Quasi-Normal Modes for spin-3/2 fields

Gerhard Harmsen

Supervisor: Prof. Alan Cornell

Collaborators: C. H. Chen, H. T. Cho, W. Naylor.

University of the Witwatersrand

gerhard.harmsen5@gmail.com [1]

2nd July 2015

[1] Gravitino fields in Schwarzschild black hole spacetimes (arXiv:1504.02579)

Overview

1. Rarita-Schwinger equation
2. QNMs
3. Black holes
   - Schwarzschild black holes
   - Potential in 4D-Schwarzschild backgrounds
4. Approximation methods
   - WKB approximation
   - Improved Asymptotic Iterative Method
5. Results
6. Further work
   - Higher dimensional black holes
   - Reissner-Nordstrom black holes

Rarita-Schwinger equation

- The relativistic field equation of spin-3/2 particles.

Rarita-Schwinger equation

$$\gamma^{\mu\nu\alpha}\nabla_\nu\Psi_\alpha = 0$$

where $\gamma^{\mu\nu\alpha} = \gamma^\mu\gamma^\nu\gamma^\alpha - \gamma^\mu g^{\nu\alpha} + \gamma^\nu g^{\nu\alpha} - \gamma^\alpha g^{\mu\nu}$.

- Gravitino is predicted to have a spin of 3/2.
- Lightest supersymmetic particle.

Quasi-Normal Modes

Black hole

2

[2] Black hole from the movie "Interstellar"

Mathematical description of QNMs

- We can determine the form of QNMs using the wave equation with a damping term.

Particle wave solution

$$\Psi_\alpha(x,t) = \phi_\alpha(x) e^{i\omega t}$$

General form for a QNM

$$\frac{d^2\phi_\alpha(\theta)}{dx^2} + \left(V(x) + \omega\right)\phi_\alpha(\theta) = 0$$

Schwarzschild black holes

- Simplest type since they are non-rotating electrically neutral.

Schwarzschild metric

$$ds^2 = -\left(\frac{r - 2GM}{r}\right) dt^2 + \left(\frac{r - 2GM}{r}\right)^{-1} dr^2 + r^2 \left(d\theta^2 + \sin^2(\theta) d\phi^2\right)$$

4D Schwarzschild backgrounds

Potential

$$V_{1,2} = \pm f(r)\frac{dW}{dr} + W^2,$$

with,

$$W = \frac{\left(j-\frac{1}{2}\right)\left(j+\frac{1}{2}\right)\left(j+\frac{3}{2}\right)\sqrt{f(r)}}{r\left(\left(j+\frac{1}{2}\right)^2 - f(r)\right)},$$

$f(r) = \left(\frac{r-2GM}{r}\right)$ and $j = 3/2, 5/2, 7/2, \ldots.$

WKB approximation

Problem type

We can use the WKB approximation to solve equations of the form,

$$\epsilon^2 \frac{d^2 y}{dx^2} = Q(x)y,$$

with $\epsilon \ll 1$ and positive.

- Assume $y(x,\epsilon) = A(x,\epsilon)e^{\frac{iu(x)}{\epsilon}}$.

WKB approximation

- We then take orders of ϵ to get our first order approximation of A.

Solution

$$-u'(x)^2A_1 + iA_0u''(x) + 2iA_0'u'(x) + Q(x)A_0 = 0,$$
$$A_0u''(x)+2A_0'u'(x) = 0,$$
$$\frac{A_0}{2\sqrt{Q(x)}}+\frac{A_0'}{\sqrt{Q(x)}} = 0,$$
$$\int \frac{1}{A_0}dA_0 = \int \frac{1}{2Q(x)}dx.$$

Where $u(x) = \pm \int_{x_0}^{x} \sqrt{Q(k)}dk$.

Improved AIM

Theory

Given λ_0 and s_0 in $C_\infty(a, b)$, then the differential

$$y'' = \lambda_0(x)y' + s_0(x)y,$$

has the general solution of:

$$y(x) = exp\left(-\int^x \alpha dt\right)\left[C_2 + C_1 \int^x exp\left(\int^t (\lambda_0(\tau) + 2\alpha(\tau))\, d\tau\right)\right],$$

for some $n > 0$.

$$\frac{s_n}{\lambda_n} = \frac{s_{n-1}}{\lambda_{n-1}} \equiv \alpha,$$

where $\lambda_k = \lambda'_{k-1} + s_{k-1} + \lambda_0\lambda_{k-1}$ and $s_k = s'_{k-1} + s_0\lambda_{k-1}$ for $k = 1, 2, ..., n$.

Results

Table: Low-lying ($n \leq l$, with $l = j - 3/2$) gravitino quasinormal mode frequencies using the WKB and the AIM methods.

		WKB		AIM
l	n	3rd Order	6th Order	150 iterations
0	0	0.3087 - 0.0902i	0.3113 - 0.0902i	0.3108 - 0.0899i
1	0	0.5295 - 0.0938i	0.5300 - 0.0938i	0.5301 - 0.0937i
1	1	0.5103 - 0.2858i	0.5114 - 0.2854i	0.5119 - 0.2863i
2	0	0.7346 - 0.0949i	0.7348 - 0.0949i	0.7348 - 0.0949i
2	1	0.7206 - 0.2870i	0.7210 - 0.2869i	0.7211 - 0.2871i
2	2	0.6960 - 0.4844i	0.6953 - 0.4855i	0.6892 - 0.4834i
3	0	0.9343 - 0.0954i	0.9344 - 0.0954i	0.9344 - 0.0954i
3	1	0.9233 - 0.2876i	0.9235 - 0.2876i	0.9235 - 0.2876i
3	2	0.9031 - 0.4835i	0.9026 - 0.4840i	0.9026 - 0.4840i
3	3	0.8759 - 0.6835i	0.8733 - 0.6870i	0.8733 - 0.6870i

N-dimensional black holes

- We have a spherical term $d\Omega_2^2 = d\theta^2 + \sin^2(\theta) d\phi^2$.
- Generalise this to $d\Omega_n^2 = d\theta^2 + \sin^2(\theta) d\Omega_{n-1}^2$.

N-dimensional black hole (non-rotating)

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega_{n-2}^2$$

Reissner-Nordstrom black holes

- This is a non-rotating electrically charged black hole.

Reissner-Nordstrom

$$ds^2 = -f(r)dt^2 + f^{-1}(r)dr^2 + r^2\left(d\theta^2 + \sin(\theta)^2 d\phi^2\right)$$

with $f(r) = \left(\frac{r^2 - 2GMr + Q^2}{r^2}\right)$.

Acknowledgements to:

UNIVERSITY OF THE WITWATERSRAND JOHANNESBURG

NITheP National Institute for Theoretical Physics

NRF