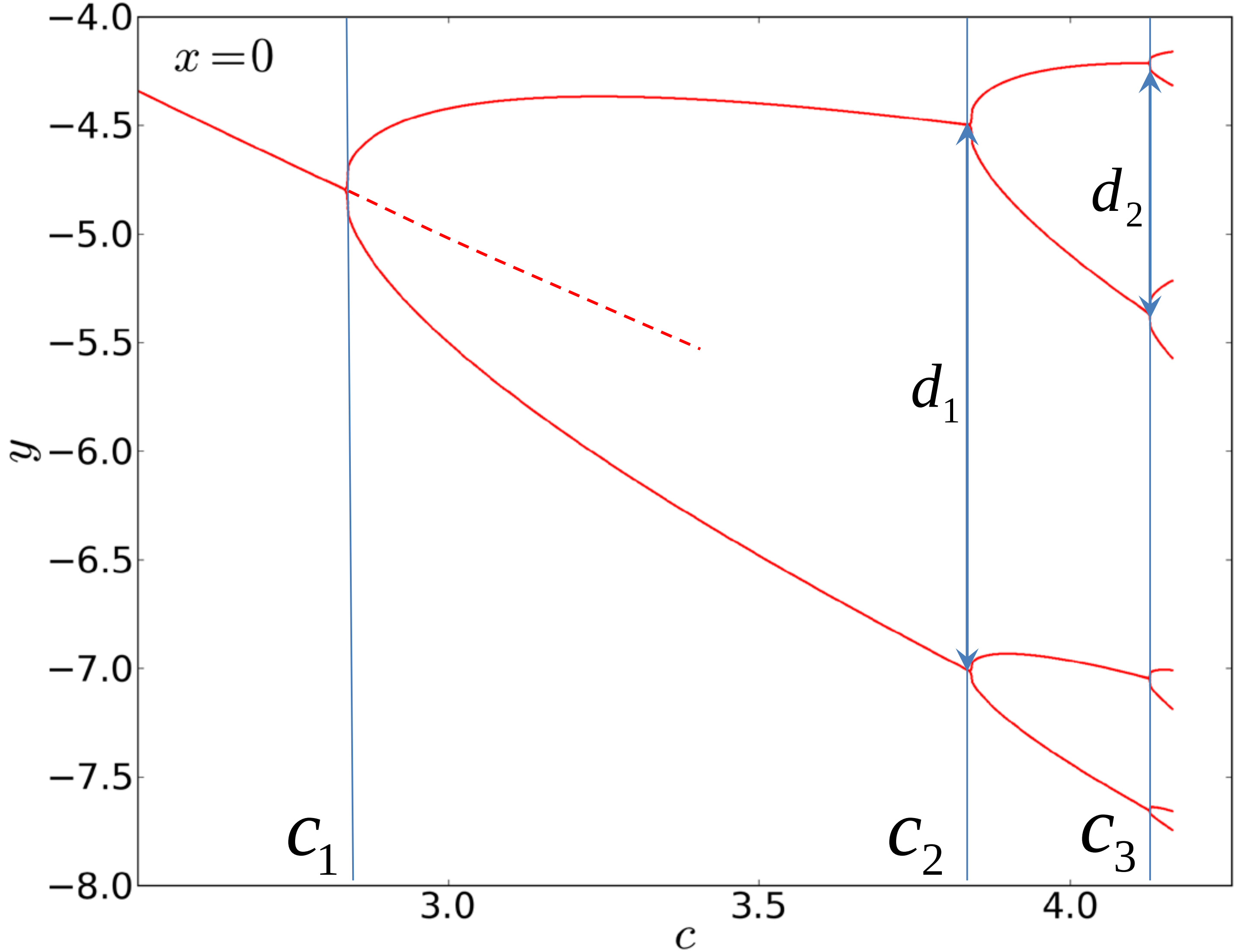
O.E. Rossler, Phys. Lett. A **57**, 397 (1976)

Period doubling




(a)
$a = b = 0.2,\ c = 2.5$
x
y
z
(b)
$a = b = 0.2,\ c = 3$
x
y
z
(c)
$a = b = 0.2,\ c = 4.0$
x
y
z
(d)
$a = b = 0.2,\ c = 4.165$
x
y
z

