NLO Rutherford Scattering and energy loss in a QGP

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Abdullah Khalil
abdullah@aims.ac.za
Supervised by
Dr. W. A. Horowitz

University of Cape Town
South Africa

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Overview

Introduction

QCD Phase Diagram

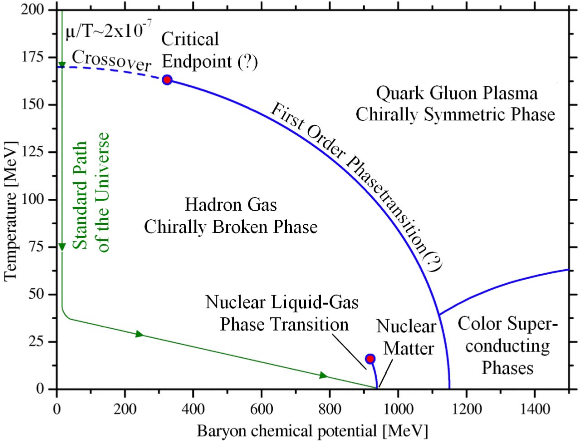


Figure : QCD phase diagram

Temperature dependence of the energy density

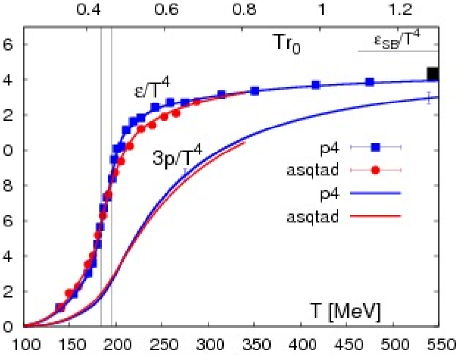


Figure : Temperature dependence of the energy density by Lattice QCD

Introduction

Physics at RHIC

At RHIC we study the dynamics of the QGP in two different limits:

Strongly coupled limit

- It is non-perturbative approach.
- Gives a good estimate for the dynamics of the particle at low $p_\perp$.

Weakly coupled limit

- It is perturbative approach, based on the asymptotic freedom of QCD.
- It describes the physics associated with high $p_\perp$.

Why weakly coupled limit?

The Lagrangian of The System

Consider the Lagrangian of an electron scattered with a fixed point charge

$$\mathscr{L} = -\frac{1}{4}\left(F^{\mu\nu}\right)^2 + \overline{\psi}\left(i\partial\!\!\!/ - m\right)\psi - e\,\overline{\psi}\gamma^\mu\psi A_\mu + e\,J_\mu A^\mu$$

Where

$$J^\mu = V^\mu \delta(\vec{x} - \vec{v}x^0)$$
$$V^\mu = (1,0)^\mu$$

Feynman Rules of The Leading Term

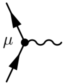
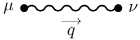
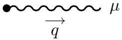
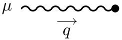
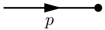
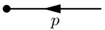
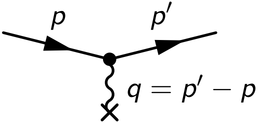
For each vertex:		$= -ie\gamma^{\mu}$
For each internal photon:		$= \frac{-ig_{\mu\nu}}{q^2+i\epsilon}$
For each incoming external photon:		$= \varepsilon_{\mu}(q)$
For each outgoing external photon:		$= \varepsilon^{*}_{\mu}(q)$
For each incoming external fermion:		$= u(p)$
For each outgoing external fermion:		$= \bar{u}(p)$
For each external source:		$= -ie\,V^{\mu}$

Figure : Feynman rules of an electron scattered with a classical potential V

The Leading Order of the differential Cross Section

Using feynman rules for leading term

$$i\mathcal{M}_0 =$$



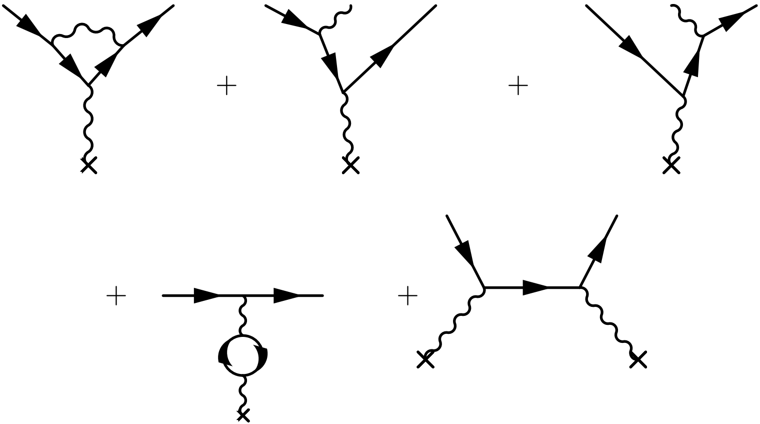
$$= \frac{i\,e^2}{q^2}\overline{u}^{s'}(p')\,\gamma^0\,u^s(p)$$

The cross section of the leading term will be

$$\left(\frac{d\sigma}{d\Omega}\right)_0 = \frac{1}{32\pi^2}\sum_{s,s'}|\mathcal{M}_0|^2 = \frac{2\alpha^2}{q^4}(2E^2 - p\cdot p')$$

Next-to-Leading Order $\mathcal{O}(\alpha^3)$

NLO Diagrams



Divergences in the NLO diagrams

The Ultra-violet divergences

- Due to loop integrals.

