Defects in the traditional analogy between the dipolar structure of a circular current and a simple electric dipole

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Abstract. It is shown that when a circular current is resolved into merged distributions of distinct Cartesian *x* and *y* component line current elements, each distribution is a complete magnetic dipole that selectively creates like Cartesian components of the magnetic torque and vector potential, plus only the magnetic field's other Cartesian components. All these are expressible in terms of a distribution's own magnetic dipolar moment, which is traditionally attributed to the whole circular current. In contrast a simple electric dipole aligned on the *z*-axis, creates its *x* and *y* electric torque components, its full cylindrically symmetric electric field and the electric scalar potential, all of which are expressible in terms of the sole electric dipolar moment. Each magnetic or electric Cartesian torque component is expressible as a cross product of a distribution's dipolar moment and one Cartesian field component parallel to an exclusive Cartesian plane perpendicularly bisecting the mutually parallel intra-dipolar displacements, while the distribution's corresponding potential vanishes in that plane. Under such special conditions, tradition compares one surviving Cartesian component of the magnetic torque or of the magnetic vector potential to respectively the electric dipole's combined *x* and *y* torque components or the whole scalar potential. Seemingly from this and the equality of the magnetic dipolar moments of the two component distributions of the cylindrically symmetric circular current, tradition incorrectly defines either of these magnetic dipolar moments as that of the entire circular current.

1. Introduction

As a follow up on the earlier paper [1], we show that the traditional analogy of the magnetic dipolar structure of a circular current to that of a simple electric dipole consisting of separated electric scalar charges of identical size but opposite signs has many short comings. This is done by evaluating for these magnetic (a circular current) and electric (axial line scalar charge) distributions of dipoles, their dipolar moments, torques in any external fields, dipolar magnetic vector and electric scalar potentials and dipolar magnetic and electric fields, on the basis that any dipole is a combination of opposite equal-sized appropriately existent monopoles. Here an elemental electric current multiplied by the free space permeability is denoted and depicted as an elemental magnetic vector charge in natural contrast to an elemental electric scalar charge and its traditional analogy of magnetic scalar charge. In this article it is observed that the contrasting vector and scalar natures of the elemental magnetic vector and electric scalar charges as well as the geometries of the distributions are the primary origin of the many interesting differences and similarities between their magnetic and electric properties, some of which are traditionally overlooked or misinterpreted. They are revealed in this step by step comparison.

2. Moments of and torques on Cartesian magnetic and electric dipoles

On a circle of radius ρ lying in the *xy*-plane and centred at the origin O in figure 1(a), an azimuthal line elemental magnetic vector charge at point P_j of radial position vector $\boldsymbol{\rho}_j$ in the j^{th} quadrant is

$$d\mathbf{Q}_j = \mathbf{I}_j\mu_0 d\ell_j \equiv \hat{\boldsymbol{\varphi}}_j I\mu_0\rho\, d\phi_j = \left(-\hat{\mathbf{x}}\sin\phi_j + \hat{\mathbf{y}}\cos\phi_j\right) I\mu_0\rho\, d\phi_j, \quad j = 1, 2, 3, 4, \tag{1}$$

where $\hat{\boldsymbol{\varphi}}_j I\mu_0$ is its line magnetic vector charge density and $\phi_j = \phi + (j-1)\frac{\pi}{2}$, as $\boldsymbol{\rho}_j$ is at an angle $0 \le \phi \le \frac{\pi}{2}$ to the *x*-axis. Hence the Cartesian elemental magnetic vector charge components

$$d\mathbf{Q}_{j_x} = \pm\hat{\mathbf{x}}dQ_a = \pm\hat{\mathbf{x}}I\mu_0\rho\sin\phi\, d\phi, \tag{2a}$$

$$d\mathbf{Q}_{j_y} = \pm\hat{\mathbf{y}}dQ_b = \pm\hat{\mathbf{y}}I\mu_0\rho\cos\phi\, d\phi. \tag{2b}$$

