

Entanglement dynamics in an oscillating bipartite Gaussian state coupled to reservoirs with different dynamics

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Abstract. An entangled bipartite Gaussian state is coupled to two thermal reservoirs, one for each particle and a harmonic oscillation is allowed between the two particles. The reservoirs are assumed to have different dynamics and to be coupled to their particles with different coupling strengths. This allows for a realistic situation where a bipartite state may be shared between two parties who “keep” their part in different environments. We solve a master equation previously derived in the non-rotating wave approximation for the system. We show the effect of a variation in the bath temperature on the entanglement, as well as that of the variation in coupling strengths.

1. Introduction

Entanglement is one of quantum mechanics’ most useful resources, yet also one of the most fragile; as such, the loss, and in some cases revival, of entanglement in a system coupled to an environment, due to said environment, has been widely studied, see e.g. [1] for an extensive review. As a continuation of previous work [2,3], where the authors studied an initially entangled bipartite system shared between two parties keeping their respective environments at the same temperature, we now examine the case where these temperatures are different. This allows for a somewhat more realistic situation, where an entangled state may be shared between two parties who are not necessarily expected to keep their environment in the same conditions.

To evolve the state, we use a pre-Lindblad master equation, derived in the Non-Rotating Wave (N.R.W.) approximation. The derivation can be found in [2–6]. The N.R.W. master equation is often well suited for systems which are expected to be strongly coupled to their environments. Other pre-Lindblad equations have been derived, using other methods. One may in particular cite that of Caldeira and Leggett [7] who used a path-integrals method, or that of Diósi [8,9] who obtained an equation valid for low temperatures.

We choose the initial state of the bipartite system to be Gaussian, since there is a formalism [10–13] which allows for a simple study of Gaussian states. These states form a class of continuous variables states which is becoming of growing importance to the field of quantum information processing, both for the ease with which they are manipulated experimentally [14] and the ease with which they can be analytically studied. The latter has led to a variety of studies of systems coupled to heat baths, such as [15–17] to cite but a few.

In the following, we will recall the master equation and solve it for the particular dynamics chosen. We will then discuss some examples of entanglement behaviour.

2. Entanglement dynamics

It was found in [3, 18, 19] that the dynamics of the entanglement in a bipartite system coupled to an environment is greatly influence by allowing an harmonic interaction between the system's particles. We study a system of two particles of equal mass, each one coupled to its own heat bath; they have coordinates x_1 and x_2 , momenta p_1 and p_2 ; ω_0 denotes the frequency of the oscillation. The overall Hamiltonian reads as

$$H = \frac{p_1^2}{2m} + \frac{p_2^2}{2m} + \frac{m\omega_0^2}{2}(x_1 - x_2)^2 + \sum_j \left\{ \frac{p_j^2}{2m_j} + \frac{m_j\omega_j^2}{2}(q_j - x_1)^2 \right\} + \sum_k \left\{ \frac{p_k^2}{2m_k} + \frac{m_k\omega_k^2}{2}(q_k - x_2)^2 \right\}. \quad (1)$$

For our study, we take all masses to be equal. The ω 's denote the frequencies of the bath oscillators. The initial state is chosen to be the Gaussian state [20, 21]

$$\Psi(x_1, x_2) = \sqrt{\frac{1}{2\pi s d}} e^{-\frac{(x_1 - x_2)^2}{4s^2}} e^{-\frac{(x_1 + x_2)^2}{16d^2}}, \quad (2)$$

where s and d denote the distance between the particles and the width of the center-of-mass system respectively. With position coupling, the master equation becomes

$$\dot{\rho} = -\frac{i}{\hbar} [H_s, \rho] - \frac{i\gamma_1}{2\hbar} [x_1, [p_1, \rho]_+] - \frac{\gamma_1 k T_1}{\hbar^2} [x_1, [x_1, \rho]] - \frac{i\gamma_2}{2\hbar} [x_2, [p_2, \rho]_+] - \frac{\gamma_2 k T_2}{\hbar^2} [x_2, [x_2, \rho]], \quad (3)$$

