The pair-model of *mono*polar and *di*polar moments of elemental electric scalar and magnetic vector charges

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Abstract. A sequel to representing elemental sources of magnetic fields as elemental magnetic *vector* charges is realizing that electric and magnetic dipole moments are different classes of moments. The distinction between *mono*polar and *di*polar moments becomes clearer when any distribution of electric *scalar* charge or magnetic *vector* charge is depicted as one or more pairs of charges with equal magnitudes. It is shown here that separation of the charges (electric scalar or magnetic vector) is essential for the very existence and other attributes of an electric or magnetic dipole, but not for a monopole. These representations are markedly different from the traditional analogous representations and notions of electric and magnetic dipole *moments* as sources of corresponding fields or potentials.

1. Introduction

The recent conception of elemental sources of magnetic fields as elemental magnetic vector charges [1–2] has many spin-offs. This paper shows electric and magnetic *di*polar moments as pairs of own *mono*polar moments of electric *scalar* and magnetic *vector* charges, and reveals faults in traditional [3–5] notions, description and terminologies.

2. Representation and classification of *mono*polar and *di*polar moments

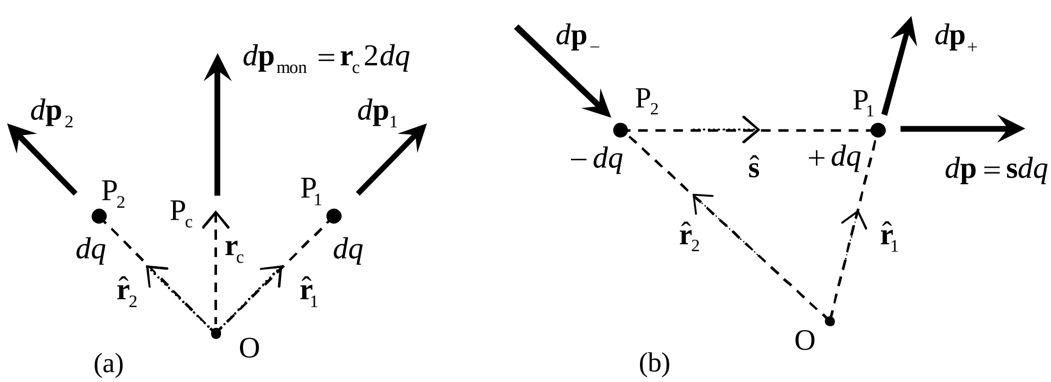

Figure 1. (a) *Mono*polar $d\mathbf{p}_{\text{mon}}$ and (b) *di*polar $d\mathbf{p}$ *collinear* moments for paired *scalar* charges.

For elemental electric *scalar* charges dq and dq at points P_1 and P_2 of positions $\mathbf{r}_1$ and $\mathbf{r}_2$, figure 1(a) shows their *mono*polar moments $d\mathbf{p}_1$ and $d\mathbf{p}_2$. Their joint moment is the *mono*polar moment

$$d\mathbf{p}_{\text{mon}} = d\mathbf{p}_1 + d\mathbf{p}_2 = \mathbf{r}_1 dq + \mathbf{r}_2 dq \equiv (\mathbf{r}_1 + \mathbf{r}_2) dq \equiv \mathbf{r}_\text{c}\, 2dq \tag{1}$$

is collinear with the position vector $\mathbf{r}_\text{c} = \frac{1}{2}(\mathbf{r}_1 + \mathbf{r}_2)$ of the *centre* P_c of its *net* charge $2dq \neq 0$. If the charges were negative in sign, each moment would be opposite in direction to the position vector.