Period Doubling Bifurcations

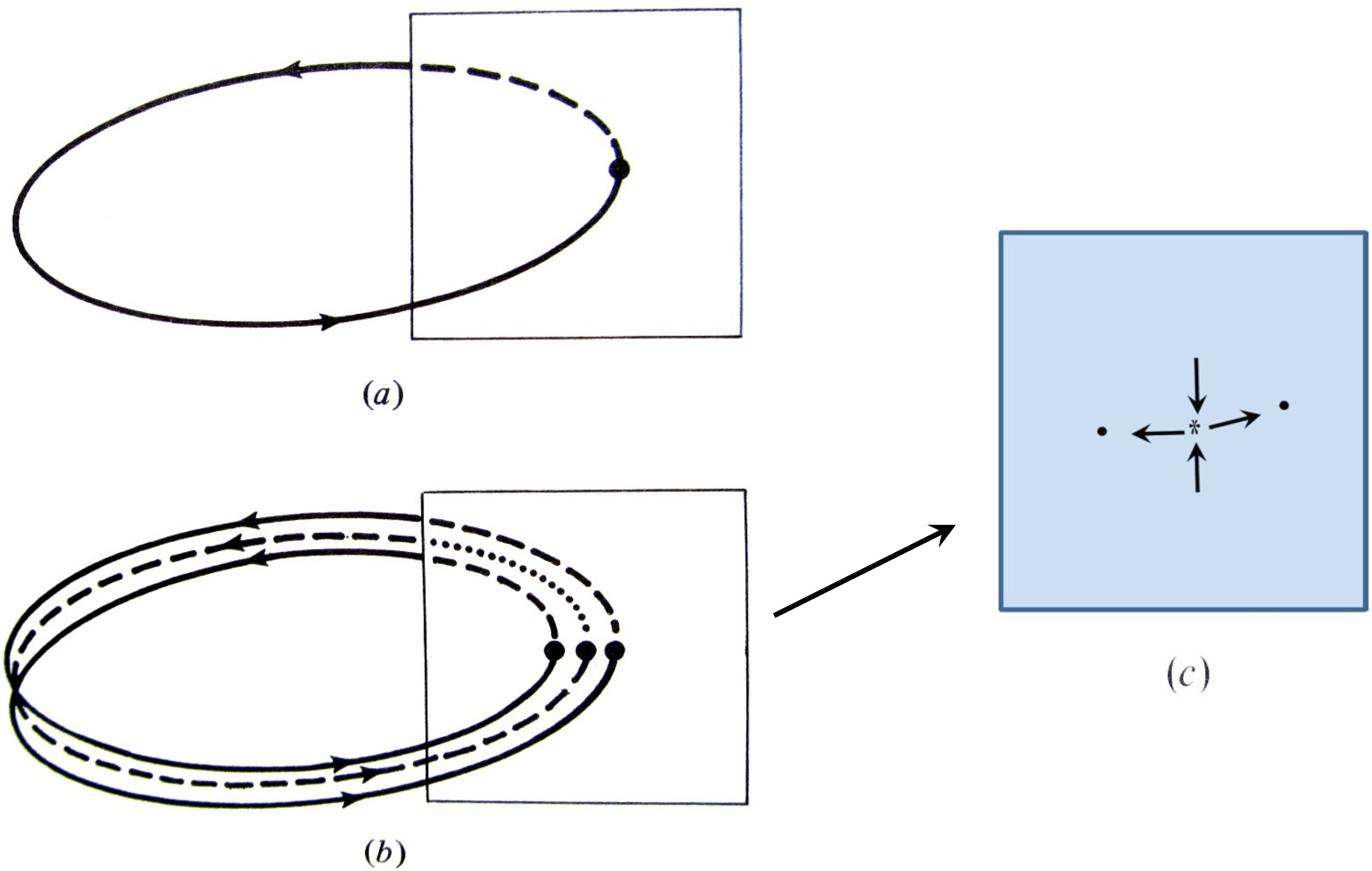


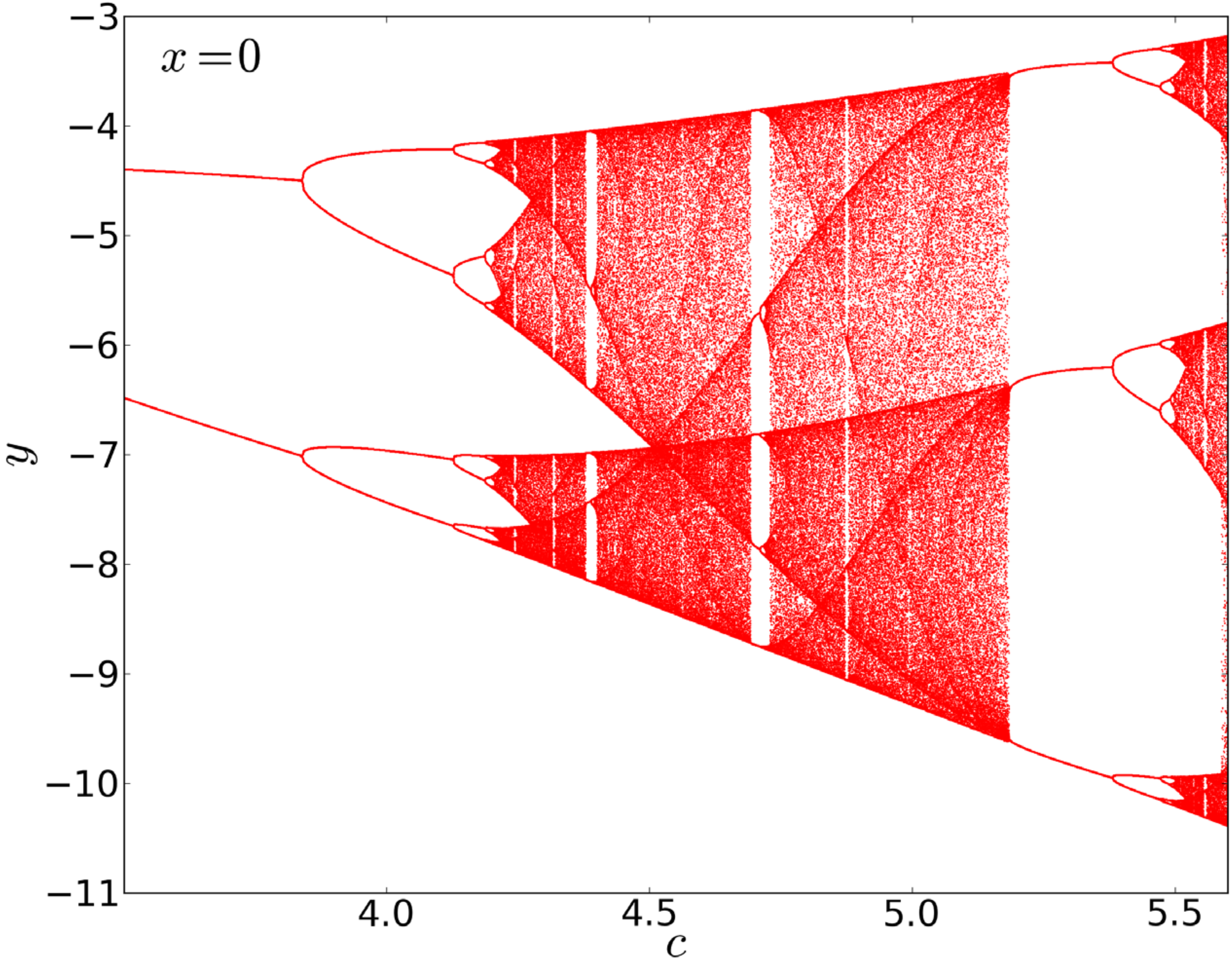