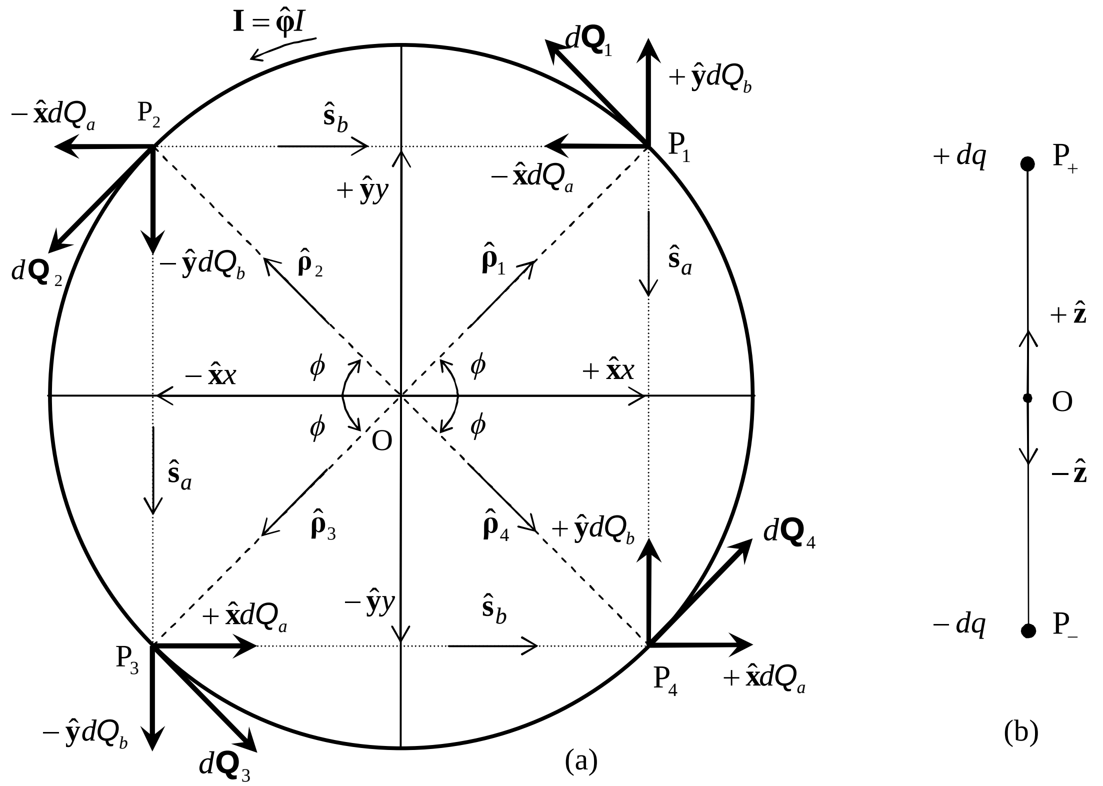

Figure 1. Pairing separated equal-sized elemental entities of opposite signs into dipoles: (a) Cartesian magnetic vector charges on an *xy*-circle and (b) electric scalar charges on a *z*-axis.

exhibit the indicated angular dependent axial line magnetic vector charge densities. When Cartesian components at P_2 are paired with the matching but oppositely directed Cartesian components at P_3 and P_1, they constitute the *a* and *b* Cartesian elemental magnetic dipoles respectively.

While in figure 1(b) at the points P_+ and P_-, of axial position vectors $\mathbf{z}_+ = +\hat{\mathbf{z}}z$ and $\mathbf{z}_- = -\hat{\mathbf{z}}z$ on the *z*-axis, are the line elemental electric scalar charges

$$dq_\pm = \pm dq = \pm\lambda\, dz. \tag{2c}$$

These form an elemental electric dipole. Here $\pm\lambda$ are electric scalar charge line densities.

The two magnetic elemental dipoles and the one electric elemental dipole have dipolar moments of

$$d\boldsymbol{m}_a = d\boldsymbol{m}_{a_+} + d\boldsymbol{m}_{a_-} = \hat{\boldsymbol{\rho}}_3\rho\times\hat{\mathbf{x}}dQ_a + \hat{\boldsymbol{\rho}}_2\rho\times(-\hat{\mathbf{x}}dQ_a) \equiv \mathbf{s}_a\times\hat{\mathbf{x}}dQ_a = \hat{\mathbf{z}}dm_a, \tag{3a}$$

