

# Miniaturization of electrostatic ion engine through ionization/acceleration coupling: corona model

**P Ferrer and MP Tchonang**

Center for Theoretical Physics, School of Physics, University of the Witwatersrand,  
Private Bag 3, Wits 2050, Johannesburg, South Africa

Email: [Philippe.Ferrer@wits.ac.za](mailto:Philippe.Ferrer@wits.ac.za), [mariust82@gmail.com](mailto:mariust82@gmail.com)

Electrostatic ion propulsion systems resist miniaturization due to constraints imposed by the size of the discharge chamber. We introduce a thruster concept where the same field is responsible for both ionization of the neutrals and acceleration of the ions, by letting the neutral propellant gas escape into a high field region through a thin, hollow needle at high electric potential. The ionization mechanism is thus reminiscent of corona ionization. Although the thruster only ionizes a small fraction of the neutral gas, the ions nevertheless impart a great deal of momentum to the plume, creating an ion wind. We propose a model to estimate the electric behavior of the system, and two further models for the obtained thrust. A comparison with experimental data shows that the models capture the dominant physical effects and give a reasonable description of the system. Apart from being about a thousand times less massive than conventional systems, the thruster, which is at the proof-of-concept stage, performed quite well during initial tests. The thruster small size and simplicity are advantageous in many situations, such as for satellite station keeping and deep space probes.

## 1. Introduction

There are numerous advantages to miniaturizing technologies for space applications. A promising propulsion technology is the gridded electrostatic ion engine. Gridded electrostatic ion engines consist of a discharge chamber for gas ionization, the accelerating grids and the neutralizer [1]. These engines are attractive for a number of reasons and they are a mature technology [2,3]. Electrostatic ion engines resist size reductions mainly due to poor down-scaling characteristics of the discharge chamber, in which the neutral propellant is ionized by electron collision [4].

The concept for the thruster proposed in this paper was conceived in order to reduce size and complexity of an engine which relies on electrostatic acceleration of ions for propulsion. Our aim in this paper is to introduce the concept for a miniaturized electrostatic thruster, to propose a mathematical model for it, and to compare some of the initial experimental findings to our model.

## 2. Principle of Operation

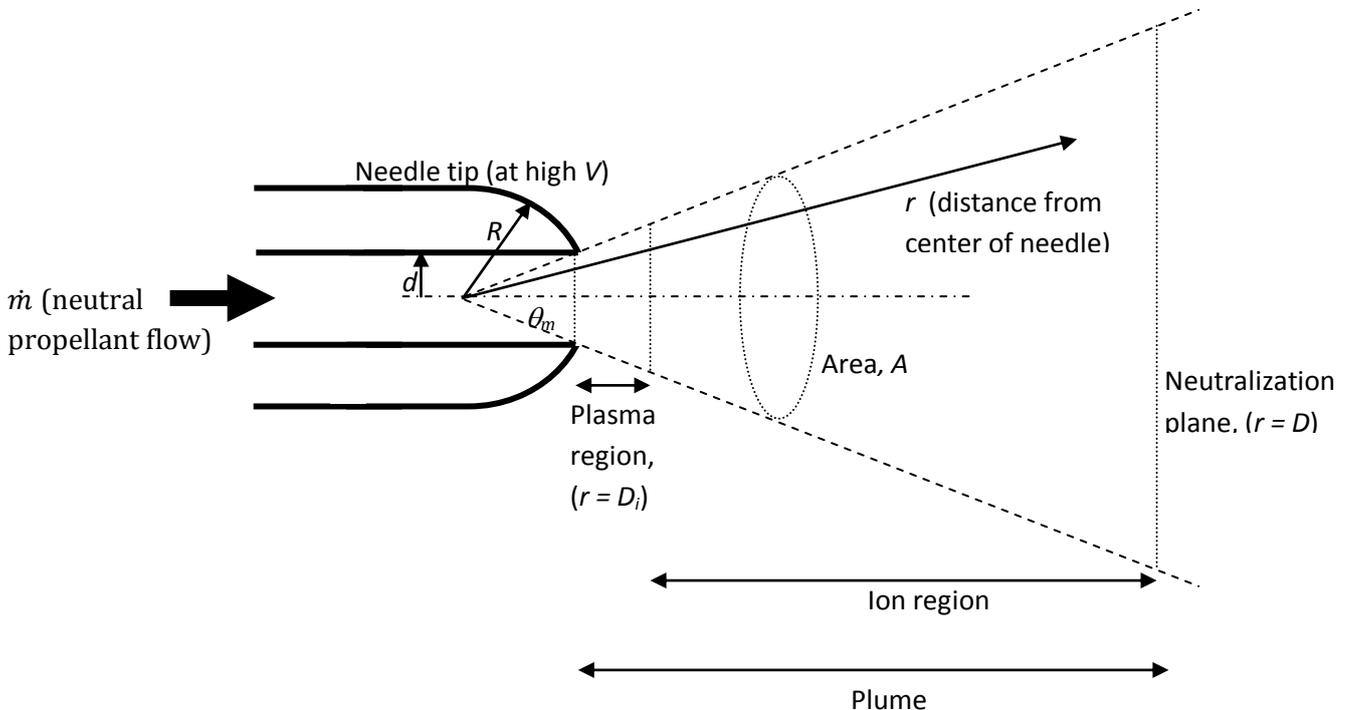
The thruster consists of a thin metal pipe (radius  $R \sim 1/10$  mm) with a rounded tip, such as a hollow needle (figure 1). A neutral gas is fed through the needle from a propellant tank. The needle is electrically connected to the positive terminal and the cathode is at some other external point relatively far away, completing the field. For a working design, a thermionic cathode is needed to neutralize the beam, but in the tests was substituted by an earthed electrode which intercepted and neutralized the ejected ions.

