A simple model of decoherence-assisted transport

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Abstract. The dynamics of a dimer under the influence of decoherence are studied. The model consists of a dimer with one level in contact with a single bosonic mode. An exact analytical expression for the density matrix of the total system is obtained. It is shown that there exist well-defined ranges of parameters for which decoherent interaction with just a single bosonic mode assists energy transfer in the dimer system. This simple and analytically solvable model for energy transfer in a dimer system is easily extendible to more complex quantum networks, and more complex environmental models.

1. Introduction

The transfer of energy and information in quantum networks plays an important role for quantum communication and quantum computation. In realistic physical situations, unavoidable interaction with the environment leads to decoherence and dissipation, processes typically associated with a destruction of coherence [1]. However, recently ultrafast spectroscopic techniques have been claimed to reveal long-lasting quantum coherence in biological systems, including in the photosynthetic light-harvesting complexes of a species of green sulphur bacteria [2,3], a species of purple bacteria [4], and two species of marine cryptophyte algae [5].

The efficiency of the energy transfer through the network of chromophores together with the evidence for quantum coherence has led to much activity proposing mechanisms for environment-assisted excitonic energy transport in quantum networks, including under the broad headings of noise-assisted transport [6-8] and oscillation-enhanced transport [9-12]. The possibility that quantum entanglement may enhance transport has also been discussed [13-15].

We show that even in the most simple analytically solvable model of a dimer coupled to a single bosonic mode, there exist well-defined ranges of parameters for which decoherent interaction with the environment assists energy transfer in the dimer system.

2. Model

In this paper we are going to study the simplest electronic energy transfer system, the dimer. The Hamiltonian of the dimer is given by $H_D = \varepsilon_1 |1\rangle\langle 1| + \varepsilon_2 |2\rangle\langle 2| + J(|1\rangle\langle 2| + |2\rangle\langle 1|)$, where ε_i are the energy levels of the dimer and J is the amplitude of transition. It is well-known that if the initial excitation is in level 1, then the maximum probability of transition to level 2 ($P_{1\to 2}(t)$) will be given by $\max(P_{1\to 2}) = J^2/(J^2 + \Delta^2)$, where Δ is half of the energy difference between the energy levels of the dimer ($\Delta = (\varepsilon_2 - \varepsilon_1)/2$). This means that only in the case $\varepsilon_1 = \varepsilon_2$

we can say that the excitation is transferred with certainty ($P_{1\to2}(t_0) = 1$) at the time $t_0 = \pi/2J$.

The aim of this paper is to show that in the general case ($\varepsilon_1 \neq \varepsilon_2$), even for the most simple analytically solvable model consisting of a dimer with one level in contact with a single bosonic mode, decoherence enhances energy transfer.

The Hamiltonian of the total system has the form

$$H = H_D + H_{BM} + H_I,$$

where in the Hamiltonian for the dimer H_D given above, we take $\varepsilon_2 > \varepsilon_1$, and the Hamiltonians of the the bosonic mode and the interaction are given by

$$
\begin{aligned}
H_{BM} &= \omega a^\dagger a, \text{ and} \\
H_I &= \gamma a^\dagger a |1\rangle\langle 1|,
\end{aligned}
$$

respectively.

The projectors $|i\rangle\langle j|$ are conveniently represented in terms of the Pauli matrices as

$$|1\rangle\langle 1| = \frac{\mathbb{1}_2 - \sigma_z}{2}, \quad |2\rangle\langle 2| = \frac{\mathbb{1}_2 + \sigma_z}{2}, \quad |2\rangle\langle 1| = \sigma^+.$$

Thus, the total Hamiltonian can be written in the following form:

$$H = \frac{\varepsilon_2 - \varepsilon_1}{2}\sigma_z + J\sigma_x + \omega a^\dagger a - \frac{\gamma}{2} a^\dagger a \sigma_z + \frac{\varepsilon_2 + \varepsilon_1}{2}\mathbb{1}_2 + \frac{\gamma}{2} a^\dagger a.$$

It is easy to find the evolution operator of the system, which is given by

$$U(t) = \sum_{n=0}^{\infty} \mathrm{e}^{-it[(\omega+\frac{\gamma}{2})n+\frac{\varepsilon_1+\varepsilon_2}{2}]} \begin{pmatrix} \cos\Omega_n t - \frac{i\Delta_n}{\Omega_n}\sin\Omega_n t & -i\frac{J}{\Omega_n}\sin\Omega_n t \\ -i\frac{J}{\Omega_n}\sin\Omega_n t & \cos\Omega_n t + \frac{i\Delta_n}{\Omega_n}\sin\Omega_n t \end{pmatrix} \otimes |n\rangle\langle n|,$$

where $\Omega_n = \sqrt{J^2 + \Delta_n^2}$ and $\Delta_n = (\varepsilon_2 - \varepsilon_1)/2 - \gamma n/2$.

