Electromagnetic Form Factors of Three-Nucleon Systems

G J Rampho[1], **S A Sofianos**[1], **S Oryu**[2] **and T Watanabe**[2]

[1] Department of Physics, University of South Africa, Pretoria 0003, South Africa
[2] Department of Physics, Tokyo University of Science, Noda, Chiba 278-8510, Japan

E-mail: `rampho@science.unisa.ac.za`

Abstract. The angular-momentum-projected and parity-projected antisymmetrized molecular dynamics is used to analyse the charge and magnetic form factors of the three-nucleon systems. Non-relativistic nuclear charge and current operators with relativistic corrections are employed. The Hamiltonian of the nuclear systems is described with a semi-realistic nucleon-nucleon potential. The results obtained are compared with some experimental data. It is found that the theoretical model describes experimental data very well at low momentum transfer.

1. Introduction

Charge and magnetic form factors are often used to test model wave functions for systems. In such tests the question of simultaneous accurate description of the form factors and the static properties of the systems is raised. The structure of few-nucleon systems and light nuclei is widely and continuously investigated, both theoretically and experimentally. Over the years a variety of theoretical methods have been developed and refined in the study of electromagnetic processes in nuclei. Very accurate wave functions for bound and scattering states in few-nucleon systems can now be constructed using realistic Hamiltonian for the systems. A demonstration of the level of accuracy in describing ground state properties of the four-nucleon system by seven different state-of-the-art methods is shown in reference [1]. These methods are continuously developing. This work focuses on the use of one of the microscopic simulation methods, the antisymmetrized molecular dynamics (AMD) in the study of electromagnetic form factors of three-nucleon systems.

The AMD approach was developed [2] from the Time-Dependent Cluster Model [3] for the study of fermionic systems. This approach combines Fermi-Dirac statistics with elementary quantum mechanics to treat the motion of particles in a system [4]. However, the model is not fully quantum mechanical and does not assume a shell structure for the system. The AMD approach was used to study the dynamics of heavy-ion collisions [5] and elastic proton-nucleus scattering [6]. Clustering in nuclei as well as angular distributions of scattered protons in proton-nucleus scattering can be well explained with the AMD model [6]. Improved AMD wave functions are shown to give good predictions of few-body systems [7, 8]. In this work the parity projected and angular momentum projected AMD approach [9] is employed.

In the next Section 2 the general formalism of the AMD approach is summarised. In this section the construction of the wave function, the equations of motion of the variable parameters and the variational technique used are briefly outlined. Results and illustrations of

the application of AMD to three-nucleon and four-nucleon systems are presented in Section 3 and in Section 4 for the charge and magnetic form factors, respectively. Conclusions drawn are given in Section 5.

2. The AMD Formalism

The antisymmetrised molecular dynamics (AMD) wave function describing a nuclear system of A nucleons is constructed as a Slater determinant

$$\Psi_{AMD}(\vec{\mathrm{S}}) = \frac{1}{\sqrt{A!}} \det[\,\phi_j(\alpha, \vec{s}_i), \chi_j(\vec{\sigma}_i), \xi_j(\vec{\tau}_i)\,] \tag{1}$$

where ϕ, χ and ξ are, respectively, the spatial, spin and isospin components of the single-particle wave functions. The spatial components are non-orthogonal Gaussian with complex variational parameters $\vec{s}_i$. The width parameter α is taken as real and free. A wave function with definite parity(π) and total angular momentum (J) with the angular momentum projection (M) is constructed from the AMD wave function as

$$\Psi^{J\pi}_{MK}(\vec{\mathrm{S}}) = \frac{1}{2} P^J_{MK}(\Omega)\,[\,1 \pm P^\pi\,]\,\Psi_{AMD}(\vec{\mathrm{S}}) \tag{2}$$

where $P^J_{MK}(\Omega)$ is the angular momentum projection operator, P^π the parity projection operator and $\vec{\mathrm{S}} \equiv \{\vec{s}_1, \vec{s}_2, \vec{s}_3, \ldots, \vec{s}_A\}$. The angular momentum projection operator is defined by [10]

$$P^J_{MK}(\Omega) = \frac{2J+1}{8\pi^2} \int d\Omega\, D^{J*}_{MK}(\Omega)\, \hat{R}(\Omega) \tag{3}$$

where $D^J_{MK}(\Omega)$ is the Wigner D-function, $\hat{R}(\Omega)$ the rotation operator and $\Omega \equiv \{\alpha, \beta, \gamma\}$ the Euler rotation angles.