In operation, the neutral propellant is fed at a mass-flow rate  $\dot{m}$  through the hollow needle which is maintained at a high positive potential,  $V$ . The propellant exits through the sharpened needle tip of inner radius  $d$  and outer radius  $R$ , where corona-like ionization takes place. The emerging plume of neutrals spreads like a cone of half-angle  $\theta_m$ , and decreases in density as  $\sim \frac{1}{r^2}$  ( $r$  being the distance from the needle tip). Free electrons from the plume move towards the needle tip, where they collide with emerging neutral propellant atoms, which they ionize, creating more electrons and ions.

The ions created close to the needle tip accelerate away, providing thrust. Hence the same electric field is responsible both for ionization of the neutrals and acceleration of the ions. The two mechanisms are coupled, allowing for great size reduction, which is the novelty of this design. A design of such small size can have many advantages: besides cost reductions, such thrusters will take very little space on a probe or satellite. They can greatly improve mission success probability due to their simplicity, and they are small enough to be stacked, so that a number of them can be on board without significantly increasing mass.

### 3. Estimate for electrical properties

The plume in figure 1 is subdivided into three regions, the plasma region, where ionization takes place, the ion region, where positively charged ions dominate, and the neutralization region, where electrons are injected into the ion stream.



**Figure 1.** Schematic of the exhaust plume. The neutral propellant exits through the needle into vacuum, giving rise to an expanding cone of gas.

We model the electric behavior of the plume by solving a related, simpler system. We consider a positive point charge at the origin, which generates an electric field similar to the one close to the

needle tip. We consider the origin to be a source of neutral gas expanding spherically, with density decreasing as  $\frac{1}{r^2}$ . The origin is encapsulated by a concentric, spherical, earthed, conducting shell at distance  $D$ . The field between the shell and the point charge increases towards the origin, where corona ionization takes place.

