

# Constraining the phase space for chameleon dark energy

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**Abstract.** A number of solutions to the dark energy problem have been proposed in literature, the simplest is the cosmological constant  $\Lambda$ . The cosmological constant lacks theoretical explanation for its extremely small value, thus dark energy is more generally modeled as quintessence scalar field rolling down a flat potential. For the quintessence scalar field to be evolving on cosmological scales to day its mass must be of order  $H_0$ , which is the present value of the Hubble constant. A scalar field  $\phi$  whose mass varies with the background energy density was proposed by Khoury and Weltman. This scalar field can evolve cosmologically while having coupling to different matter fields of order unity. Such a scalar field also couples to photons in the presence of an external magnetic field via the  $\phi F^2$  interaction, where  $F$  stands for the electromagnetic field strength tensor. The chameleon-photon coupling of this nature causes a conversion of photons to light Chameleon( $\phi$ ) particles and vice versa. In this work we investigate this effect on pulsars, and we constrain parameters space of this theory.

## 1. Introduction

One of the most intriguing discoveries in modern cosmology is the accelerating expansion of the universe. This acceleration is said to be caused by a fluid component of the universe which has negative pressure, this form of energy density is called dark energy (DE). Cosmological observations such as supernovae luminosity-distance measurements [6], and the cosmic microwave background anisotropy [7] suggests that DE forms about 70% of the energy density of the universe. A number of solutions to the dark energy problem have been proposed in literature, the simplest is the cosmological constant  $\Lambda$ . The cosmological constant lacks theoretical explanation for its extremely small value, thus dark energy is more generally modeled as quintessence scalar field rolling down a flat potential [5]. For the quintessence scalar field to be evolving on cosmological scales to day its mass must be of order  $H_0$ , which is the present value of the Hubble constant.

A scalar field  $\phi$  whose mass varies with the background energy density was proposed by Khoury and Weltman [3]. This scalar field can evolve cosmologically while having coupling  $\beta_m$  to different matter fields of order unity. On Earth for example, the mass of  $\phi$  is sufficiently large i.e  $\mathcal{O}(1\text{mm}^{-1})$ , while on the field its Compton wavelength is typically hundreds of astronomical units(AU). As a result  $\phi$  evades equivalence principle(EP) tests and fifth force constraints from laboratory experiments. The dependence of this scalar fields mass on its environment has given it the name "chameleon scalar field".

Such a scalar field also couples to photons in the presence of an external magnetic field via the  $\phi F^2$  interaction term, where  $F$  stands for the electromagnetic field strength tensor [4]. The

chameleon-photon coupling of this nature causes a conversion of photons to light chameleon particles and vice versa. These effects are similar to those of Axion-like particles(ALPs) which interact with light. This conversion needs two photons and one scalar particle in order to take place, as a result it happens when a photon or a scalar field passes through an external electromagnetic field [2].

This paper is organized in the following way, section(2) we discuss the chameleon scalar field theory, in section(3) we look at how to calculate the chameleon-photon system oscillation probability, in section(4) we calculate the oscillation probability in pulsars, in section(5) it's conclusions, in section(6) it's future work and acknowledgements followed by references.

## 2. Chameleon theory

We consider the following chameleon action  $S$  adapted from [1],

$$S = \int d^4x \sqrt{-g} \left( \frac{1}{2} M_{Pl}^2 R - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) - \frac{e^{\beta_\gamma \phi / M_{Pl}}}{4} F^{\mu\nu} F_{\mu\nu} \right) + \int d^4x \left( \mathcal{L}_m(e^{2\beta_m^i \phi / M_{Pl}} g_{\mu\nu}, \psi_m^i) \right) \quad (1)$$

where  $S$  is the sum of the normalized vacuum Einstein-Hilbert action( $S_{EH}$ ) by equation (2), in which  $R$  is the Ricci scalar,  $M_{Pl} = (8\pi G)^{-1/2}$  is the reduced Planck mass and  $g$  is the determinant of the metric tensor  $g_{\mu\nu}$  [9].