The single nucleon wave functions are given by

$$\psi_i(\vec{r}_j) = \left(\frac{2\alpha}{\pi}\right)^{2/4} \exp\left[-\alpha\left(\vec{r}_j - \frac{\vec{s}_i(t)}{\sqrt{\alpha}}\right)^2 + \frac{1}{2}\vec{s}_i^2(t)\right] \otimes \chi_i \otimes \xi_i \tag{4}$$

where $\chi_i \otimes \xi_i$ are the fixed spin-isospin states of the i-th nucleon. These states are compactly expressed in the form $\kappa_i = \{\,N\!\uparrow\ \text{or}\ N\!\downarrow\,\}$ for nucleon with spin-up or spin-down. The Gaussian width parameter α is a real constant and the variational parameter $\vec{s}(t)$ is complex. The time-dependent variational principle [5]

$$\delta \int_{t_1}^{t_2} \frac{\langle\,\Psi(\vec{S})\,|\,i\hbar\frac{\partial}{\partial t} - H\,|\,\Psi(\vec{S})\,\rangle}{\langle\,\Psi(\vec{S})\,|\,\Psi(\vec{S})\,\rangle}\, dt = 0 \tag{5}$$

with the constraints

$$\delta\Psi(t_1) = \delta\Psi(t_2) = \delta\Psi^*(t_1) = \delta\Psi^*(t_2) = 0 \ . \tag{6}$$

is used to determine the dynamical equations for the variational parameters. The resulting equations can be transformed into the form [11]

$$\frac{d\vec{s}_i}{dt} = -\mu\,\frac{\partial E_0^{J\pm}(\vec{S}, \vec{S}^*)}{\partial \vec{s}_i^*} \qquad , \qquad \frac{d\vec{s}_i^*}{dt} = -\mu\,\frac{\partial E_0^{J\pm}(\vec{S}, \vec{S}^*)}{\partial \vec{s}_i} \tag{7}$$

where μ is an arbitrary positive real constant and

$$E_0^{J\pm}(\vec{S}, \vec{S}^*) = \frac{\langle\,\Psi^{J\pm}_{MK}(\vec{\mathrm{S}})|\,H\,|\Psi^{J\pm}_{MK}(\vec{\mathrm{S}})\,\rangle}{\langle\,\Psi^{J\pm}_{MK}(\vec{\mathrm{S}})|\Psi^{J\pm}_{MK}(\vec{\mathrm{S}})\,\rangle} \ . \tag{8}$$

the variational energy of the nucleus. Solving these equations minimizes E_0 and determines the variational parameters. The Hamiltonian of the system is given by

$$H \;=\; -\sum_i \frac{\hbar^2}{2\,M_i} \nabla_i^2 + \frac{1}{2} \sum_{i\neq j} \left[\, V_{NN}(\vec{r}_{ij}) + V_C(\vec{r}_{ij}) \,\right] \tag{9}$$

where M_i is the mass of nucleon i, V_{NN} the two-body nuclear potential and V_C the Coulomb potential. In this work the AV4 NN potential with the $V_{C1}(r)$ Coulomb component is used [12]. The evaluation of the components of the energy expectation values is explained in Ref. [13].

3. Charge Form Factor

In elastic electron-nucleus scattering the charge distribution in the nucleus is inferred from the induced electric transitions in the nucleus. The charge form factor is determined as expectation values of the nuclear charge operator. The AMD nuclear charge form factor is given by

$$Z\, F_{ch}(\vec{q}) \;=\; \frac{\langle\, \Psi_{MK}^{J_i\pm} \,|\, \rho(\vec{q}) \,|\, \Psi_{MK}^{J_i\pm} \,\rangle}{\langle\, \Psi_{MK}^{J_i\pm} \,|\, \Psi_{MK}^{J_i\pm} \,\rangle} \tag{10}$$

where Z is the charge on and $\rho(\vec{q})$ the charge operator of the nucleus with $\vec{q}$ as the momentum transferred to the nucleus by the electron. In the plane wave impulse approximation (PWIA) the nuclear charge operator is formed by the superposition of the individual nucleon charge operators. The nuclear charge operator is given by [13]