The system is spherically symmetric, hence the Poisson equation for the radial component is

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial V}{\partial r} \right) = \frac{-\rho_{ion}}{\epsilon_0} \quad (1)$$

The solution to (1) is the space charge corrected radial electric field with the needle tip at its origin [5],

$$\frac{dV_r}{dr} = \left( \frac{3i}{2C_1} \right)^{\frac{2}{3}} \frac{1}{r^2} \left( r + \frac{2C_1}{3i} (RV_0)^{\frac{3}{2}} - R \right)^{\frac{2}{3}}, \quad (2)$$

where

$$C_1 = 2\pi\epsilon_0(1 - \cos(\theta_m))C_0 \text{ and } C_0 = \left( \frac{Ke^2}{2M^2n_0^2\sigma^2} \right)^{\frac{1}{4}}$$

For the current – voltage relationship, we integrate (3) from  $R$  to  $D$ , which results in

$$\begin{aligned} V - V_0 = & \left( \frac{-(D-C_2)^{\frac{2}{3}}}{D} + \frac{1}{3b} \ln \left[ \frac{\frac{(D+C_2)^{\frac{2}{3}}}{b^2} - \frac{2(D+C_2)^{\frac{1}{3}}}{b} + 1}{\frac{(D+C_2)^{\frac{2}{3}}}{b^2} + \frac{(D+C_2)^{\frac{1}{3}}}{b} + 1} \right] + \frac{\sqrt{12}}{3b} \tan^{-1} \left[ \frac{2}{\sqrt{3}} \frac{(D+C_2)^{\frac{1}{3}}}{b} + \frac{1}{\sqrt{3}} \right] - \frac{1}{3b} \ln \left[ \frac{\frac{(R+C_2)^{\frac{2}{3}}}{b^2} - \frac{2(R+C_2)^{\frac{1}{3}}}{b} + 1}{\frac{(R+C_2)^{\frac{2}{3}}}{b^2} + \frac{(R+C_2)^{\frac{1}{3}}}{b} + 1} \right] - \right. \\ & \left. \frac{\sqrt{12}}{3b} \tan^{-1} \left[ \frac{2}{\sqrt{3}} \frac{(R+C_2)^{\frac{1}{3}}}{b} + \frac{1}{\sqrt{3}} \right] \right) \left( \frac{3i}{2C_1} \right)^{\frac{2}{3}}, \quad (4) \end{aligned}$$

$$\text{where } C_2 = \frac{2C_1}{3i} (RV_0)^{\frac{3}{2}} - R \text{ and } b = C_2^{\frac{1}{3}}$$

The  $C_2$  terms contain the current. This relationship will be tested against experiment.

#### 4. Estimation of the thrust

We derive the thrust using two different methods, the first makes use of energy conservation while the second uses the more conventional approach of integrating the force contribution of the charges on the needle by recoil.

##### 4.1 Estimate for the thrust using “vector heating” (VH)

As an ion current develops in the plume, the effective resistance can be obtained from the measured current and voltage. Keeping to this model, the ionic current collides with neutrals and “heats” them via collision in a direction away from the needle (hence the directional character). The total power  $P$  into the plume has two main components, the ionization power  $P'$  and the power used to heat the plume,  $H$ . We write

$$P = P' + H, \quad (5)$$

where  $H$  is given simply by:

$$H = \dot{m}C(T_2 - T_1) \quad (6)$$

Here,  $C$  is the heat capacity at constant pressure,  $T_1$  the temperature of the cold exhaust and  $T_2$  the final temperature.

Next, using  $\bar{v} = \sqrt{\frac{3k_B T}{M}}$  (where  $k_B$  is Boltzmann's constant,  $T$  the gas temperature,  $M$  the molecular mass and  $\bar{v}$  the average molecular velocity in the gas) and the thrust equation,  $F = \dot{m}(v_2 - v_1)$ , we obtain

$$F = \dot{m} \sqrt{\frac{3k_B}{M}} \left\{ \left[ \frac{P-P'}{\dot{m}C} + T_1 \right]^{1/2} - T_1^{1/2} \right\} \quad (7)$$

This equation will be tested against the measured thrust. Despite its simple derivation, it gives very reasonable results.

#### 4.2 Estimate for the thrust from the electrostatic method (ES)

We generalized a procedure used by a number of authors to calculate thrust using electrostatic interactions [6, 7, 8].