$$S_{EH} = \int dx^4 \sqrt{-g} \frac{1}{2} M_{Pl}^2 R, \quad (2)$$

The second and the third terms give the scalar field  $\phi$  action, with a potential  $V(\phi)$  which is of a runaway form, the potential of this form is required to have the following characteristics given in [3].

$$\lim_{\phi \rightarrow +\infty} V(\phi) = 0, \quad \lim_{\phi \rightarrow +\infty} \frac{V_{,\phi}}{V} = 0, \quad \lim_{\phi \rightarrow +\infty} \frac{V_{,\phi\phi}}{V_{,\phi}} = 0 \dots, \\ \text{as well as} \\ \lim_{\phi \rightarrow 0} V(\phi) = \infty, \quad \lim_{\phi \rightarrow 0} \frac{V_{,\phi}}{V} = \infty, \quad \lim_{\phi \rightarrow 0} \frac{V_{,\phi\phi}}{V_{,\phi}} = \infty \dots,$$

The scalar field action is as follows,

$$S_\phi = \int dx^4 \sqrt{-g} \left( -\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right), \quad (3)$$

and finally we have

$$\mathcal{L}_\gamma = -\frac{e^{\beta_\gamma \phi / M_{Pl}}}{4} F^{\mu\nu} F_{\mu\nu}, \quad (4)$$

and

$$\mathcal{L}_m(e^{2\beta_m^i \phi / M_{Pl}} g_{\mu\nu}, \psi_m^i) \quad (5)$$

which are the photon-scalar and matter-scalar interaction Lagrangians respectively. The  $\psi_m^i$  are matter fields which couple to  $\phi$  by a conformal coupling of the form  $g_{\mu\nu}^i = e^{2\beta_m^i \phi / M_{Pl}} g_{\mu\nu}$  [3], in which  $\beta_m^i$  and  $\beta_\gamma$  are dimensionless matter-chameleon and photon-chameleon coupling constants respectively. To obtain the equation of motion for  $\phi$  we vary the action  $S$  with respect to  $\phi$ , baring in mind that  $g_{\mu\nu}^i = e^{2\beta_m^i \phi / M_{Pl}} g_{\mu\nu}$ , where  $i$  stands for different matter fields, this gives the following equation of motion.

$$\square \phi = V_{,\phi}(\phi) + \frac{\beta_\gamma e^{\beta_\gamma \phi / M_{Pl}}}{4 M_{Pl}} F^{\mu\nu} F_{\mu\nu} + \frac{\beta_m^i}{M_{Pl}} e^{\beta_m^i \phi / M_{Pl}} \rho_m^i \quad (6)$$

Equation (6) can be written in a Klein-Gordon form as  $\square \phi = -\frac{\partial V_{eff}}{\partial \phi}$  where  $V_{eff}$  is the effective chameleon potential shown below.

$$V_{eff}(\vec{x}, \phi) = V(\phi) + e^{\frac{\beta_m \phi}{M_{Pl}}} \rho_m + e^{\frac{\beta_\gamma \phi}{M_{Pl}}} \rho_\gamma \quad (7)$$

in which  $\rho_\gamma = (F^{\mu\nu} F_{\mu\nu})/4 = (|\vec{B}|^2 - |\vec{E}|^2)/2$  is the electromagnetic field Lagrangian density, and  $\rho_m$  is the matter density. We assume the fiducial exponential potential used in chameleon dark energy theories given by the following equation [8] and  $\kappa > 0$ .

$$V(\phi) = M_\Lambda^4 \exp \left[ \kappa \left( \frac{M_\Lambda}{\phi} \right)^n \right] \quad (8)$$

which we can approximate by an inverse power law potential if we assume that  $M_\Lambda/\phi \ll 1$ .

$$V(\phi) \approx M_\Lambda^4 \left[ 1 + \kappa \left( \frac{M_\Lambda}{\phi} \right)^n \right] \quad (9)$$