Universality of period doubling

$$\delta = \lim_{n\to\infty} \frac{c_n - c_{n-1}}{c_{n+1} - c_n} = 4.6692$$

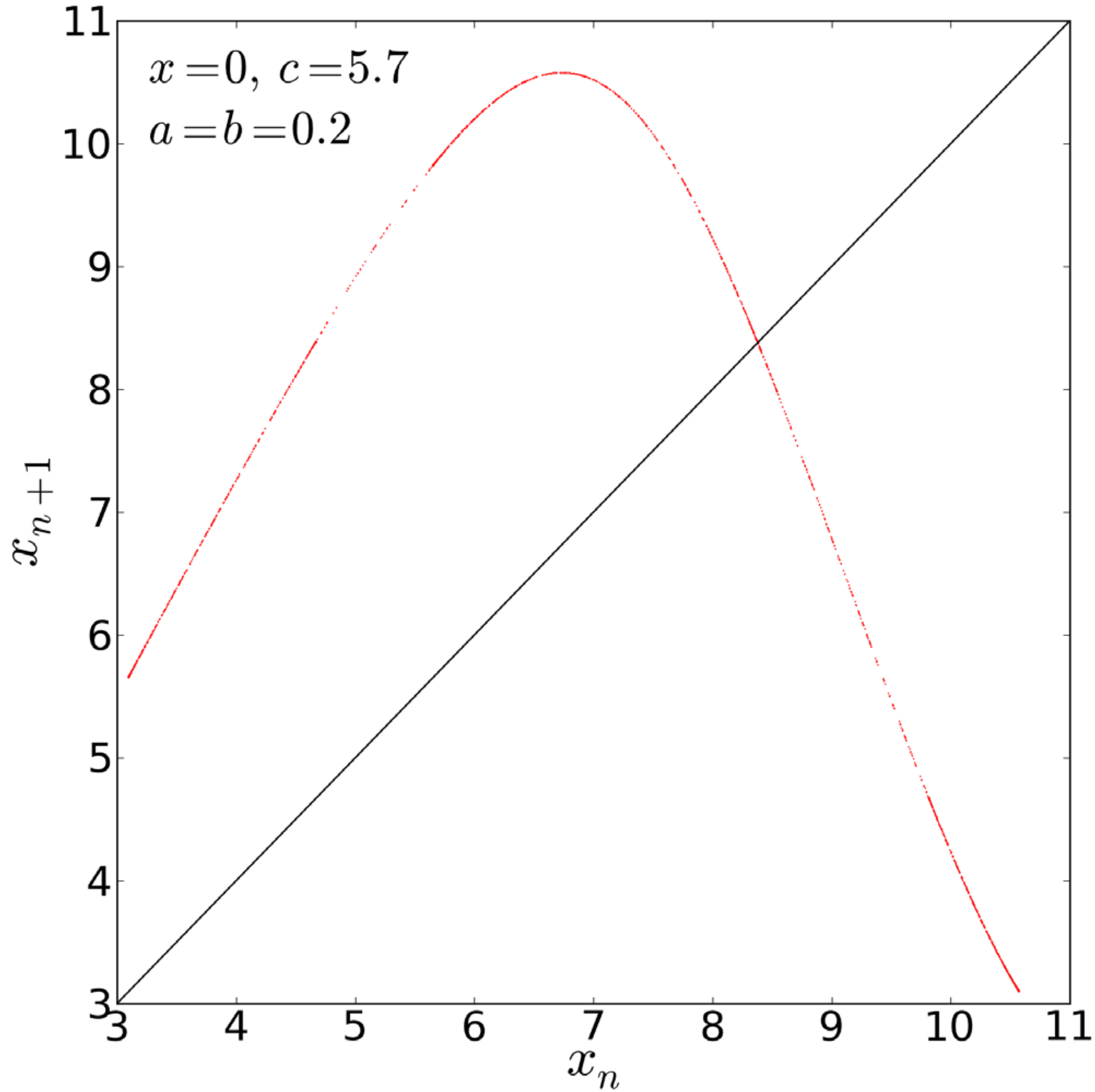
$$\alpha = \lim_{n\to\infty} \frac{d_n}{d_{n+1}} = 2.5029$$

Sequence of periodic windows: 6,5,3, ...