$$\rho(\vec{q}) \;=\; \sum_{k=1}^{A} \left[\frac{q}{Q}\, G_{Ek}^N(Q^2) - \frac{2\,G_{Mk}^N(Q^2) - G_{Ek}^N(Q^2)}{4\,M_N^2\,\sqrt{1+\tau}}\, \mathrm{i}\vec{\sigma}_k \cdot \vec{q} \times \vec{p}_k \right] \exp(\,\mathrm{i}\,\vec{q}\cdot\vec{r}_k\,) \tag{11}$$

where M_N is the proton mass, $\tau = Q^2/4\,M_N^2$, $Q^2 = q^2 - \omega^2$, $\omega = \sqrt{q^2 + M_N^2} - M_N$ and G_E^N (G_M^N) the nucleon Sachs electric (magnetic) form factor. For the Sachs form factors the phenomenological parametrisation derived in reference [14] is adopted. The transitions are between states of definite angular momentum. The general multipole analysis of nuclear charge form factors is given by [15]

$$F_{ch}(\vec{q}) \;=\; \sqrt{4\pi}\, \sum_{L=0}^{\leq 2J} \langle JJL0|JJ\rangle\, F_L^\rho(q)\, Y_{L0}^*(\hat{\mathbf{q}}) \tag{12}$$

where $Y_{LM}^*(\hat{\mathbf{q}})$ are the spherical harmonics, L the nuclear orbital angular momentum and $\langle JJL0|JJ\rangle$ Clebsch-Gordan coefficients. The summation is over even values of L only. For three-nucleon systems $J_i = \frac{1}{2}$. The intrinsic charge form factor is obtained by dividing the calculated charge form factor by the contributions of the center-of-mass [13].

The ground-state charge form factors of the ^{3}H and ^{3}He nuclei are calculated in the impulse approximation. In this approximation the nucleons inside the target nucleus are assumed not to interact with one another during the nuclear interaction with the electron [16]. Then the electron interacts with independent nucleons inside the nucleus. The results obtained for the calculated charge form factors are displayed in Figure. 1 and Figure. 2. The presented charge form factors are normalised such that $F_{ch}(0) = 1$. In these figures the results of the theoretical predictions of the AMD formalism are compared with the IA theoretical results presented in reference [17]. The calculations of [17] use the Faddeev approach with the SdT NN potential [18] including the three-body force and employs standard parametrisation of nucleon electromagnetic form factors. For low momentum transfers, up to the first diffraction minimum, predictions of
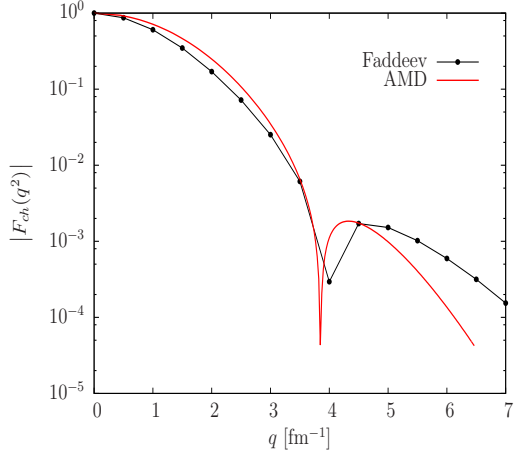
Figure 1. The AMD charge form factor of the ^{3}H nucleus compared with the IA results of reference [17].

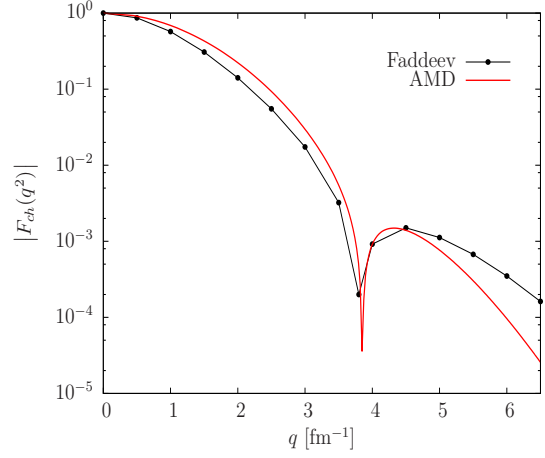**Figure 2.** The AMD charge form factor of the ^{3}He nucleus compared with the IA results of reference [17].

the AMD are similar to those of reference [17]. For momentum transfers greater than the first diffraction minimum the AMD results are lower than those of reference [17]. The AMD model predicts values of the first diffraction minimum for the ^{3}H and ^{3}He nuclei to be at 3.82 fm^{-1}. These predictions are consistent with the predictions of other theoretical models using standard nucleon-nucleon potentials [19]. However, it is known that the overestimation of the position of the diffraction minimum indicates the underestimation of the nuclear charge radius.