where T_1 and T_2 are the temperatures of bath 1 and 2 respectively, γ_1 and γ_2 the coupling constants of bath 1 and 2 respectively and $[,]_+$ represents the anti-commutator. The 1 indices denote particle 1 and bath 1, the 2 indices, particle 2 and bath 2. Writing the density matrix in position representation, $\rho(x_1, x_2; y_1, y_2)$, we get

$$\begin{aligned} \frac{\partial \rho}{\partial t} = & \frac{i\hbar}{2m} \left(\frac{\partial^2}{\partial x_1^2} - \frac{\partial^2}{\partial y_1^2} + \frac{\partial^2}{\partial x_2^2} - \frac{\partial^2}{\partial y_2^2} \right) \rho \\ & - \frac{m\omega_0^2}{2\hbar} ((x_1 - x_2)^2 - (y_1 - y_2)^2) \rho \\ & - \frac{\gamma_1}{2m} (x_1 - y_1) \left(\frac{\partial}{\partial x_1} - \frac{\partial}{\partial y_1} \right) \rho - \frac{\gamma_1 k T_1}{\hbar^2} (x_1 - y_1)^2 \rho \\ & - \frac{\gamma_2}{2m} (x_2 - y_2) \left(\frac{\partial}{\partial x_2} - \frac{\partial}{\partial y_2} \right) \rho - \frac{\gamma_2 k T_2}{\hbar^2} (x_2 - y_2)^2 \rho. \end{aligned} \quad (4)$$

This equation can be solved to obtain, after some algebra,

$$\begin{aligned} \tilde{P} = \exp[& -\mathcal{A}_1 q_1^2 - \mathcal{A}_2 q_2^2 - \mathcal{E} q_1 q_2 - \mathcal{B}_1 z_1^2 - \mathcal{B}_2 z_2^2 - \mathcal{D} z_1 z_2 \\ & - \mathcal{C}_{11} z_1 q_1 - \mathcal{C}_{22} z_2 q_2 - \mathcal{C}_{12} z_1 q_2 - \mathcal{C}_{21} z_2 q_1], \end{aligned} \quad (5)$$

where

$$\begin{aligned}
\mathcal{A}_1 = & (\epsilon_+ + 4k\chi_z^1)\delta_1^{+2} + (\epsilon_+ + 4k\chi_z^2)\delta_1^{-2} - (2\epsilon_- - 8k\vartheta_1)\delta_1^+\delta_1^- \\
& + \left(\frac{\epsilon_+}{4(\epsilon_+^2 - \epsilon_-^2)} + 4k\chi_q^1\right)\theta_1^{-2} + \left(\frac{\epsilon_+}{4(\epsilon_+^2 - \epsilon_-^2)} + 4k\chi_q^2\right)\theta_1^{+2} \\
& + \left(\frac{\epsilon_-}{2(\epsilon_+^2 - \epsilon_-^2)} + 8k\vartheta_2\right)\theta_1^-\theta_1^+ \\
& + 8k(\delta_1^+\theta_1^-\Lambda_{11} + \delta_1^-\theta_1^+\Lambda_{22} + \delta_1^+\theta_1^+\Lambda_{12} + \delta_1^-\theta_1^-\Lambda_{21})
\end{aligned}$$

and all the remaining coefficients are as bulky.

A more detailed solution will be found in [22].