The Infra-red divergences

- Emission or absorption of massless photons.

The collinear divergences

- Emission or absorption of a massless photons collinearly with a massless electron.

UV divergence cancellation

Renormalization

We follow the renormalization steps:

1. We define the Lagrangian in terms of the bare parameters
$$\mathscr{L}_0 = -\frac{1}{4}\left(F_0^{\mu\nu}\right)^2 + \overline{\psi}_0\left(i\partial\!\!\!/ - m_0\right)\psi_0 - e_0\,\overline{\psi}\gamma^\mu\psi A_{0\mu} + e_0\,J_{0\mu}A_0^\mu$$
2. We renormalize the bare fields (ψ_0 and A_0^μ) and the bare parameters (e_0 and m_0) by defining the renormalization parameters Z_ψ, Z_A, Z_e and Z_m
$$\begin{aligned} \psi_0 &= Z_\psi^{\frac{1}{2}}\psi \\ A_0^\mu &= Z_A^{\frac{1}{2}}A^\mu \\ Z_\psi m_0 &= Z_m m \\ e_0 Z_\psi Z_A^{\frac{1}{2}} &= Z_e e \end{aligned}$$

Renormalization Procedure Cont...

3. We expand the renormalization patrameters in terms of the counter terms

$$
\begin{aligned}
Z_\psi &= 1 + \delta_\psi \\
Z_A &= 1 + \delta_A \\
Z_e &= 1 + \delta_e \\
Z_m &= 1 + \delta_m
\end{aligned}
$$

4. We rewrite the Lagrangian in terms of the Renormalized fields and parameters (ψ, A, m and e and the counter terms)

$$
\begin{aligned}
\mathscr{L} = &-\frac{1}{4}\left(F^{\mu\nu}\right)^2 + \overline{\psi}\left(i\partial\!\!\!/ - m\right)\psi - e\,\overline{\psi}\gamma^\mu\psi A_\mu + e\,J_\mu A^\mu \\
&- \frac{1}{4}\delta_A\left(F^{\mu\nu}\right)^2 + \overline{\psi}\left(i\delta_\psi\,\partial\!\!\!/ - m\,\delta_m\right)\psi - e\,\delta_e\,\overline{\psi}\gamma^\mu\psi A_\mu + e\,J_\mu A^\mu
\end{aligned}
$$

Renormalization

Feynman Rules of The Renormalized Lagrangian

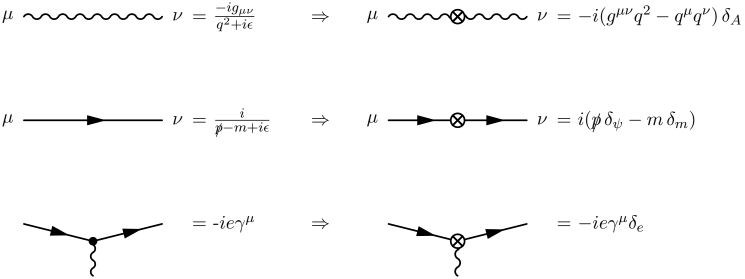


Figure : Feynamn rules of the renormalized QED

Renormalization

Renormalization Tools

- Dimensional Regularization to regularize the U.V divergences, which requires Introducing the mass scale μ.

$$\int \frac{d^4 k}{(2\pi)^4} \to \int \frac{d^d k}{(2\pi)^d} \qquad \Rightarrow \qquad e \to e\,\mu^{\frac{4-d}{2}}$$

- Mass Regularization (m_γ, m) to regularize both IR and collinear divergences.

$$\frac{-ig_{\mu\nu}}{k^2} \to \frac{-ig_{\mu\nu}}{k^2 + m_\gamma^2}$$

- $\overline{MS}$ Renormalization Scheme.

Why did we use $\overline{MS}$?

On-shell VS. $\overline{MS}$

On-shell renormalization scheme:

- We use the renormalization conditions to tame the UV divergence.
- The physical quantities are the renormalized ones.
- The differential cross section diverges as we send the mass of the electron (m_e) to be zero.

$\overline{MS}$ renormalization scheme:

- We choose the counter terms such that it removes the $(\frac{1}{\epsilon} + \log(4\pi) - \gamma_E)$ term.
- The renormalized parameters are not necessarily the physical ones and the value of the residue is no longer one.
- The differential cross section is finite as we send the electron mass (m_e) to be zero.