4. Magnetic Form Factors

Magnetisation density distribution in nuclei are determined from magnetic transitions involving transverse nuclear currents. The transition amplitude, the nuclear magnetic form factor, is calculated as

$$\mu_A \, F_{mag}(\vec{q}) \;=\; \frac{\langle\, \Psi_{M_f K_f}^{J\pm} \,|\, \vec{\mu}(\vec{q}) \,|\, \Psi_{M_i K_i}^{J\pm} \,\rangle}{\sqrt{\mathcal{N}_{M_f K_f}^{J\pm} \, \mathcal{N}_{M_i K_i}^{J\pm}}} \tag{13}$$

where $\vec{\mu}(\vec{q})$ is the magnetisation density operator and μ_A the nuclear magnetic dipole moment. Following reference [20] the PWIA transverse nuclear magnetisation density operator is given by

$$\vec{\mu}(\vec{q}) \;=\; \frac{Q}{2\,M_p\,q} \sum_{k=1}^{A} \Big[\, G_{Ek}^{N}(Q^2)\, \vec{\ell}_k - \mathrm{i}\, G_{Mk}^{N}(Q^2)\, \vec{q} \times \vec{\sigma} \,\Big] \exp(\,\mathrm{i}\, \vec{q} \cdot \vec{r}_k\,) \tag{14}$$

where $\vec{\ell}_N$ is the nucleon orbital angular momentum. The multipole expansion of nuclear magnetic form factor has the form [15]

$$F_{mag}(\vec{q}) \;=\; \frac{\sqrt{4\,\pi}}{\langle JJ10|JJ\rangle} \sum_{L=0}^{\leq 2J} \langle JJL0|JJ\rangle \Big[\, F_{LL-1}^{\mu}(q)\, \mathbf{Y}_{LL-1}^{0*}(\hat{\mathbf{q}}) + F_{LL+1}^{\mu}(q)\, \mathbf{Y}_{LL+1}^{0*}(\hat{\mathbf{q}}) \,\Big] \tag{15}$$

where the summation is over odd values of L,

$$\mathbf{Y}_{LM}^{0*}(\hat{\mathbf{q}}) \;=\; \sum_{m} \langle Mm1-m|L0\rangle\, Y_{Mm}(\hat{\mathbf{q}})\, \hat{\mathbf{e}}_m \;\;. \tag{16}$$

are the vector spherical harmonics and $\hat{\mathbf{e}}_m$ spherical unit vectors. The general form of the nuclear magnetic transition multipole operator can be derived as in reference [21] for a given nuclear current operator. The intrinsic magnetic form factor of the systems is obtained by factoring-out the contributions of the center-of-mass from equation 15.
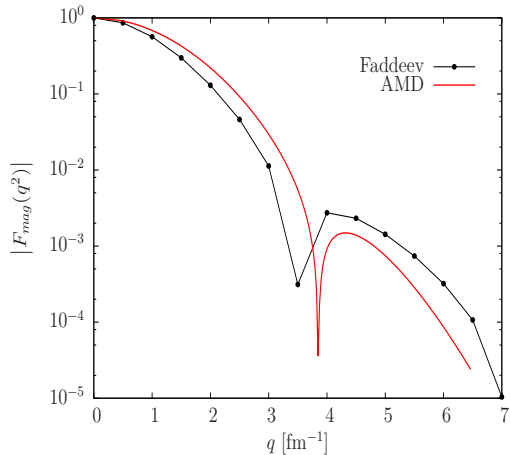

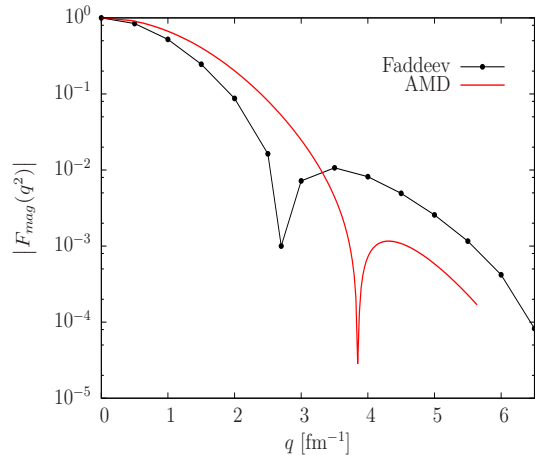**Figure 3.** The AMD magnetic form factor of the ^{3}H nucleus compared with the IA results of reference [17].