3. Observations and concluding remarks

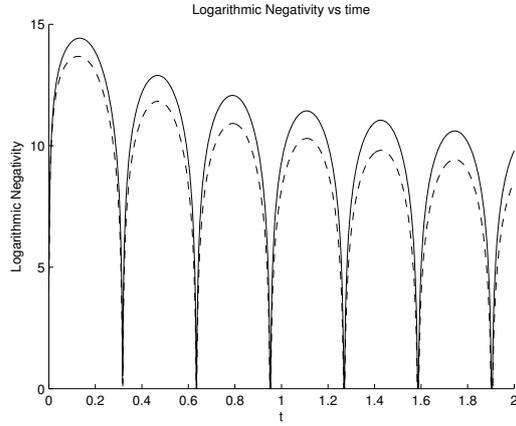
The following figures illustrate some of the possible evolutions of the entanglement. It should be noted that the behaviours displayed here are obtained in the under-damped case, i.e. when the harmonic potential is greater than either of the coupling constants. We limit our study to this particular class of behaviours since it yields the most striking observations [2,3]. Figure 1a, Figure 2a and Figure 3a illustrate the short time behaviour of the entanglement, whereas Figure 1b, Figure 2b and Figure 3b show the long time behaviour for the same parameters as their counterparts'. It is easily observed at first glance that the behaviour of the entanglement is remarkably similar in all three pairs of figures. The entanglement oscillates with damping oscillations, over the full range of its value until a certain time. After that time, the oscillations do not decrease all the way down to 0; eventually the entanglement oscillates over a stable range. Upon closer examinations of Figure 1a, Figure 2a and Figure 3a, one can see that the range of entanglement is less when the temperatures are higher (Figure 3a) or when the coupling constants are stronger (Figure 1a, Figure 2a). This shows that, as one would expect, the temperatures and the coupling constants are crucial to the survival of the entanglement in the system we study. Closer examinations of Figure 1b, Figure 2b and Figure 3b yields similar observations. More precisely, the time at which the oscillations cease to reach the x-axis is longer if the coupling constants are almost equal and the temperatures are equal, but the upper end of the range is lower. Figure 4 illustrates how increasing or decreasing s , the distance between the particle influences the evolution of entanglement. In particular, larger entanglement is obtained for smaller values of s . This suggests that if one was to "stretch" the system, the entanglement it contained would be greatly reduced. One may conclude that the coupling constants γ_1 and γ_2 , the temperatures of the baths T_1 and T_2 and the separation between the variables s may be finely tuned to delay the destruction of the entanglement by dissipation.

Acknowledgements

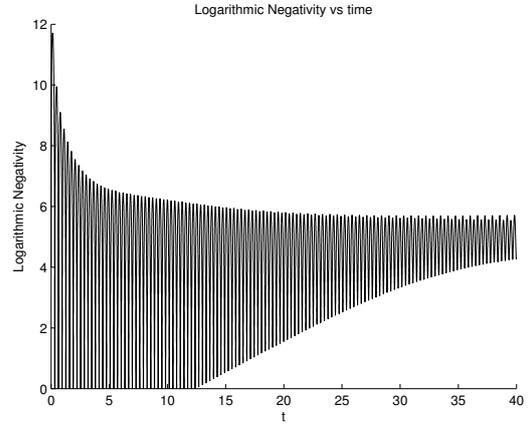
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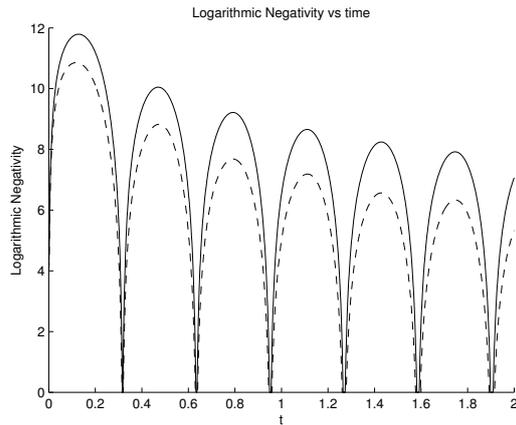