Mass and Residue Corrections

Full Electron Propagator

The Fourier transform of the two point correlation function of the electron self energy is given by

$$\int d^4x \, \langle\Omega| \, T(\psi(x)\bar{\psi}(0)) \, |\Omega\rangle \, e^{ip\cdot x} = \frac{i}{\not{p} - m - \Sigma(\not{p})} \, .$$

This means that the pole is shifted by $\Sigma(\not{p})$, so the renormalized mass is not the physical mass and the residue of this pole is no longer one.

The Physical Mass and Residue Correction

The physical mass can be given by the position of the pole

$$\left(\not{p} - m - \Sigma(\not{p})\right)\Big|_{\not{p}=m_e} = 0$$

Which implies

$$m_e = m\left[1 + \frac{\alpha}{4\pi}\left(4 + 3\log\left(\frac{\mu^2}{m^2}\right)\right) + O(\alpha^2)\right]$$

The inverse of the residue is given by

$$\begin{aligned} R^{-1} &= \frac{d}{d\not{p}}\left(\not{p} - m - \Sigma(\not{p})\right)\Big|_{\not{p}=m_e} \\ &= 1 - \frac{\alpha}{4\pi}\left[2\log\left(\frac{m^2}{m_\gamma^2}\right) - \log\left(\frac{\mu^2}{m^2}\right) - 4\right] + \mathcal{O}(\alpha^2) \end{aligned}$$

We should multiply the amplitude by $R^{1/2}$ for each external leg, which means that we multiply the differential cross section by R^2.

IR and collinear divergences cancellation

BN Vs. KLN

There are two main theorems describing the cancellation of the IR and collinear divergences:

The Bloch-Nordsiek theorem

- One should sum over the emitted soft photons (i.e Photons with energy less than the experimental energy resolution (Δ)) to cancel the IR divergences!

Kinoshita-Lee-Neunberg (KLN) theorem

- One should sum over both emitted and absorbed hard photons within a cone of an angle less than the experimental angular resolution (δ) to get rid of the collinear divergences!

The NLO correction to the differential Cross Section

The final formula will be

$$
\begin{aligned}
\left(\frac{d\sigma}{d\Omega}\right) &= \frac{1}{32\pi^2}\sum_{s,s'}\left(R^2|\mathcal{M}_0|^2 + \mathcal{M}_0^*\mathcal{M}_V + \mathcal{M}_V^*\mathcal{M}_0 + \mathcal{M}_0^*\mathcal{M}_P\right.\\
&\qquad\left. + \mathcal{M}_p^*\mathcal{M}_0 + \mathcal{M}_0^*\mathcal{M}_{BO} + \mathcal{M}_{BO}^*\mathcal{M}_0 + |\mathcal{M}_B|^2\right)\\
&= \left(\frac{d\sigma}{d\Omega}\right)_0 \left\{1 + \frac{\alpha}{\pi}\left[\log\left(\frac{\Delta^2}{E^2}\right)\left(1 - \log\left(\frac{\delta^2 E^2}{-q^2}\right)\right)\right.\right.\\
&\qquad\left. - \frac{3}{2}\log\left(\frac{\delta^2 E^2}{-q^2}\right) + \log\left(\frac{\delta^2 E^2}{m^2}\right)\left(\frac{2\Delta}{E} - \frac{\Delta^2}{2E^2}\right)\right]\\
&\qquad\left. - \frac{\pi^2}{3} + \frac{5}{36}\right\} + \frac{\pi\alpha^3 E}{p\,Q\,q^2}(p - Q) + O(\alpha^4)\,.
\end{aligned}
$$

Comments

There are two main comments on the previous results:

- There are two collinear divergences (ignored by LN paper) that have not been cancelled yet!
- We used a combination between the BN and KLN theorems! which provide a question about the consistency of such a treatment.

There are some suggestions to overcome the problems states above respectively:

- We will check the calculations of the soft bremsstrahlung emission beyond the Eikonal approximation.
- We will check including the disconnected diagrams for the initial state soft bremsstrahlung divergences cancellation to stay in the spirit of the KLN theorem.

Conclusion

- All U.V, I.R and the collinear divergences has been cancelled by using $\overline{MS}$ renormalization scheme, the BN and KLN theorems.
- the treatment of applying both BN and KLN theorems separately to get rid of the IR and collinear divergences is inconsistent.
- We use the more general theorem (KLN), however Including the absorption of soft photons will double the IR divergences. So a further work needs to be done to get rid of these extra infinities. One suggestion is to look at the disconnected diagrams.
- After the cancellation of all the infinities we expect a result for the differential cross section to be finite and valid up to arbitrary large momentum exchange.
- We have used a very simple and powerful renormalization scheme which can be used for the QCD calculations as we deal with the light quarks (nearly zero mass).

Thank you!