The effective mass of small fluctuations about the minimum of  $V_{eff}$  is given by  $m_{eff}(\phi) = \sqrt{\frac{\partial^2 V_{eff}}{\partial \phi^2}}$ , which gives

$$m_{eff}(\phi_{min}) = \sqrt{n(n+1)\kappa^{-\left(\frac{1}{n+1}\right)} M_\Lambda^{n+4} \left( \frac{\beta_m \rho_m + \frac{\beta_\gamma B^2}{2}}{n M_\Lambda^{n+4} M_{Pl}} \right)^{\frac{n+2}{n+1}} + \frac{\beta_m^2 \rho_m}{M_{Pl}^2} + \frac{\beta_\gamma^2 B^2}{2 M_{Pl}^2}}. \quad (10)$$

When we vary the action in (1) with respect to  $A^\mu$  which is the electromagnetic 4-potential, it can be shown that the photon fields equation of motion is (here  $A^\mu = (\psi, \vec{A})$ ) [8])

$$\frac{\beta_\gamma}{M_{Pl}} \vec{\nabla} \phi \times (\vec{\nabla} \times \vec{A}) + \square \vec{A} = 0. \quad (11)$$

### 3. Oscillation probability for the chameleon-photon system

The Chameleon-Photon oscillation Probability ( $P_{\gamma \leftrightarrow \phi}$ ) is derived by considering the Chameleon-Photon system shown below, which is obtained from equations (6) and (11), where  $B$  is the magnetic field strength,  $\omega$  is the energy,  $\omega_p$  is the plasma frequency,  $m_\phi$  is the chameleon mass and  $z$  is the direction of propagation. The photon and the chameleon fields are indicated by  $\vec{\Psi}_\gamma$  and  $\Psi_\phi$  respectively. The photon-chameleon coupling factor is given by  $\frac{1}{M} = \frac{\beta_\gamma}{M_{Pl}}$ .

$$\left[ \omega^2 + \partial_z^2 + \begin{pmatrix} -\omega_p^2 & \frac{B\omega}{M} \\ \frac{B\omega}{M} & -m_\phi^2 \end{pmatrix} \right] \begin{bmatrix} \vec{\Psi}_\gamma \\ \Psi_\phi \end{bmatrix} = 0 \quad (12)$$

The system in equation(12) can be solved by diagonalizing the matrix below

$$A = \begin{pmatrix} \Delta_\gamma & \Delta_M \\ \Delta_M & \Delta_\phi \end{pmatrix} \quad (13)$$

with  $\Delta_\gamma = -\omega_p^2$ ,  $\Delta_M = \frac{B\omega}{M}$  and  $\Delta_\phi = -m_\phi^2$ . Therefore  $A$  can be diagonalized by a rotation matrix

$$P = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}, \quad (14)$$

such that  $A = PDP^{-1}$ , with

$$D = \begin{pmatrix} \lambda_+ & 0 \\ 0 & \lambda_- \end{pmatrix}, \quad (15)$$

where  $\lambda_\pm$  are the eigenvalues of the matrix  $A$  given by

$$\lambda_\pm = -\frac{(\omega_p^2 + m_\phi^2)}{2} \pm \frac{\sqrt{(\omega_p^2 - m_\phi^2)^2 + 4B^2\omega^2/M^2}}{2} \quad (16)$$

$$= \Lambda \pm \Omega \quad (17)$$

Now we define

$$k_{\pm} = \sqrt{\omega^2 + \lambda_{\pm}} \quad (18)$$

Consider a plane wave solution in the primed fields traveling in the  $z$ -direction of the following form

$$\begin{bmatrix} \vec{\Psi}'_{\gamma}(z, t) \\ \Psi'_{\phi}(z, t) \end{bmatrix} = \begin{pmatrix} e^{i(\omega t - k_+ z)} & 0 \\ 0 & e^{i(\omega t - k_- z)} \end{pmatrix} \begin{bmatrix} \vec{\Psi}'_{\gamma}(0, t) \\ \Psi'_{\phi}(0, t) \end{bmatrix} \quad (19)$$