Figure 1(b) shows the *mono*polar moments $d\mathbf{p}_+$ and $d\mathbf{p}_-$ of $+dq$ and $-dq$, as well as their sum

$$d\mathbf{p} = d\mathbf{p}_+ + d\mathbf{p}_- = \mathbf{r}_1(+dq) + \mathbf{r}_2(-dq) = \mathbf{r}_1 dq - \mathbf{r}_2 dq \equiv \mathbf{s} dq \tag{2}$$

It is a *di*polar moment, which is collinear with the intra dipolar displacement $\mathbf{s} = \mathbf{r}_1 - \mathbf{r}_2$ drawn from $-dq$ to $+dq$. Traditionally an integral of any mixture of (1) and (2) would qualify as an electric "dipole" moment [3–5]. Yet an integral of $\mathbf{r}dm$ is simply called a moment of the total mass m [6].

In stark contrast to tradition (Gilbert model of 1600 or Dirac magnetic monopole of 1931) [3–5], figure 2(a) shows parallelogram representations of *areal* (area-like) monopolar moments $d\boldsymbol{m}_1$ and $d\boldsymbol{m}_2$ of identical elemental magnetic vector charges $d\mathbf{Q}$ and $d\mathbf{Q}$ at P_1 and P_2. Their sum is

$$d\boldsymbol{m}_{\text{mon}} = d\boldsymbol{m}_1 + d\mathbf{m}_2 = \mathbf{r}_1 \times d\mathbf{Q} + \mathbf{r}_2 \times d\mathbf{Q} \equiv (\mathbf{r}_1 + \mathbf{r}_2) \times d\mathbf{Q} \equiv \mathbf{r}_\text{c} \times 2d\mathbf{Q} \tag{3}$$

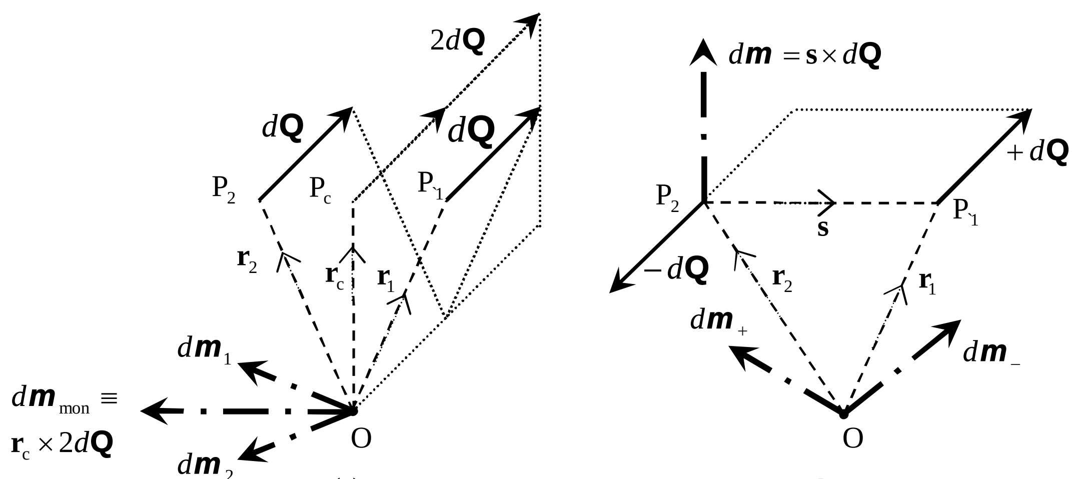

Figure 2. (a) *Mono*polar $d\boldsymbol{m}_{\text{mon}}$ and (b) *di*polar $d\boldsymbol{m}$ *areal* moments for paired *vector* charges.