The total force exerted by the needle tip on  $N$  charges is

$$F_T = NqE \quad (8)$$

Using (3) for the effective field and integrating from  $R$  to  $D$  gives the total thrust,

$$F_T = \frac{i}{c_0} \left( \frac{3i}{2c_1} \right)^{\frac{1}{3}} \left\{ \frac{-a}{r} + \frac{1}{6b^2} \ln \left[ \frac{a^2 - \frac{2a}{b} + 1}{\frac{a^2}{b^2} + \frac{2a}{b} + 1} \right] + \frac{\sqrt{12}}{6b^2} \tan^{-1} \left[ \frac{2}{\sqrt{3}} \frac{a}{b} + \frac{1}{\sqrt{3}} \right] \right\} \Bigg|_R^D, \quad (9)$$

with  $a = (r + C_2)^{\frac{1}{3}}$  and  $b = C_2^{\frac{1}{3}}$

This equation will be tested against the data, and should give results similar to (7). It will be seen, however, that (9) underestimates the thrust.

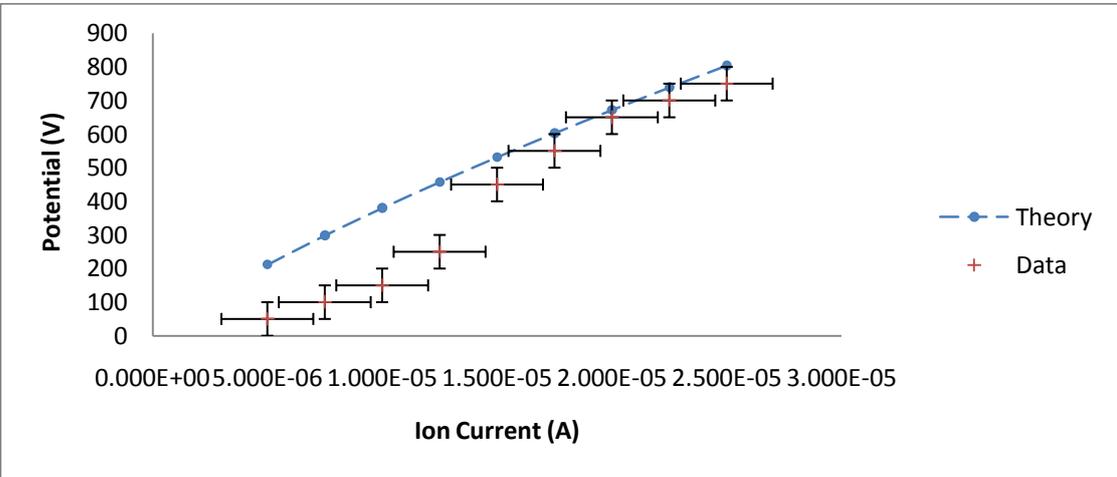
## 5. Comparison and Discussion

Experimental results obtained by [9] are compared with the theoretical prediction in figures 2 for the electrical behaviour of the thruster. The graph in figure 3 compares the two thrust models, (VH) from (7), and (ES) from (9).

