Probing quantum gravity through strong gravitational lensing

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Abstract. We report the use of Einstein rings to reveal the quantized and dynamical states of spacetime in a region of impressed gravitational field as predicted by the Nexus Paradigm of quantum gravity. This in turn reveals the orbital speeds of objects found therein and the radius of curvature of the quantized spacetime. Similarities between the nexus graviton and the singular isothermal sphere (SIS) in the cold dark matter (CDM) paradigm are highlighted. However unlike the singular isothermal sphere, the nexus graviton does not contain singularities or divergent integrals. This provides a viable solution to the core cusp problem. In this work, data from a sample of fifteen Einstein rings published on the CfA-Arizona Space Telescope Lens Survey (CASTLES) website is used to probe the quantized properties of spacetime.

Keywords: (Quantum Gravity; Gravitational Lensing; Dark Matter; Quantum Vacuum; Graviton)

1. Introduction

In 1937, Zwicky [1] provided the first reproducible evidence of the presence of unseen matter in the Coma Cluster group of galaxies by applying the classic virial theorem. In the same paper he also suggested that the gravitational field of galaxy clusters is expected to deflect the light observed from background galaxies. Today, numerous observations [2,3,4,5,6] have confirmed the presence of Zwicky's dark matter and most studies [7,8,9,10] have used gravitational lensing to explore its distribution in cluster groups. Strong lensing allows the determination of important physical parameters such as the total mass of the lensing object without any assumptions on the dynamics. Einstein rings (ER) are particularly important in constraining the mass within the Einstein radius with great accuracy. Currently, it is thought that dark matter is the source of much of the lensing potential. Here we report the use of ERs to calculate the quantum state of spacetime in the presence of the baryonic mass. A remarkable feature of the Nexus Paradigm of quantum gravity is that under certain critical conditions, the intrinsic curvature of spacetime in the *n*-th quantum state is the source of deflection and not the curvature due to the presence of baryonic mass. This feature is clearly illustrated in the Bullet Cluster [11].

2. Theoretical Background

The nature of dark matter along with that of dark energy is one of the most perplexing unsolved problems in astrophysics and has largely divided the astrophysical community into some suggesting a modification of the law of gravity [12,13,14,15] and others suggesting the presence of unseen baryonic or non-baryonic matter [16,17,18,19]. The Nexus Paradigm of quantum gravity [20, 21] is a part of a third approach that seeks to explain dark matter and dark energy as an effect from quantum gravity. In this paradigm, spacetime is a nexus graviton field with 10^{60} eigenstates.

A nexus graviton in the *n*-th quantum state as described in Ref:[20-21] is a spherically symmetric pulse or wave packet of four-space with the following components

$$\Delta x_n^\mu = \frac{2r_{HS}}{n\pi}\gamma^\mu \int_{-\infty}^{\infty} sinc(k^\mu x_\mu)e^{ikx}dk^\mu$$

$$= \gamma^\mu \int_{-\infty}^{\infty} a_{nk}\varphi_{(nk\mu)}dk^\mu \qquad (1)$$

Here $\gamma^\mu$are the Dirac matrices, r_{HS} is the Hubble radius, $\varphi_{(nk\mu)} = sinc(k^\mu x_\mu)e^{ikx}$ are Bloch energy eigenstate functions $kx = k^\mu x_\mu$, $k^\mu = \frac{n\pi}{r_{HS}^\mu}$, $n = \pm 1, \pm 2 \dots 10^{60}$ and the metric $ds^2 = g_{(n)\mu\nu}dx^\mu x^\nu$ is the intensity of the pulse computed from the the inner product of Eqn(1). Percieving the metric equation as the intensity of a four- pulse forms the basis of the nexus graviton formulation of space-time.

