# Application of Kalman filtering for the determination of a VLBI terrestrial reference frame

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# Kalman filter for TRF determination

- Time series representation  $\rightarrow$  capture short-term variations
- Short-term stability by restrictive stochastic model
- Predictions by extrapolating the functional model
- TRF easy to update & real-time capable
- Kalman filter software for multi-technique TRFs (KALREF) developed at NASA JPL
  - Wu et al., 2015
  - JTRF2014
- Software at GFZ: VLBI only focus on different modeling approaches
  - Soja et al. (JoG, submitted)









# Input VLBI data

- 4239 IVS VLBI sessions between 1980 and 2013
  - 4 or more telescopes participating
  - Spanning a polyhedron with a volume of more than 10<sup>15</sup> m<sup>3</sup>
- 104 stations out of 143 stations considered
  - Regular observations over more than 1 year
- Session-wise station coordinates XYZ
  - NNT+NNR w.r.t. ITRF2008 for all stations with ITRF2008 coordinates









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# Kalman filter setup

- Kalman filter & smoother
- States updated for every VLBI session (usually every 1-4 d)
- Breaks in position and/or velocity
  - Earthquakes, equipment changes
- Output:
  - Filtered and smoothed XYZ time series
  - Average: XYZ at reference epoch, velocities, amplitude + phase of annual signals
- Datum by 12 parameter transformation (scale not changed)
  - Average coordinates & velocities w.r.t. ITRF2008 for selected datum stations

$$\tilde{x}_{k} = F_{k} x_{k-1}$$

$$\tilde{P}_{k} = F_{k} P_{k-1} F_{k}^{T} + Q_{k}$$

$$F_{k} = \begin{bmatrix} 1 & dt & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2\cos\left(2\pi \frac{dt}{T}\right) & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$



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# Comparison of linear TRFs

- Kalman filter in "linear mode" (zero noise)
  - Transformation to ITRF2008 based on 10 stable core stations
- Consistent global solution (with VIE\_GLOB)
  - Stacking of normal equations

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NNT+NNR w.r.t. ITRF2008 for same 10 core stations



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# Velocity differences w.r.t. ITRF2008

- RMS values of the velocity differences of linear Kalman filter and least-squares VLBI TRF solutions
- 64 stations
  - present in all 3 solutions
  - with observation history > 3 years
  - without breaks
- 21 stations
  - out of the 64 stations
  - participated in more than 200 sessions

		64 stations				21  stations  (> 200  obs.)			
RMS [mm/yr]	R	Е	Ν	3D	R	Ε	Ν	3D	
KF VTRF	3.91	1.43	1.53	4.44	1.42	0.67	0.32	1.60	
LS VTRF	6.59	2.23	2.35	7.34	1.87	0.70	0.44	2.05	





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## Process noise of station coordinates

- Assumption: irregular station coordinate variations due to unmodeled NTAL, NTOL & CWSL displacements
- Time series of NTAL, NTOL & CWSL
  - Downloaded from massloading.net (Petrov, 2015), resolution 6 h
  - Sum calculated, trend & annual signal removed
- Assuming random walk (RW) processes → computation of power spectral densities (PSDs) of driving white noise





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### Station coordinate time series

- Solutions with different functional and stochastic models
  - Linear, linear+annual, RW, RW+annual, integrated RW
- RW solutions: applying noise model from loading displacements
  - Scaled by factor 1, 1/10, 1/100





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# Velocity comparison of TRF solutions

- RMS values of the velocity differences of Kalman filter TRF solutions w.r.t. the linear Kalman filter TRF solution
- 74 stations
  - with observation history > 3 years
  - without breaks
- 22 stations
  - out of the 74 stations
  - participated in more than 200 sessions

	74 stations				22  stations  (> 200  obs.)			
RMS [mm/yr]	R	Е	Ν	3D	R	Ε	Ν	3D
Linear + annual	1.65	0.64	0.52	1.85	0.03	0.01	0.01	0.04
RW noise scaling $0.1 + annual$	3.52	0.88	2.61	4.47	0.30	0.12	0.09	0.34
RW noise scaling 0.1	1.26	0.56	1.48	2.02	0.31	0.13	0.10	0.35
IRW $10^{-4} \text{ mm}^2/\text{day}^3$	1.26	0.39	0.61	1.45	0.28	0.09	0.19	0.35





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# Earthquake handling

- Post-seismic motion captured by KF setups with process noise
  - Unscaled noise from loading model best right after Earthquake
  - At other times maybe too noisy  $\rightarrow$  dynamic scaling of noise desirable





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# Time-dependent noise after earthquakes

- Exponential scaling of noise  $q_{rw}$  after earthquakes
  - Applied during period T after earthquakes
  - $\Delta t$  from 0 to T,  $\alpha$  from  $\alpha_0$  to 1





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# ITRF2014 & JTRF2014

- Based on GNSS, VLBI, SLR & DORIS SINEX files
- Combination at the parameter level
  - VLBI normal equations inverted before combination
- Datum: SLR origin, ITRF2008 orientation, VLBI+SLR scale
- ITRF2014: least squares estimation
  - Linear + post-seismic (+ annual + semi-annual)
- JTRF2014: Kalman filter & smoother
  - Linear + annual + semi-annual
  - Process noise for non-linear & non-harmonic signals
  - Weekly time steps







# Comparison VTRF vs. JTRF/ITRF

• TSUKUB32





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## Comparison VTRF vs. JTRF/ITRF

GFZ input

• TSUKUB32



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#### Recapitulation

- Kalman filtering successfully used to create VLBI TRFs
  - Linear KF TRF comparable to standard TRFs (velocities & transformation parameters)
  - Time series representation recovery of non-linear signals
- Stochastic model station-dependent and time-variable
  - Noise from unmodeled elastic displacements
  - Scaling of noise for post-seismic activity
- Comparison to ITRF2014 and JTRF2014
  - Promising agreement during post-seismic periods
  - Investigations to be extended...







# References

- Altamimi et al., 2011: ITRF2008: an improved solution of the international terrestrial reference frame. J Geod. 85(8), 457-473. doi:10.1007/s00190-011-0444-4
- Petrov, 2015:. The International Mass Loading Service, http://arxiv.org/abs/1503.00191.
- Soja et al., 2016: Determination of a Terrestrial Reference Frame via Kalman Filtering of Very Long Baseline Interferometry Data. Journal of Geodesy, submitted.
- Wu et al., 2015: KALREF—A Kalman filter and time series approach to the International Terrestrial Reference Frame realization. J. Geophys. Res. Solid Earth, 120, 3775–3802. doi:10.1002/2014JB011622.











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