$$d\boldsymbol{m}_b = d\boldsymbol{m}_{b_+} + d\boldsymbol{m}_{b_-} = \hat{\boldsymbol{\rho}}_1\rho \times \hat{\mathbf{y}} dQ_b + \hat{\boldsymbol{\rho}}_2\rho \times (-\hat{\mathbf{y}} dQ_b) \equiv \mathbf{s}_b \times \hat{\mathbf{y}} dQ_b = \hat{\mathbf{z}} dm_b, \tag{3b}$$

$$d\mathbf{p} = d\mathbf{p}_+ + d\mathbf{p}_- = \hat{\mathbf{z}} z dq + (-\hat{\mathbf{z}} z)(-dq) = \mathbf{s}_\mathrm{e} dq \equiv \hat{\mathbf{z}} dp \tag{3c}$$

where their Cartesian intra dipolar displacements or Cartesian dipolar orientation vectors are

$$\mathbf{s}_a = -\hat{\mathbf{y}} s_a = -\hat{\mathbf{y}} 2\rho \sin\phi, \qquad \mathbf{s}_b = \hat{\mathbf{x}} s_b = \hat{\mathbf{x}} 2\rho\cos\phi, \qquad \mathbf{s}_\mathrm{e} = \hat{\mathbf{z}} s_\mathrm{e} = \hat{\mathbf{z}} 2z. \tag{4}$$

The (sub) subscripts $+$ and $-$ in (3a) to (3c), and in subsequent discussions below, signify entities for or due to opposite elemental magnetic vector or electric scalar charges, that is, respective monopoles.

Integrating (3a) and (3b) from $\phi = 0$ to $\phi = \pi$ and (3c) from $z = 0$ to $z = z$ yield the moments

$$\boldsymbol{m}_a = \hat{\mathbf{z}} \mu_0 I \pi \rho^2 \equiv \hat{\mathbf{z}} m_a, \qquad \boldsymbol{m}_b = \hat{\mathbf{z}} \mu_0 I \pi \rho^2 \equiv \hat{\mathbf{z}} m_b, \qquad \mathbf{p} = \hat{\mathbf{z}} \lambda z^2 \equiv \hat{\mathbf{z}} p. \tag{5}$$

Each one is equivalent to a charge density size multiplied by an area vector of a real or unreal surface. Yet the circular current's magnetic dipolar moment is twice the (Kennelly) traditional value [2–8] as:

$$\boldsymbol{m} = \boldsymbol{m}_a + \boldsymbol{m}_b = \hat{\mathbf{z}} 2\mu_0 I \pi \rho^2 \equiv 2\boldsymbol{m}_a \equiv 2\boldsymbol{m}_b. \tag{6}$$

In external magnetic $\mathbf{H}$ and electric $\mathbf{E}$ fields the Cartesian elemental magnetic and electric dipoles are characterized by paired magnetic forces $\pm d\mathbf{F}_{\mathrm{m}_a}$, $\pm d\mathbf{F}_{\mathrm{m}_b}$ and electric forces $\pm d\mathbf{F}_\mathrm{e}$. As coupled moments of the forces acting on the elemental dipoles, the magnetic and electric torques are:

$$d\boldsymbol{\tau}_a = d\boldsymbol{\tau}_{a_+} + d\boldsymbol{\tau}_{a_-} \equiv \mathbf{s}_a \times d\mathbf{F}_{\mathrm{m}_a} = -\hat{\mathbf{y}} s_a \times \left(\hat{\mathbf{x}} dQ_a \times \mathbf{H}\right) = -\hat{\mathbf{y}} s_a \times \left(\hat{\mathbf{z}} dQ_a H_y - \hat{\mathbf{y}} dQ_a H_z\right), \tag{7a}$$

$$d\boldsymbol{\tau}_b = d\boldsymbol{\tau}_{b_+} + d\boldsymbol{\tau}_{b_-} \equiv \mathbf{s}_b \times d\mathbf{F}_{\mathrm{m}_b} = +\hat{\mathbf{x}} s_b \times \left(\hat{\mathbf{y}} dQ_b \times \mathbf{H}\right) = +\hat{\mathbf{x}} s_b \times \left(\hat{\mathbf{x}} dQ_b H_z - \hat{\mathbf{z}} dQ_b H_x\right), \tag{7b}$$

$$d\boldsymbol{\tau}_\mathrm{e} = d\boldsymbol{\tau}_{\mathrm{e}_+} + d\boldsymbol{\tau}_{\mathrm{e}_-} = \mathbf{s}_\mathrm{e} \times d\mathbf{F}_\mathrm{e} = +\hat{\mathbf{z}} s_\mathrm{e} \times \left(dq\mathbf{E}\right) = +\hat{\mathbf{z}} s_\mathrm{e} \times \left(\hat{\mathbf{z}} dq E_z + \hat{\mathbf{y}} dq E_y + \hat{\mathbf{x}} dq E_x\right). \tag{7c}$$