The norm squared of the four- momentum of the *n*-th state graviton is

$$(\hbar)^2 k^\mu k_\mu = \frac{E_n^2}{c^2} - \frac{3(nhH_0)^2}{c^2} = 0 \qquad (2)$$

where H_0 is the Hubble constant ($2.2 \times 10^{-18}\ s^{-1}$) and can be expressed in terms of the cosmological constant, Λ as

$$\Lambda_n = \frac{E_n^2}{(hc)^2} = \frac{k_n^2}{(2\pi)^2} = n^2\Lambda \qquad (3)$$

The curvature of spacetime in the *n*-th quantum state is then expressed as

$$G_{(n)\mu\nu} = n^2\Lambda g_{\mu\nu} \qquad (4)$$

where $G_{(n)\mu\nu}$ is the Einstein tensor of spacetime in the *n*-th state. The dark energy arises from the emission of a ground state graviton such that Eqn. (4) becomes

$$G_{(n)\mu\nu} = (n^2 - 1)\Lambda g_{\mu\nu} \qquad (5)$$

If the graviton field is perturbed by the presence of baryonic matter then Eqn.(5) becomes

$$G_{(n)\mu\nu} = kT_{\mu\nu} + (n^2 - 1)\Lambda g_{\mu\nu} \qquad (6)$$

From Ref.[20] the solution to Eqn. (4) is computed as

$$ds^2 = -\left(1 - \left(\frac{2}{n^2}\right)\right)dt^2 + \left(1 - \left(\frac{2}{n^2}\right)\right)^{-1}dr^2 + r^2(d\theta^2 + sin^2\theta d\varphi^2) \qquad (7)$$

In the above equation n = 1 state refers to the ground state of spacetime and is the quantum state of space inside a black hole event horizon. The state in close proximity to this event horizon is the *n*=2 state. There are no singularities in Eqn. (7).

For weak gravitational fields that characterize distances ranging from solar system to cosmic scales, the solution to Eqn.(6) for an aggregation of baryonic matter *M(r)* within a radius *r*, as provided in Ref.[20] is expressed as

$$\frac{d^2r}{dt^2} = \frac{GM(r)}{r^2} + H_0 v_n - H_0 c \qquad (8)$$

Here *c* is the speed of light.

The first term on the right is the Newtonian gravitational acceleration, the second term is a radial acceleration induced by spacetime in the *n*-th quantum state and the final term is acceleration due to dark energy. The dynamics becomes strongly non-Newtonian when

$$\frac{GM(r)}{r^2} = H_0 c = \frac{v_n^2}{r} \qquad (9)$$

These are conditions in which the spacetime curvature due to baryonic matter is annulled by that due to the presence of dark energy. Under such conditions

$$r = \frac{v_n^2}{H_0 c} \tag{10}$$

Substituting for r in Eqn.(9) yields

$$v_n^4 = GM(r)H_0 c \tag{11}$$

This is the baryonic Tully – Fisher relation. The conditions permitting the dark energy to cancel out the curvature due to baryonic matter leave quantum gravity as the unique source of curvature. Thus condition (9) reduces Eqn.(8) to

$$\frac{d^2r}{dt^2} = \frac{dv_n}{dt} = H_0 v_n \tag{12}$$

From which we obtain the following equations of galactic and cosmic evolution

$$r_n = \frac{1}{H_0} e^{(H_0 t)} (GM(r)H_0 c)^{\frac{1}{4}} \tag{13}$$

$$v_n = e^{(H_0 t)} (GM(r)H_0 c)^{\frac{1}{4}} \tag{14}$$

$$a_n = H_0 e^{(H_0 t)} (GM(r)H_0 c)^{\frac{1}{4}} \tag{15}$$

Here r_n is the radius of curvature of spacetime in the n-th quantum state (which is also the radius of the n-th state nexus graviton), v_n the radial velocity of objects embedded in that spacetime, and a_n, their radial acceleration within it. The amplification of the radius of curvature with time explains the existence of ultra-diffuse galaxies and the spiral shapes of most galaxies (see Refs: [20-21]). The increase in radial velocity with time explains why early type galaxies composed of population II stars are fast rotators. Eqn.(15) explains late time cosmic acceleration which began once condition (9) was satisfied or equivalently from Eqn.(6), when the density of baryonic matter was at the same value as that of dark energy. Thus condition (9) also explains the Coincidence Problem.