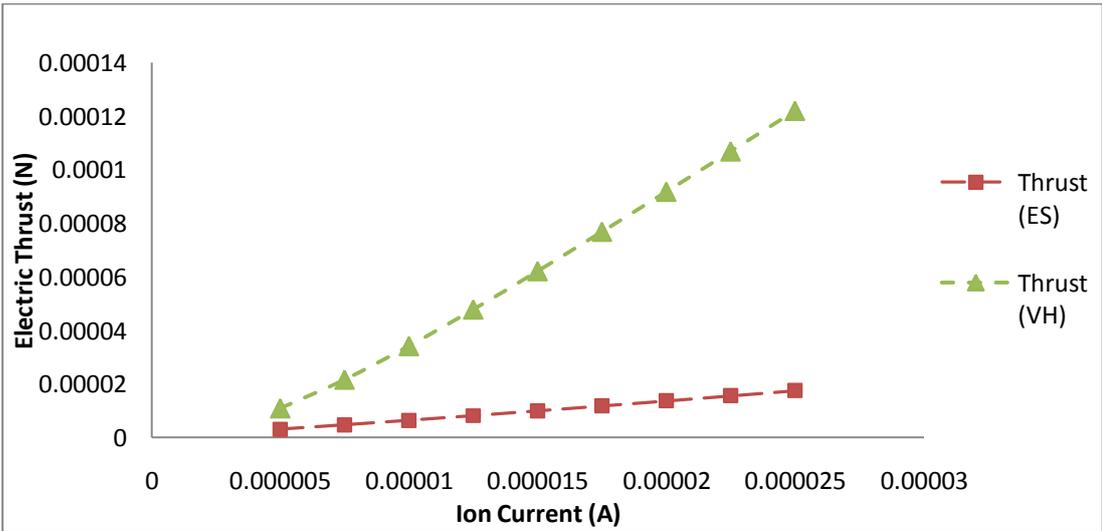
In figure 2, the data and the model for electric behavior are shown. From the number of our approximations, basic trends and order of magnitude agreement was acceptable. The model did

describe the trend and fell within the error bars of some of the points. The model therefore seems to capture the main physical effects.

Both models describe only the part of the thrust arising from electric power input, and have no free parameters. The ES model underestimates the thrust given by the VH model by a factor of roughly 10 on figure 3. The reason for this is unclear. Since the VH model estimates the thrust reasonably, two possibilities for the failure of the ES model are that, first, the ion charge distribution is not described accurately enough, and/or a dominant ion section was left out; or, second, the extra thrust has an origin other than electrostatic (perhaps a pressure effect from the residual gas). A more accurate set of measurements is necessary to compare the thrust given by the models to that obtained experimentally and to come to a definite conclusion.



**Figure 2.** Comparison of data with model for the electrical behavior of the thrusters.



**Figure 3.** Comparison of the vector heating (VH) and electrostatic (ES) thrust models.

## 6. Conclusion

We investigated a mechanism which could significantly decrease the size and complexity of electrostatic propulsion systems. A model for the electric behavior was compared to the preliminary data, and showed reasonable agreement. For the thrust, the VH model provides results that are more close to experimental results. The ES model, which has been used on numerous occasions, seems to be falling short. The reason for this cannot be determined from the available data. Although the models capture the dominant physics, more realistic approximations will certainly improve the model.

The main aim to the present was establishing its basic feasibility, operational principles and modeling. Numerous issues must be studied next, so that a comparison to existing systems becomes possible.

## References

- [1] Stuhlinger E 1964 *Ion Propulsion for Space Flight* McGraw-Hill New York  
Goebel D M and Katz I 2008 *Fundamentals of Electric propulsion – Ion and Hall thrusters* John Wiley & Sons New York
- [2] Sutton G P and Biblarz O 2001 *Rocket Propulsion Elements Seventh Edition* John Wiley & Sons New York
- [3] Brophy J R 2000 NASA's deep space 1 Ion Engine ICIS-01-20.1
- [4] Mahalingam S 2007 Particle based plasma simulation for an ion engine discharge chamber *PhD submitted at Wright State University Engineering*
- [5] Tchonang M 2011 Modeling of the Corona Ionization Space Propulsion System *Masters Dissertation University of the Witwatersrand*
- [6] Wilson J, Perkins D H and Thompson W K 2009 An Investigation of Ionic Wind Propulsion *NASA/TM—215822*
- [7] Christenson E A and Moller P S 1967 Ion neutral propulsion in atmospheric media *AIAA journal* **Volume 5 Number 10** pp. 1768-73
- [8] Robinson M May 1961 Movement of Air in the Electronic Wind of Corona Discharge *AIEE Transactions* **Volume 80** pp. 143-50
- [9] Lafleur T 2007 Corona discharge as an ionization mechanism for electrostatic propulsion applications *Honors project University of the Witwatersrand*