Each triple vector product in (7a) and (7b) yields one Cartesian magnetic torque component, while the duo vector product in (7c) yields two Cartesian electric torque components. These torque components

$$\begin{aligned}
d\boldsymbol{\tau}_{\mathrm{m}_x} &= d\boldsymbol{\tau}_a = \hat{\mathbf{z}} s_a dQ_a \times \hat{\mathbf{y}} H_y \equiv \left\{\mathbf{s}_a \times \hat{\mathbf{x}} dQ_a\right\} \times \hat{\mathbf{y}} H_y = d\boldsymbol{m}_a \times \hat{\mathbf{y}} H_y \neq d\boldsymbol{m}_a \times \mathbf{H}, \\
d\boldsymbol{\tau}_{\mathrm{m}_y} &= d\boldsymbol{\tau}_b = \hat{\mathbf{z}} s_b dQ_b \times \hat{\mathbf{x}} H_x \equiv \left\{\mathbf{s}_b \times \hat{\mathbf{y}} dQ_b\right\} \times \hat{\mathbf{x}} H_x = d\boldsymbol{m}_b \times \hat{\mathbf{x}} H_x \neq d\boldsymbol{m}_b \times \mathbf{H}, \\
d\boldsymbol{\tau}_{\mathrm{e}_x} &= \hat{\mathbf{z}} s_\mathrm{e} dq \times \hat{\mathbf{y}} E_y \equiv \left\{\mathbf{s}_\mathrm{e} dq\right\} \times \hat{\mathbf{y}} E_y = d\mathbf{p} \times \hat{\mathbf{y}} E_y \neq d\mathbf{p} \times \mathbf{E}, \\
d\boldsymbol{\tau}_{\mathrm{e}_y} &= \hat{\mathbf{z}} s_\mathrm{e} dq \times \hat{\mathbf{x}} E_x \equiv \left\{\mathbf{s}_\mathrm{e} dq\right\} \times \hat{\mathbf{x}} E_x = d\mathbf{p} \times \hat{\mathbf{x}} E_x \neq d\mathbf{p} \times \mathbf{E}
\end{aligned} \tag{8}$$

show matched formats and inequalities. Using (5) and (6), the total magnetic and electric torques are

$$\boldsymbol{\tau}_\mathrm{m} = \boldsymbol{\tau}_a + \boldsymbol{\tau}_b = \boldsymbol{m}_a \times \hat{\mathbf{y}} H_y + \boldsymbol{m}_b \times \hat{\mathbf{x}} H_x \equiv \boldsymbol{m}_a \times \mathbf{H} \equiv \boldsymbol{m}_b \times \mathbf{H} \neq \boldsymbol{m} \times \mathbf{H}, \tag{9}$$

$$\boldsymbol{\tau}_\mathrm{e} = \boldsymbol{\tau}_{\mathrm{e}_x} + \boldsymbol{\tau}_{\mathrm{e}_y} = \mathbf{p} \times \hat{\mathbf{y}} E_y + \mathbf{p} \times \hat{\mathbf{x}} E_x \equiv \mathbf{p} \times \mathbf{E} \equiv \mathbf{p} \times \mathbf{E} \neq 2\mathbf{p} \times \mathbf{E}. \tag{10}$$

The contrasts in (7a) to (10) nullify the traditional analogy between magnetic and electric torques, and especially the taking of $\boldsymbol{m}_a$ or $\boldsymbol{m}_b$ [2–8] as the circular current's total magnetic dipolar moment. The second equivalences in (9) and (10) highlight the duo magnetic and lone electric dipolar distributions.