(a)
(b)
(c)



$x=0$
y
c
−3
−4
−5
−6
−7
−8
−9
−10
−11
4.0
4.5
5.0
5.5



$x=0,\ c=5.7$
$a=b=0.2$
11
10
9
8
7
6
5
4
3
3
4
5
6
7
8
9
10
11
x_{n+1}
x_n

Previous work

Optimization method for finding periodic orbits:

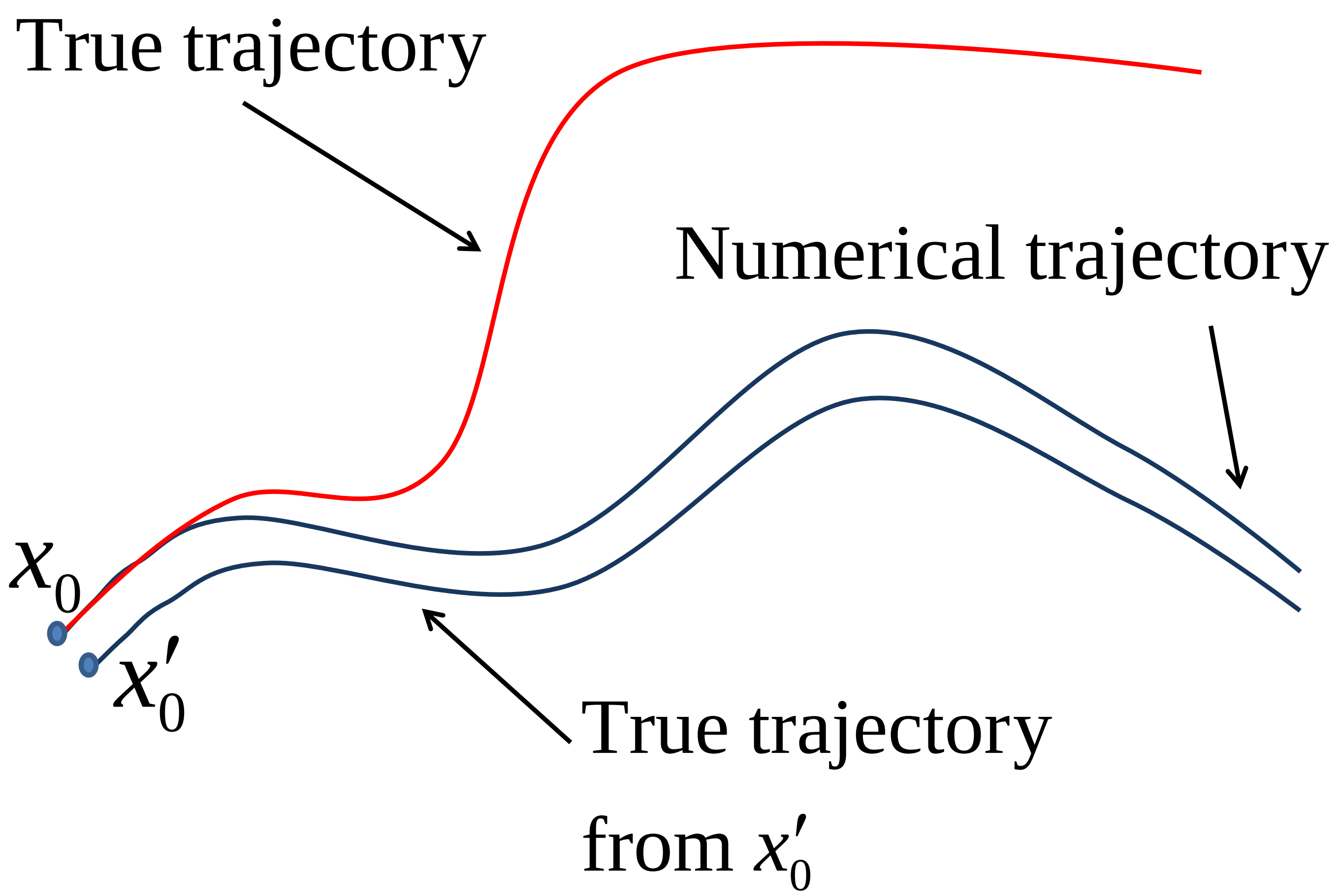
W. Dednam and A.E. Botha, Engineering with Comp. **31**, 126 (2015)

Conjecture:

For any initial condition (x_0, y_0, z_0) there exists real non-zero parameters defining a Rössler system for which the solution through (x_0, y_0, z_0) is periodic.