We let the transfer matrix  $\mathcal{T}(z, t)$  be calculated as follows

$$\begin{aligned} \mathcal{T}(z, t) &= \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} e^{i(\omega t - k_+ z)} & 0 \\ 0 & e^{i(\omega t - k_- z)} \end{pmatrix} \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \\ &= e^{i\omega t} \begin{pmatrix} [e^{-ik_+ z} \cos^2 \theta + e^{-ik_- z} \sin^2 \theta] & \frac{1}{2} \sin 2\theta [e^{-ik_+ z} - e^{-ik_- z}] \\ \frac{1}{2} \sin 2\theta [e^{-ik_+ z} - e^{-ik_- z}] & [e^{-ik_+ z} \sin^2 \theta + e^{-ik_- z} \cos^2 \theta] \end{pmatrix} \end{aligned} \quad (20)$$

For propagation we require  $k_+^2 > 0$  and  $k_-^2 > 0$  such that the chameleon-photon oscillation probability becomes

$$P_{\gamma \leftrightarrow \phi} = \frac{1}{2} \sin^2 2\theta [1 - \cos([\Delta k] z)] \quad (21)$$

where  $\Delta k = k_- - k_+$ , and when substituting back for  $\sin^2 2\theta$  we obtain the probability as follows;

$$P_{\gamma \leftrightarrow \phi} = \frac{4B^2 \omega^2 \sin^2([\Delta k] z/2)}{M^2(m_{\phi}^2 - \omega^2)^2 + 4B^2 \omega^2}. \quad (22)$$

#### 4. Chameleon-photon oscillation probability in pulsars

A RQNS RX J0822-4300 was observed with the Australian Telescope Compact Array(ATCA; Fraster, Brooks and Whiteoak 1992) at 1.4GHz and they observed no radio emission with the limiting flux for the telescope of  $\sim 7\text{mJy}$ . The expected radio flux for a pulsar of this type having magnetic field of  $B = 3.4 \times 10^{12}\text{G}$ , the period of 75.5ms which is associated with a young( $< 5000\text{yr}$ ) SNR is 14.9Jy at this observed frequency. The surface magnetic field strength of a rotating neutron star is given as follows,

$$B^2 = \frac{3Ic^3}{8\pi^2 R^6} P\dot{P} \quad (23)$$

Where  $c$  is the speed of light,  $I$  is the moment of inertia,  $R$  is the radius from the center and  $P$  is the rotational period, it is well known that for a canonical pulsar with mass  $M \approx 1.4M_{\odot}$  and a radius of  $R = 10\text{km}$  the magnetic field has a lower limit  $B > 3.2 \times 10^{19} \left(\frac{P\dot{P}}{s}\right)^{1/2} \text{G}$ . Now using the measured value of  $B$  we can see that  $(P\dot{P})^{1/2} < 1.06 \times 10^{-7}$  from which using equation(23) we can find the value of  $I = 9.9 \times 10^{38} \text{m}^3 \text{s}^2$ . Finally we get the equation for the B field strength in terms  $R$  and  $\Theta$  as follows

$$B(R, \Theta) = \frac{3.4 \times 10^{24}}{R^3} [2 \cos(\Theta) \hat{r} + \sin(\Theta) \hat{\Theta}] \text{G} \quad (24)$$

We can approximate the electron number density given in([4]) by the following equation.

$$N_e = 7 \times 10^{-2} \text{cm}^{-3} \left( \frac{B}{\text{Gauss}} \right) \left( \frac{\text{sec}}{P} \right) \quad (25)$$