(a) from $t = 0$ to $t = 2$

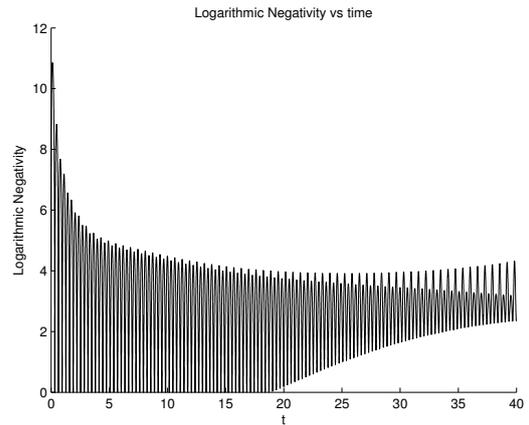


(b) from $t = 0$ to $t = 40$

Figure 1: Logarithmic negativity plotted versus time for temperatures $T_1 = 1$ and $T_2 = 1$ with coupling constants $\gamma_2 = 0.8$, full line : $\gamma_1 = 0.1$, dashed line : $\gamma_1 = 0.9$



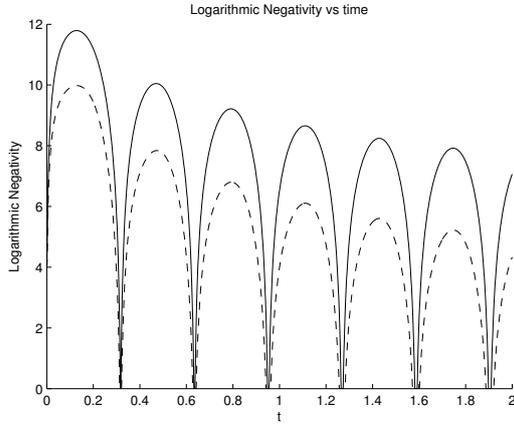
(a) from $t = 0$ to $t = 2$



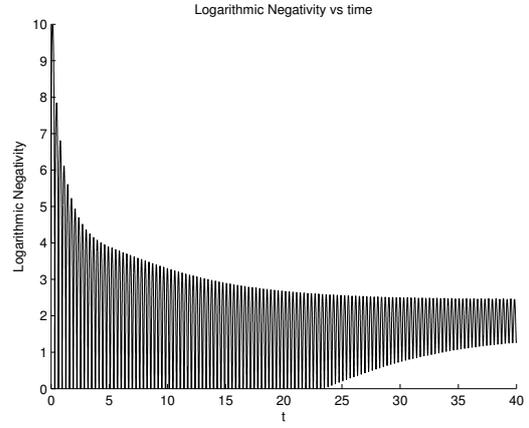
(b) from $t = 0$ to $t = 40$

Figure 2: Logarithmic negativity plotted versus time for coupling constants $\gamma_1 = 0.79$ and $\gamma_2 = 0.8$ with temperatures $T_2 = 1$, full line : $T_1 = 1$, dashed line : $T_1 = 3$

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(a) from $t = 0$ to $t = 2$



(b) from $t = 0$ to $t = 40$

Figure 3: Logarithmic negativity plotted versus time for coupling constants $\gamma_1 = 0.79$ and $\gamma_2 = 0.8$ with temperatures full line : $T_1 = T_2 = 1$, dashed line : $T_1 = T_2 = 3$

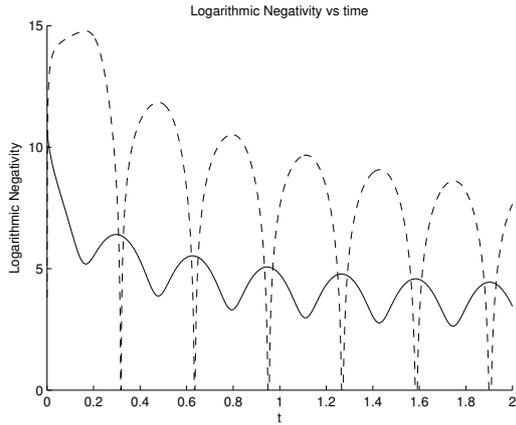


Figure 4: Logarithmic negativity plotted versus time for coupling constants $\gamma_1 = 0.79$, $\gamma_2 = 0.8$, temperatures $T_1 = 1$ and T_2 , $d = 2$ and : full line $s = 0.1$, dashed line $s = 15$

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