3. Dipolar magnetic vector and electric scalar potentials and associated fields

When the magnetic vector and electric scalar charge distributions in figure 1 are the sources of magnetic vector and electric scalar potentials, as well as the associated magnetic and electric fields at a field point P, primed symbols signify the charges and their positions. Thus, in figure 2 the field point P in a $z\rho$- or $r\theta$-plane is displaced from origin O by $\mathbf{r} = \hat{\mathbf{r}} r$, and from elemental source Cartesian magnetic vector charges at points P'_j, $j = 1, 2, 3$ on a circle of radius ρ' in the $\rho\phi$- or $r\phi$-plane by

$$\mathbf{R}_j = \hat{\mathbf{R}}_j R_j = \hat{\mathbf{R}}_j f_j^{\frac{1}{2}} r = \hat{\mathbf{r}} r - \hat{\boldsymbol{\rho}}'_j \rho' = \hat{\mathbf{z}} z + \hat{\boldsymbol{\rho}} \rho - \hat{\boldsymbol{\rho}}'_j \rho', \quad j = 1, 2, 3, \tag{11}$$

where $\rho = r\sin\theta$ and the geometrical factors f_j are functions of r, θ, ϕ, ρ', ϕ'. While in figure 3, displacements of P from electric scalar charges $+\mathrm{d}q'$ and $-\mathrm{d}q'$ at P'_+ and P'_- on the z-axis are

$$\mathbf{R}_\pm = \hat{\mathbf{R}}_\pm R_\pm = \hat{\mathbf{R}}_\pm f_\pm^{\frac{1}{2}} r = \hat{\mathbf{r}} r - \mathbf{z}'_\pm = \hat{\mathbf{r}}\left(r \mp \tfrac{1}{2} s'_\mathrm{e} \cos\theta\right) \pm \hat{\boldsymbol{\theta}} \tfrac{1}{2} s'_\mathrm{e} \sin\theta, \tag{12}$$