If the two magnetic vector charges are $+d\mathbf{Q}$ and $-d\mathbf{Q}$, that is, of same magnitude but *opposite* directions, their joint moment is shown in figure 2(b) as a figurative parallelogram based on $\mathbf{s}$, with its "area" vector being

$$d\boldsymbol{m} = d\boldsymbol{m}_+ + d\boldsymbol{m}_- = \mathbf{r}_1 \times (+d\mathbf{Q}) + \mathbf{r}_2 \times (-d\mathbf{Q}) = \mathbf{r}_1 \times d\mathbf{Q} - \mathbf{r}_2 \times d\mathbf{Q} \equiv \mathbf{s} \times d\mathbf{Q} \tag{4}$$

Torque, area vector and angular momentum are areal moments of the vectors force, displacement and linear momentum respectively. However, note that when the opposite sides of a parallelogram are regarded as displacements in opposite directions, then its area vector becomes the areal moment of either pair of opposite sides. Naturally the areal moment of both pairs is ***twice*** the area vector [1].

3. Geometric expansion of electric scalar and magnetic vector potentials

Figure 3 displays positions P_1' and P_2' of two *identical* electric scalar charges, with the field point P in free space (permittivity ε_0) displaced from $\mathrm{P}_i',\, i = 1, 2$ by $\hat{\mathbf{R}}_i R_i = \mathbf{r} - \mathbf{r}_i'$. Similarly, figure 4 shows that for electric scalar charges of *opposite* sign but same magnitude. In these two situations the electric scalar potentials at P are respectively

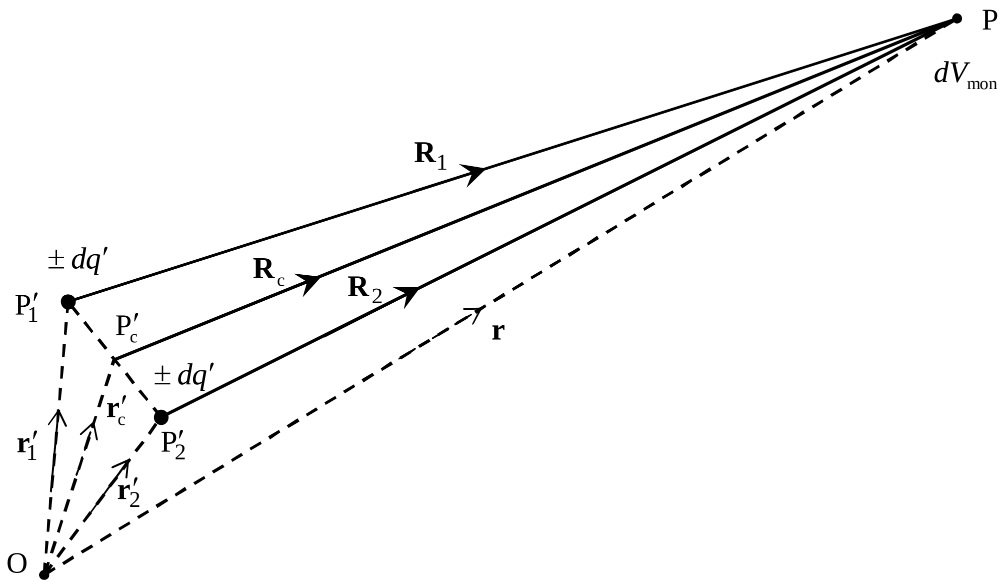

Figure 3. Position vector $\mathbf{r}'_{\text{c}} = \frac{1}{2}(\mathbf{r}'_1 + \mathbf{r}'_2)$ of *centre* P'_{c} and displacement $\mathbf{R}_{\text{c}} = \mathbf{r} - \mathbf{r}'_{\text{c}}$ of P from P'_{c}.