W. Dednam and A.E. Botha, SAIP Conference Proceedings (2014)

Shadowing



S.M. Hammel et al., J. Complexity **3**, 136 (1987)

Computer Assisted 'Proof'

n	α	x'_0	y'_0	z'_0	T	a	b	c
1	1	-0.322	0.283	-0.827	6.285	-0.01832876699	-37.861974659	45.448602039
2	1	0.083	0.498	0.756	6.282	0.01659280123	34.3119795334	45.496814976
3	1	0.271	-0.033	-0.492	6.284	-0.01074867143	-22.256338664	45.495734090
4	1	-0.843	-0.154	-0.338	6.283	-0.00756927443	-15.661740755	45.483293583
5	1	0.920	-0.101	-0.062	6.285	-0.00133598606	-2.7619826425	45.463987246
6	10	0.274	-0.843	-0.019	6.280	-0.00346652925	-9.6387456311	53.299901841
7	10	-0.431	0.429	-0.132	6.287	-0.02714795455	-75.519091787	52.954713982
8	10	-0.762	0.094	-0.023	6.283	-0.00504377136	-13.929997631	52.959008822
9	10	0.721	0.212	0.532	6.303	0.09058831072	242.356061832	52.926271854
10	10	0.212	-0.008	-0.946	6.412	-0.18855060455	-483.78516338	53.074125658
11	100	0.370	0.525	-0.489	9.221	-0.72151348243	-2913.5197570	96.641076651
12	100	0.465	-0.177	0.111	6.506	0.27456932042	1.25094211568	35.351207642
13	100	0.045	-0.881	0.206	1.670	0.28422944418	0.06378881294	25.148204118
14	100	0.448	-0.071	0.533	7.016	0.41997047657	5480.91338691	148.08093025
15	100	0.177	-0.786	0.321	3.795	0.35076508274	0.24268923009	27.447807055

Difficult case

$$\dot{x} = -y - z$$

$$\dot{y} = x + ay$$

$$\dot{z} = b + z(x - c)$$

Consider the case when x_0 is large and negative, and $y_0 = -z_0$, with z_0 large and positive.