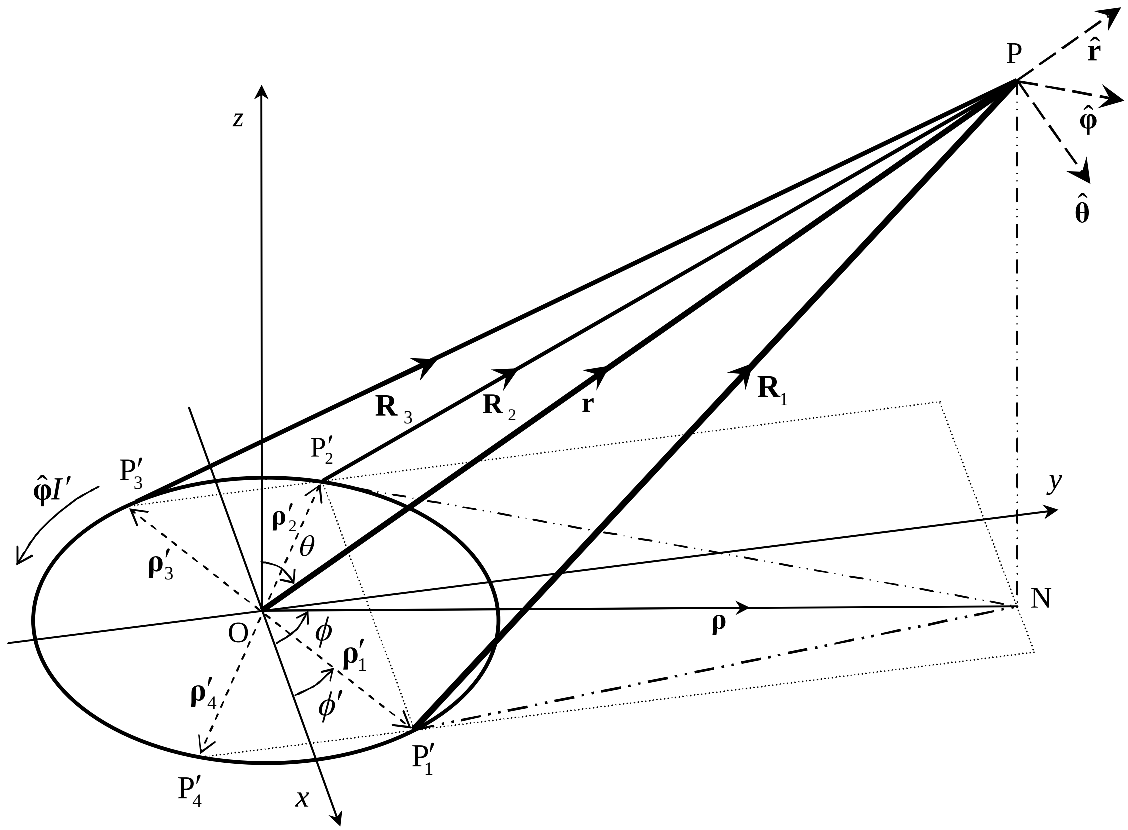

Figure 2: A field point P in a $z\rho$-plane and magnetic source points P_1', P_2', P_3', P_4' on a $\rho\phi$-circle.

where $s_\mathrm{e}' = 2z'$, with the geometrical factors $f_\pm$ being dependent only on r, θ, z'.

Below, each ensuing first order approximation for $\rho' << r$ or $z' << r$ is after binomial expansion of $f_j^{-\frac{n}{2}}$ or $f_\pm^{-\frac{n}{2}}$, where $n = 1$ for potentials and $n = 3$ for fields, and application of equations (3a) to (3c). Then the two Cartesian elemental magnetic dipoles on the circle in figure 2 and the electric dipole on the z-axis in figure 3 will generate at P the magnetic vector and electric scalar potentials

$$d\mathbf{A}_a = d\mathbf{A}_{a_+} + d\mathbf{A}_{a_-} = \left(f_3^{-\frac{1}{2}} - f_2^{-\frac{1}{2}}\right)\frac{\hat{\mathbf{x}}\,dQ_a'}{4\pi\mu_0 r} \approx -\hat{\mathbf{x}}\sin\phi\sin\theta\,\frac{s_a' dQ_a'}{4\pi\mu_0 r^2} \equiv -\hat{\mathbf{x}}\sin\phi\sin\theta\,\frac{dm_a'}{4\pi\mu_0 r^2}, \quad (13a)$$

$$d\mathbf{A}_b = d\mathbf{A}_{b_+} + d\mathbf{A}_{b_-} = \left(f_1^{-\frac{1}{2}} - f_2^{-\frac{1}{2}}\right)\frac{\hat{\mathbf{y}}\,dQ_b'}{4\pi\mu_0 r} \approx +\hat{\mathbf{y}}\cos\phi\sin\theta\,\frac{s_b' dQ_b'}{4\pi\mu_0 r^2} \equiv +\hat{\mathbf{y}}\cos\phi\sin\theta\,\frac{dm_b'}{4\pi\mu_0 r^2}, \quad (13b)$$

$$dV = dV_+ + dV_- = \left(f_+^{-\frac{1}{2}} - f_-^{-\frac{1}{2}}\right)\frac{dq}{4\pi\varepsilon_0 r} \approx + \quad \cos\theta\,\frac{s_\mathrm{e}' dq}{4\pi\varepsilon_0 r^2} \equiv + \quad \cos\theta\,\frac{dp'}{4\pi\varepsilon_0 r^2}. \quad (13c)$$

Each of $d\mathbf{A}_a$, $d\mathbf{A}_b$ and dV has a Cartesian plane where it alone is exactly zero. This is traditionally [2–4] misjudged as absence of a related magnetic dipolar moment, which wrongly implies that the latter depends on point P. There is no such inference for the electric dipolar moment! Integrating, and then changing to spherical unit vectors, shows that $\mathbf{A}_a$ and $\mathbf{A}_b$ have all three spherical components:

$$\mathbf{A}_a = \left[-\left(\hat{\mathbf{r}}\sin\theta + \hat{\boldsymbol{\theta}}\cos\theta\right)\sin\phi\cos\phi + \hat{\boldsymbol{\varphi}}\sin^2\phi\right]\frac{m_a'\sin\theta}{4\pi\mu_0 r^2}, \quad (14a)$$

$$\mathbf{A}_b = \left[+ \left(\hat{\mathbf{r}} \sin\theta + \hat{\boldsymbol{\theta}} \cos\theta \right) \sin\phi \cos\phi + \hat{\boldsymbol{\varphi}} \cos^2\phi \right] \frac{m_b' \sin\theta}{4\pi\mu_0 r^2}, \tag{14b}$$