$$dV_{\text{mon}} = dV_1 + dV_2 = \frac{\pm dq'}{4\pi\varepsilon_0 r}\left(f_1 + f_2\right) \equiv f_{\text{mon}} dV_{\text{or}} \tag{5}$$

and

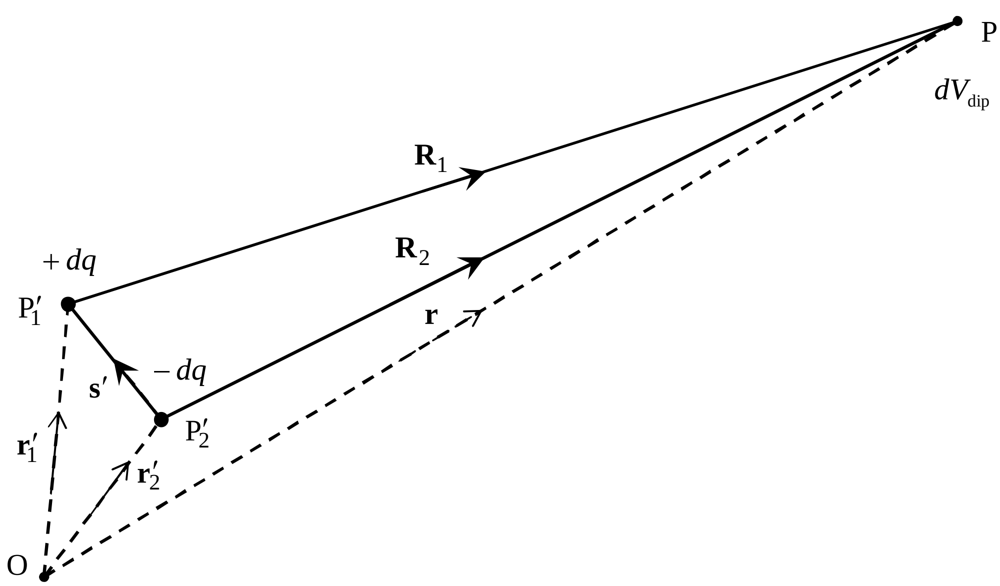

Figure 4. Intra source displacement $\mathbf{s}' = \mathbf{r}'_1 - \mathbf{r}'_2$ and displacements $\mathbf{R}_1$, $\mathbf{R}_2$ of P from P'_1 and P'_2.

$$dV_{\text{dip}} = dV_+ + dV_- = \frac{dq'}{4\pi\varepsilon_0 r}\left(f_1 - f_2\right) \equiv f_{\text{dip}} dV_{\text{or}} \tag{6}$$

where

$$dV_{\mathrm{or}} = \frac{dq'}{4\pi\varepsilon_0 r} \tag{7}$$

is the potential at P due to a *single* positive elemental electric scalar charge dq' at the origin O,

$$f_{\mathrm{mon}} = \pm\left(f_1 + f_2\right) = \pm\left(\frac{r}{R_1} + \frac{r}{R_2}\right) \tag{8}$$

the *mono*polar geometrical enhancement factor, and

$$f_{\mathrm{dip}} = f_1 - f_2 = \frac{r}{R_1} - \frac{r}{R_2} \tag{9}$$

the *di*polar geometrical enhancement factor. As $r > r_i'$, a binomial expansion of R_i^{-1} gives

$$f_i \equiv \frac{r}{R_i} = \left\{1 + \frac{\mathbf{r}_i' \cdot \hat{\mathbf{r}}}{r^1} + \frac{3(\mathbf{r}_i' \cdot \hat{\mathbf{r}})^2 - r_i'^2}{2r^2} + \frac{5(\mathbf{r}_i' \cdot \hat{\mathbf{r}})^3 - 3(\mathbf{r}_i' \cdot \hat{\mathbf{r}})\, r_i'^2}{2r^3} + \,.\,.\,.\,.\right\} \approx \left\{1 + \frac{\mathbf{r}_i' \cdot \hat{\mathbf{r}}}{r}\right\} \tag{10}$$

Substituting (10) into (5) shows that the potential at P due to a *single* $\pm\, dq'$ at P_i', can be created by placing *at* O that $\pm\, dq'$ *plus* an infinite series of its *fractions*, whose extreme values are ordered as

$$\pm\frac{r_i'}{r}dq',\ \pm\frac{r_i'^2}{r^2}dq',\ \pm\frac{r_i'^3}{r^3}dq',\ .\ .\ .\ . \tag{11}$$

Thus, the traditional naming of successive terms in such expansions as *monopole*, *dipole*, *quadrupole*, *octupole* and so on [3–5], is inapt. Indeed, it is absurd to speak of the *monopole term*, the *dipole term*, the *quadrupole term*, the *octupole term* and so on, of a point electric scalar charge (a traditional prototype monopole).