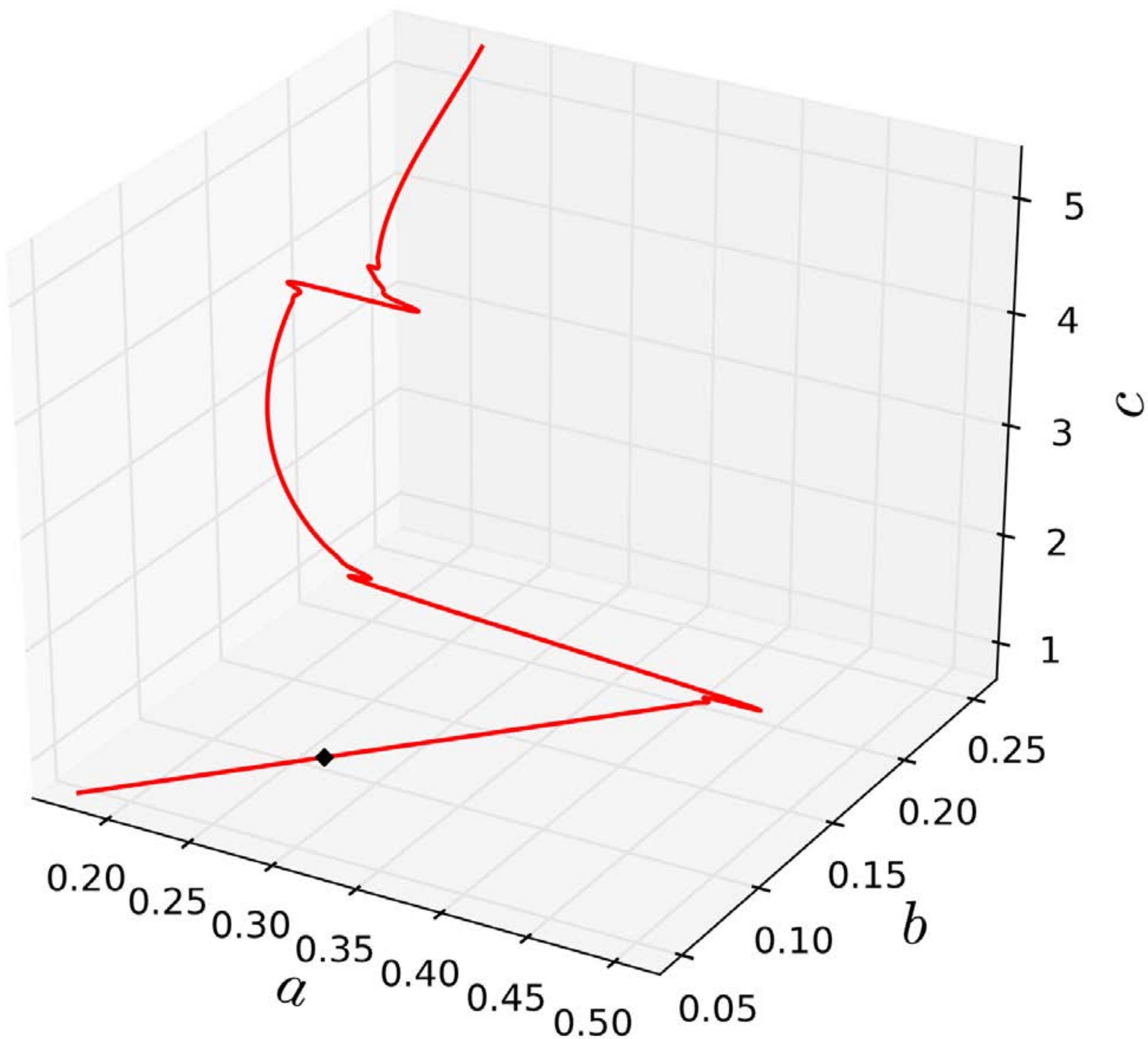
$$z(t) = \frac{b}{c - x} + \left(z_0 - \frac{b}{c - x} \right) e^{-(c-x)t}$$

Impossible case?