Since we are interested in the probability of transition $P_{1\to2}(t)$ in the dimer, we assume the dimer is initially in the state $|\psi_D^0\rangle = |1\rangle$, while the initial state of the bosonic mode is the thermal state

$$\rho_{\mathrm{BM}}^0 = \frac{1}{Z}\mathrm{e}^{-\beta\omega a^\dagger a} = (1 - \mathrm{e}^{-\beta\omega})\mathrm{e}^{-\beta\omega a^\dagger a}.$$

The state of the dimer at time t is then given by

$$\rho_D(t) = \mathrm{tr}_{\mathrm{BM}}(U(t)|1\rangle\langle 1| \otimes \rho_{\mathrm{BM}}^0 U^\dagger(t)).$$

The probability of transition is then found to be

$$
\begin{aligned}
P_{1\to2}(t) &= \mathrm{tr}_{\mathrm{BM}}(\langle 2|U(t)|1\rangle \rho_{BM}^0 \langle 1|U^\dagger(t)|2\rangle) \\
&= \sum_{n=0}^{\infty}(1 - \mathrm{e}^{-\beta\omega})\frac{J^2}{\Omega_n^2}\sin^2\Omega_n t\, \mathrm{e}^{-\beta\omega n} \\
&= (1 - \mathrm{e}^{-\beta\omega})\sum_{n=0}^{\infty}\frac{J^2}{J^2 + (\frac{\varepsilon_2-\varepsilon_1}{2} - \frac{\gamma n}{2})^2}\sin^2\left(t\sqrt{J^2 + (\frac{\varepsilon_2 - \varepsilon_1}{2} - \frac{\gamma n}{2})^2}\right)\mathrm{e}^{-\beta\omega n}.
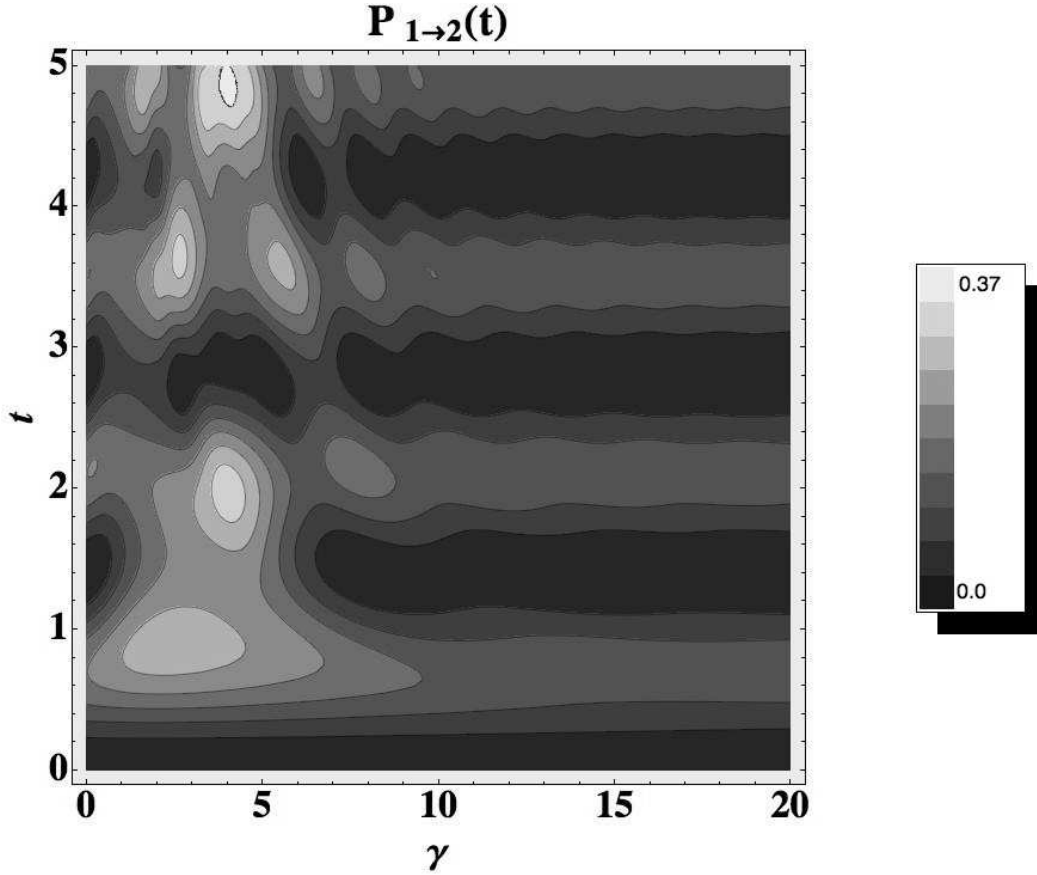\end{aligned}
$$

Figure 1. Probability of transition $P_{1\rightarrow2}(t)$ in the dimer system coupled to just one bosonic mode as a function of time and coupling constant γ to the bosonic mode. The parameters are chosen to be $J = 1$, $\Delta = (\varepsilon_2 - \varepsilon_1)/2 = 2$, $\beta = 1$ and $\omega = 1$.