$$V = \frac{p' \cos\theta}{4\pi\varepsilon_0 r^2}. \tag{14c}$$

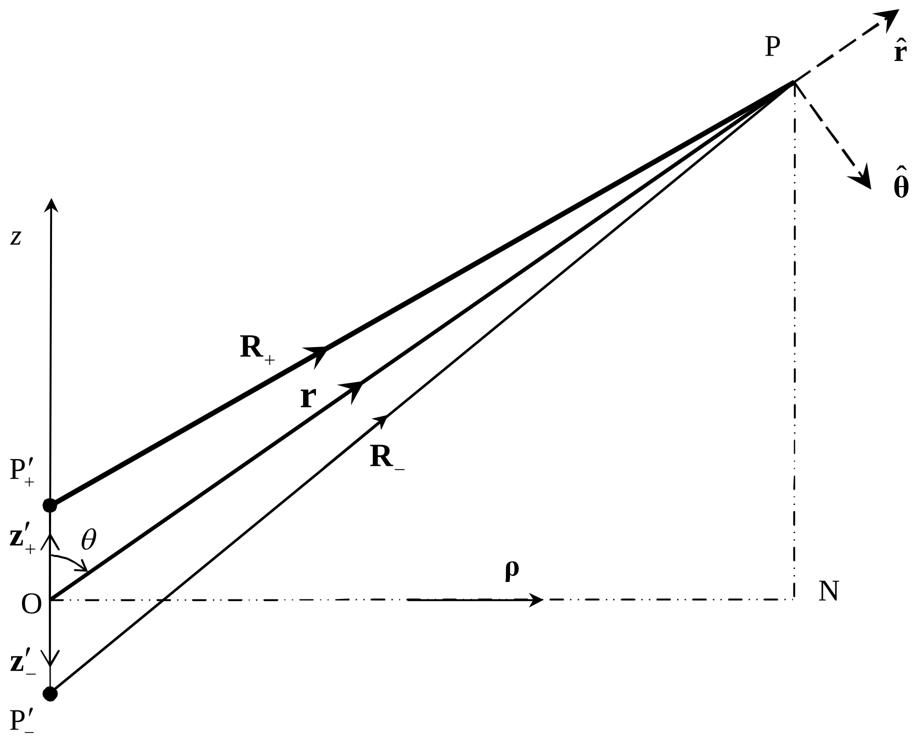

Figure 3: A field point P in a $z\rho$- or $r\theta$-plane and electric source points P_+' and P_-' on the z axis.

Due to relations in (5), the total magnetic vector potential is interchangeably expressed exclusively in terms of either m_a' or m_b' and thus acquires cylindrical symmetry:

$$\mathbf{A} = \mathbf{A}_a + \mathbf{A}_b = \hat{\boldsymbol{\varphi}} \frac{m_a' \sin\theta}{4\pi\mu_0 r^2} \equiv \hat{\boldsymbol{\varphi}} \frac{m_b' \sin\theta}{4\pi\mu_0 r^2}. \tag{15}$$

Similarly, the fields at point P due to the two distinct Cartesian elemental magnetic dipoles in figure 2 and the electric dipole in figure 3 are

$$d\mathbf{H}_a = d\mathbf{H}_{a_+} + d\mathbf{H}_{a_-} = \frac{\hat{\mathbf{x}} dQ_a'}{4\pi\mu_0 r^3} \times \left(\mathbf{R}_3 f_3^{-\frac{3}{2}} - \mathbf{R}_2 f_2^{-\frac{3}{2}} \right)$$
$$\approx \left[\hat{\mathbf{y}} 3 \sin\phi \cos\theta \sin\theta + \hat{\mathbf{z}} \left(1 - 3\sin^2\phi \sin^2\theta \right) \right] \frac{s_a' dQ_a'}{4\pi\mu_0 r^3}, \tag{16a}$$

$$d\mathbf{H}_b = d\mathbf{H}_{b_+} + d\mathbf{H}_{b_-} = \frac{\hat{\mathbf{y}} dQ_b'}{4\pi\mu_0 r^3} \times \left(\mathbf{R}_1 f_1^{-\frac{3}{2}} - \mathbf{R}_2 f_2^{-\frac{3}{2}} \right)$$
$$\approx \left[\hat{\mathbf{x}} 3 \cos\phi \cos\theta \sin\theta + \hat{\mathbf{z}} \left(1 - 3\cos^2\phi \sin^2\theta \right) \right] \frac{s_b' dQ_b'}{4\pi\mu_0 r^3}, \tag{16b}$$