Applying the approximation in (10) to equations (8) and (9) respectively yields

$$f_{\mathrm{mon}} \equiv \frac{\pm 2r}{R_{\mathrm{c}}} \approx \pm 2\left\{1 + \frac{\mathbf{r}_{\mathrm{c}}' \cdot \hat{\mathbf{r}}}{r}\right\} \tag{12}$$

and

$$f_{\mathrm{dip}} \approx +\left\{0 + \frac{\mathbf{s}' \cdot \hat{\mathbf{r}}}{r}\right\} \tag{13}$$

The value of f_{mon} varies with $\mathbf{r}_{\mathrm{c}}'$ but stays close to ± 2 and its sign is simply the common sign of the electric scalar charges, so that from (5) and (7), dV_{mon} vanishes only at $r = \infty$. The value of f_{dip} varies with $\mathbf{s}'$ but stays close to ± 0, and its sign is that of the product $\mathbf{s}' \cdot \hat{\mathbf{r}}$. Thus dV_{dip} has a relatively reduced magnitude. Except for $\hat{\mathbf{r}} \perp \mathbf{s}'$, the approximation (13) is invalid for all P in the plane perpendicularly bisecting $\mathbf{s}'$, where dV_{dip} is always exactly zero. This fault is never mentioned.

Using the transformations $\pm\, dq \to dm$, $\varepsilon_0 \to \xi_0$, $dV \to d\psi$ and $\pm\, dq \to \pm d\mathbf{Q}$, $\varepsilon_0 \to \mu_0$, $dV \to d\mathbf{A}$, respectively yield harmonized equations (*gravitativity* $\xi_0 < 0$, permeability $\mu_0 > 0$) [7] for the gravitational scalar $d\psi$ and magnetic vector $d\mathbf{A}$ potentials. Note that although $r'dq'$ is the magnitude of a collinear moment, $r'd\mathbf{Q}'$ is neither an areal moment nor its magnitude.

Now let us examine two extreme configurations of *mono*poles and *di*poles. From the above, when $\mathbf{r}_1 = \mathbf{r}_2$ or $\mathbf{r}_1' = \mathbf{r}_2'$, both the moment and potential of a monopole survive; but these die out for a dipole as $\mathbf{s} = \mathbf{0}$ and $f_{\mathrm{dip}} = 0$. On the other hand, if $\mathbf{r}_1 = -\mathbf{r}_2$ or $\mathbf{r}_1' = -\mathbf{r}_2'$, both the moment and potential of a dipole are nonzero quantities as $\mathbf{s} \neq \mathbf{0}$ and $f_{\mathrm{dip}} \neq 0$; whereas for a monopole, the moment vanishes but its potential remains as $\mathbf{r}_{\mathrm{c}} = \mathbf{0}$ and $f_{\mathrm{mon}} = \pm 2$.

4. Conclusion

Any dipole (electric or magnetic) has equal amounts of *separated* charges with opposite signs or directions, creates a position-*independent* (collinear or areal) moment and a relatively reduced magnitude of the electric scalar or magnetic vector potential as the 0^{th} order term is absent. The spatial separation of charges is a requirement of the existence of a dipole. A monopole is a pair of equal amounts of charges with the same sign or direction, has a centre of non-zero *net* charge, a position-*dependent* moment and an enhanced potential which is dominated by the 0^{th} order term and never vanishes except at $r = \infty$. Faults in the traditional notions and terminologies of electric and magnetic dipole moments have also been revealed.

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