In figure 1 and figure 2 the probability of transition in the dimer system as a function of time and coupling constant to the single bosonic mode is presented. One can see that the maximum of the probability of transition is achieved for the non-zero interaction with a decoherent environment. This means that in the general case of non-zero temperature, decoherence as a result of coupling to even a single bosonic mode assists energy transfer in the dimer system. Note that in figure 2 the temperature is higher by a factor of 10, which means that the effect is present in a wide range of temperatures.

3. Conclusion

In conclusion, we have studied a dimer system in contact with a decoherent environment consisting of just a single bosonic mode. We have shown that there exist parameter ranges for which interaction with the bosonic mode assists energy transfer in a dimer. These results motivate further study of more complicated energy transfer networks and more complicated models of environments.
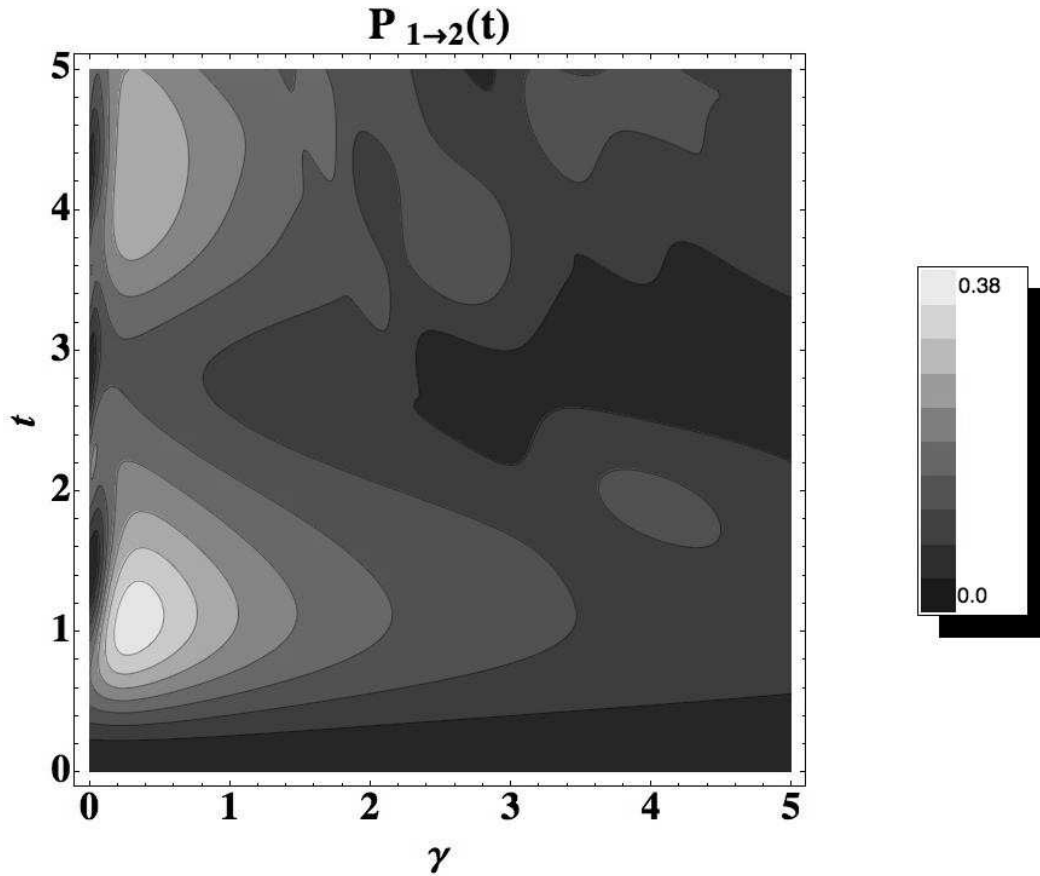
Figure 2. Probability of transition $P_{1\to2}(t)$ in the dimer system coupled to just one bosonic mode as a function of time and coupling constant γ to the bosonic mode. The parameters are the same as in Fig. 1, except with higher temperature, namely $\beta = 0.1$.

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