Figure 4. The AMD magnetic form factor of the ^{3}He nucleus compared with the IA results of reference [17].

The results obtained for the calculated magnetic form factors of the three-nucleon systems are displayed in Figure. 3 and Figure. 4. In these figures the AMD form factors are compared with the theoretical IA results of reference [17]. The calculated form factors are normalised such that $F_{mag}(0) = 1$. As can be seen the AMD form factors are greater in magnitude than those of reference [17] at low momentum transfer, for both systems. At momentum transfer greater than the diffraction minimum the AMD results are less than those of reference [17]. As a result the AMD predicts a larger value for the diffraction minimum.

5. Concluding Remarks

Electromagnetic form factors of three-nucleon and four-nucleon systems were investigated using electron-nucleus scattering. The properties were determined in the non-relativistic formalism within the impulse approximation. Conventional forms of the one-body nuclear charge and current operators were employed. Nucleon electromagnetic form factors parametrised using recent experimental nucleon-nucleon scattering data were used. Except for the incomplete treatment of many-body current effects, this approach is realistic. Firstly, the charge monopole and the magnetic dipole transitions in the nuclei were determined from elastic electron scattering. These transitions are used to extract information about ground-state charge and magnetisation distributions in the nuclei. The obtained results were compared with predictions obtained using other theoretical methods and with selected experimental data. The AMD approach generates PWIA charge form factors for few-nucleon systems similar to those predicted by other theoretical models. The calculated charge form factors for the nuclei are very close to the experimental form factors for values of momentum transfers less than the first diffraction minimum. For values of momentum transfers greater than the diffraction minimum the theoretical predictions underestimate the experimental data and overestimate the position of the diffraction minimum. In the case of the magnetic form factors the theoretical results generally underestimate the experimental data as well as the position diffraction minimum. The calculated AMD ground-state charge and magnetisation distribution in the nuclei are consistent with the results obtained

with other conventional theoretical methods. The deviations of the theoretical results from experimental data can be explained by the limitations of the electromagnetic operators used. Therefore the AMD wave functions can describe nuclear states as accurately as most conventional theoretical methods.

Acknowledgments

References

[1] Kamada H *et al* 2001 *Phys. Rev.* **C 64** 044001
[2] Horiuchi H 1991 *Nucl. Phys.* **A 522** 257c
[3] Caurier E, Grammaticos B and Sami T 1982 *Phys. Lett.* **B 109** 150
[4] Feldmeier H 1990 *Nucl. Phys.* **A 515** 147
[5] Ono A, Horiuchi H, Maruyama T and Ohnishi A 1992 *Prog. Theor. Phys.* **87** 1185
[6] Tanaka E I, Ono A, Horiuchi H, Maruyama T and Engel A 1995 *Phys. Rev.* **C 52** 316
[7] Togashi T, Katō K 2007 *Prog. Theor. Phys.* **117** 189
[8] Watanabe T and Oryu S 2006 *Prog. Theor. Phys.* **116** 429
[9] Kanada-En'yo Y, Horiuchi H and Ono A 1995 *Phys. Rev.* **C 52** 628
[10] Peierls R E and Yoccoz J 1957 *Proc. Phys. Soc.* **A 70** 381
[11] Ono A and Horiuchi H 2004 *Prog. Part. Nucl. Phys.* **53** 501
[12] Wiringa R B and Pieper S C 2002 *Phys. Rev. Lett.* **89** 182501-1
[13] Rampho G J 2010 *PhD Thesis*, University of South Africa, unpublished.
[14] Friedrich J and Walcher Th 2003 *Eur. Phys. J.* **A 17** 607
[15] Uberall H 1971 *Electron Scattering from Complex Nuclei*, Part A (Academic, New York)
[16] Chew G F and Wick G C 1952 *Phys. Rev.* **85** 636
[17] Hadjimichael E, Goulard B and Bornais R 1983 *Phys. Rev.* **C 27** 831
[18] De Tourreil R, Rouben B and Sprung D W L 1975 *Nucl. Phys.* **A 242** 445
[19] Kloet W M and Tjon J A 1974 *Phys. Lett.* **B 49** 419
[20] Ciofi degli Atti C 1980 *Prog. Part. Nucl. Phys.* **3** 163
[21] Willey R S 1963 *Nucl. Phys.* **40** 529