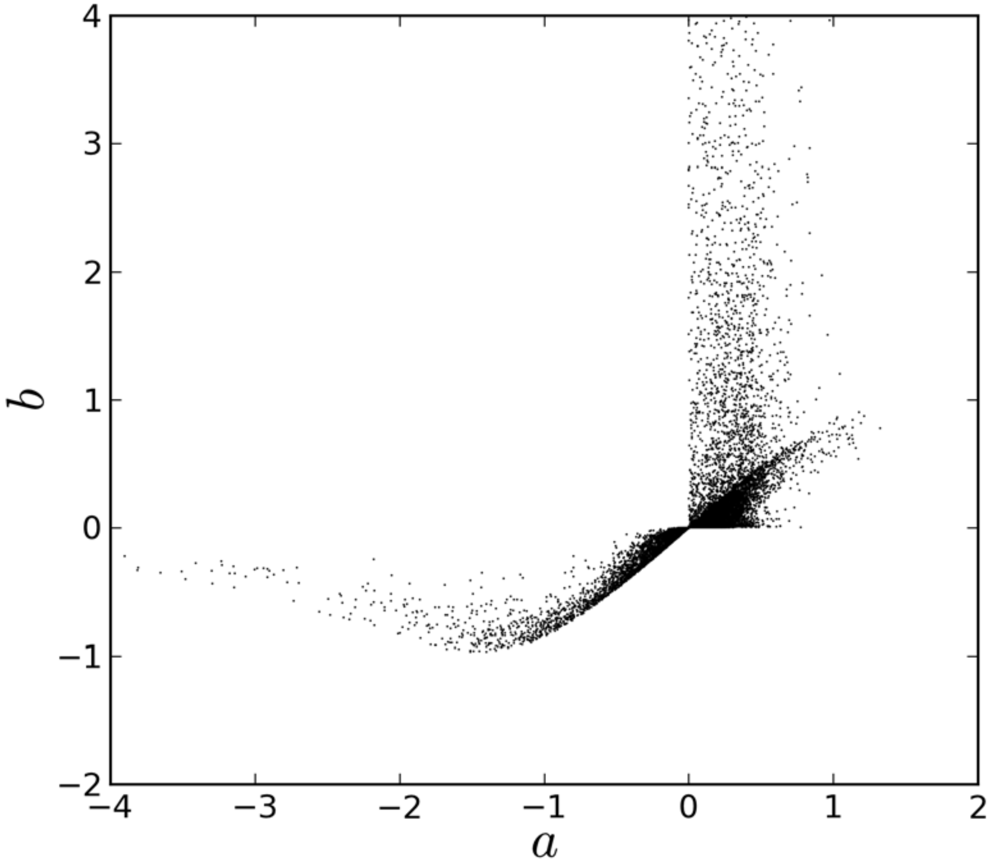
$x = -0.431$

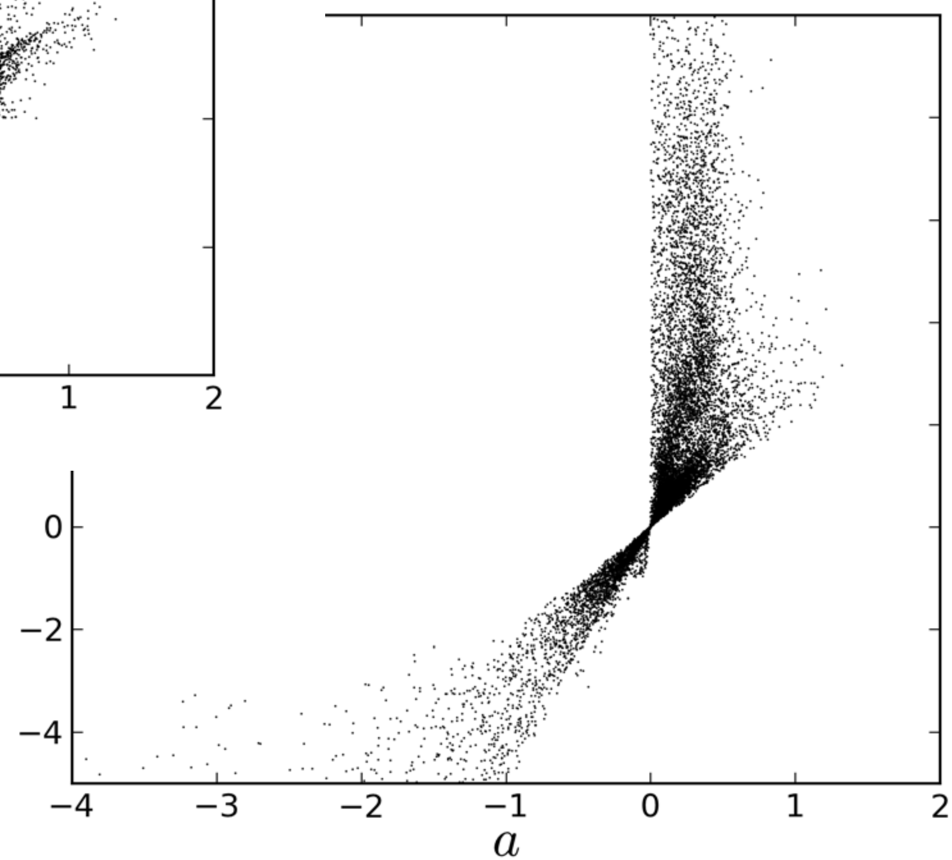
$y = 0.429$

$z = 0.533$



Clustering





Conclusions and questions

- Hypothesis of the possible global existence of periodic orbits has prompted several new questions about a different kind of period doubling route to chaos and clustering in the parameter space.

- Pointed out a different course of possible investigation: is there still universality in period doubling routes to chaos which have always one point in common?

- What kind of bifurcations occur in this case?