$$d\mathbf{E} = d\mathbf{E}_+ + d\mathbf{E}_- = \frac{dq'}{4\pi\varepsilon_0 r^3}\left(\mathbf{R}_{e_+} f_+^{-\frac{3}{2}} - \mathbf{R}_{e_-} f_-^{-\frac{3}{2}}\right) \approx \left(\hat{\mathbf{r}} 2\cos\theta + \hat{\boldsymbol{\theta}}\sin\theta\right)\frac{s'_e dq'}{4\pi\varepsilon_0 r^3}. \qquad (16c)$$

Integrating (16a) to (16c), and changing from Cartesian to spherical unit vectors, shows that, while $\mathbf{E}$ does not vary with ϕ, each of $\mathbf{H}_a$ and $\mathbf{H}_b$ has all three spherical components and varies with ϕ:

$$\mathbf{H}_a = \left[\hat{\mathbf{r}}\cos\theta + \hat{\boldsymbol{\theta}}\left(3\sin^2\phi - 1\right)\sin\theta + \hat{\boldsymbol{\varphi}} 3\cos\phi\sin\phi\cos\theta\sin\theta\right]\frac{m'_a}{4\pi\mu_0 r^3}, \qquad (17a)$$

$$\mathbf{H}_b = \left[\hat{\mathbf{r}}\cos\theta + \hat{\boldsymbol{\theta}}\left(3\cos^2\phi - 1\right)\sin\theta - \hat{\boldsymbol{\varphi}} 3\cos\phi\sin\phi\cos\theta\sin\theta\right]\frac{m'_b}{4\pi\mu_0 r^3}, \qquad (17b)$$

$$\mathbf{E} = (\hat{\mathbf{r}} 2\cos\theta + \hat{\boldsymbol{\theta}}\sin\theta)\frac{p'}{4\pi\varepsilon_0 r^3}. \qquad (17c)$$

Also due to (5), when expressed exclusively in terms of either m'_a or m'_b, the circular current's total magnetic field $\mathbf{H}$ acquires cylindrical symmetry similar to that of $\mathbf{E}$ in (17c):

$$\mathbf{H} = \mathbf{H}_a + \mathbf{H}_b = \left(2\hat{\mathbf{r}}\cos\theta + \hat{\boldsymbol{\theta}}\sin\theta\right)\frac{m'_a}{4\pi\mu_0 r^3} \equiv \left(2\hat{\mathbf{r}}\cos\theta + \hat{\boldsymbol{\theta}}\sin\theta\right)\frac{m'_b}{4\pi\mu_0 r^3}. \qquad (18)$$

Clearly this similarity cannot justify the tradition [2–8] of taking either $\boldsymbol{m}'_a$ or $\boldsymbol{m}'_b$ as the circular current's only magnetic dipolar moment. Again the traditional analogy fails. In fact equations (17a) to (18) can also be obtained indirectly from equations (14a) to (15) by applying the relations

$$\mathbf{H} = \nabla \times \mathbf{A} \qquad \text{and} \qquad \mathbf{E} = -\nabla V. \qquad (19)$$

4. Conclusions

It has been shown that traditional analogies between the structures and torques of electric and magnetic dipoles are deceptively erroneous. A circular current is resolvable into merged distributions of distinct Cartesian *x* and *y* component line current elements; each distribution creates like Cartesian components of the azimuthal magnetic torque and vector potential, plus only the magnetic field's other Cartesian components, all of which are expressible in terms of the distribution's own magnetic dipolar moment. Equality allows interchangeable use of the magnetic dipolar moments of these two complementary distributions and thus gives cylindrical symmetry to the circular current's magnetic properties, so that tradition incorrectly assigns either of these magnetic dipolar moments to the whole circular current. In contrast a simple electric dipole aligned on the *z*-axis, creates its *x* and *y* electric torque components, its full cylindrically symmetric electric field and the related electric scalar potential, all of which are expressible in terms of the sole electric dipolar moment.

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