From this we can get the plasma frequency as follows

$$\omega_p = \sqrt{\frac{4\pi\alpha N_e}{m_e}} \quad (26)$$

in which  $m_e$  is the mass of the electron and  $\alpha$  is the fine structure constant. Radio emission altitude in pulsar magnetosphere is found to be in the following range of  $Re$  between

$$Re_{min} = 32 \times 10^4 \left( \frac{f}{10^9} \right)^{-0.35} \left( \frac{\dot{P}}{10^{-15}} \right)^{0.04} \left( \frac{P}{s} \right)^{0.25} \text{ m} \quad (27)$$

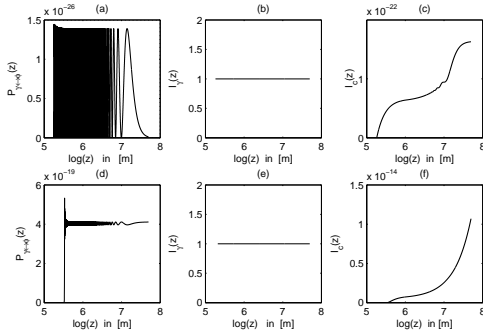
and

$$Re_{max} = 48 \times 10^4 \left( \frac{f}{10^9} \right)^{-0.17} \left( \frac{\dot{P}}{10^{-15}} \right)^{0.1} \left( \frac{P}{s} \right)^{0.35} \text{ m} \quad (28)$$

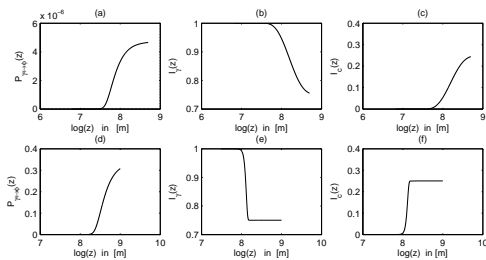
where  $Re$  is measured from the center of the neutron star,  $f$  is the photon frequency and  $P$  is the period of rotation.

#### 4.1. Weak vs strong coupling

In figure one below the chameleon to photon oscillation probability, normalized photon flux and the chameleon flux are shown for weak coupling ( $\beta_\gamma = 10^1$  and  $10^3$ ), whereas in figure two the chameleon to photon oscillation probability, normalized photon flux and the chameleon flux are shown for strong coupling ( $\beta_\gamma = 10^{12}$  and  $10^{16}$ ). Figure one(b and e) shows no observable change in the photon flux due to  $\phi$ -photon interaction, while figure two(b and e) shows significant reduction of the photon flux  $\sim 25\%$ , which can be observed.



**Figure 1.** Figure a,b and c shows  $P_{\gamma \leftrightarrow \phi}$ , photon flux( $I_\gamma$ ) and chameleon flux( $I_c$ ) vs the log of distance from the emission region in the pulsar magnetosphere. For  $\beta_\gamma = 10$  and figure d,e and f shows  $P_{\gamma \leftrightarrow \phi}$ , photon flux( $I_\gamma$ ) and chameleon flux( $I_c$ ) vs the log of distance from the emission region in the pulsar magnetosphere. For  $\beta_\gamma = 10^3$ .



**Figure 2.** Figure a,b and c shows  $P_{\gamma \leftrightarrow \phi}$ , photon flux( $I_\gamma$ ) and chameleon flux( $I_c$ ) vs the log of distance from the emission region in the pulsar magnetosphere. For  $\beta_\gamma = 10^{12}$  and figure d,e and f shows  $P_{\gamma \leftrightarrow \phi}$ , photon flux( $I_\gamma$ ) and chameleon flux( $I_c$ ) vs the log of distance from the emission region in the pulsar magnetosphere. For  $\beta_\gamma = 10^{16}$ .

## 5. Conclusion

It is evident from figures one and two that in the strong coupling regime, chameleon-photon oscillation is efficient compared to the weak coupling regime.

## 6. Future work

We need to calculate the oscillation probability at different photon frequencies to be able to see the effect on pulsar spectra. And propose an observation experiment for the SKA/MeerKat telescopes.

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