3. Gravitational lensing and quantum gravity

In the context of the Nexus Paradigm, a gravitational field is a region of spacetime in a low quantum state or a zone with a low vacuum expectation value

$$\langle \psi_n(r_i) | T_{00} | \psi_n(r_j) \rangle = n^2 \rho_{DE} \tag{16}$$

where ψ_n is the wave function of the quantum vacuum in the n-th quantum state and ρ_{DE} is the density of dark energy. From Ref.[20], the flow of spacetime in each quantum state of curvature r_n, induces a constant radial speed onto any test particle embedded within it of

$$v_n = H_0 r_n = c/n \tag{17}$$

The deflection of light through gravitational lensing by spacetime in the n-th quantum state is

$$\alpha = (\theta - \beta) D_{ls}/D_s = 4/n^2 \tag{18}$$

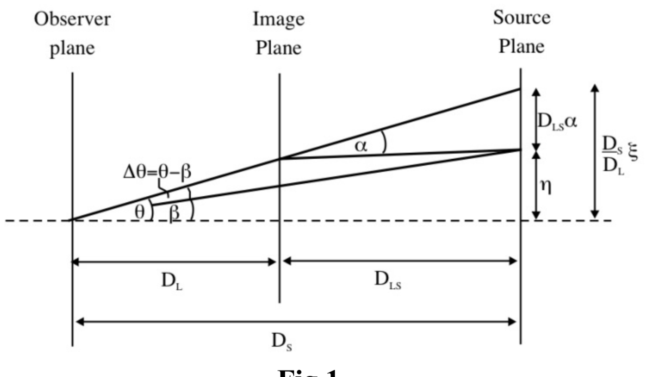

Fig.1

From which we obtain

$$n = 2\sqrt{\frac{D_s}{D_{ls}(\theta - \beta)}} \tag{19}$$

Thus the orbital speed of a particle embedded in the *n*-th quantum state is

$$v_n = c\sqrt{\frac{D_{ls}(\theta-\beta)}{4D_s}} = e^{(Ht)}(GM(r)H_0c)^{\frac{1}{4}} \quad (20)$$

In the case where β= 0 an Einstein ring forms and Eqn.(20) becomes

$$v_n = c\sqrt{\frac{D_{ls}\theta_E}{4D_s}} = e^{(Ht)}(GM(r)H_0c)^{\frac{1}{4}} \quad (21)$$

Here we observe similarities with the cold dark matter (CDM) model where the dispersion velocities for a SIS are calculated from

$$\sigma = c\sqrt{\frac{D_{ls}\theta_E}{4\pi D_s}} \quad (22)$$

Hence Eqn.(21) is the link between the CDM paradigm and the baryonic Tully-Fisher relation. In such a scenario r_n, would denote the radius of the CDM halo. However, in the Nexus Paradigm, the CDM is vacuum energy as described by Eqn.(16).Moreover Eqn.(7) describing the nexus graviton has no singularity nor divergences found in the SIS model.

If the radial velocity and baryonic mass content in the lensing system are known then one can calculate the time *t*, which has elapsed since the lensing system became a system in dynamical equilibrium as stipulated by condition (9).

4. Results

We apply Eqn.(17) and Eqn.(21) to determine the value of *n*, r_n, v_n and a_n for a spherically symmetric lensing system from a sample of fifteen ERs lenses published on the CASTLES website (www.cfa.havard.edu/castles/) . The calculations assume a flat universe in which H_0=69.6 km/s/Mpc Ω_Λ=0.714 Ω_M= 0.286 .The results are displayed in Table 1.

Table 1

Lens	D_{ls}/D_s	θ_E/asc	n	r_n/Mpc	v_n/kms^{-1}	$a_n/10^{-10}$m/s^{-2}
JVA B1938+666	0.50	0.44	1948	2.27	154.0	0.0034
B0218+357	0.17	0.16	5508	0.80	54.5	0.0012
PG1115+080	0.64	1.16	1054	4.19	284.6	0.0063
B1608+656	0.35	1.14	1268	3.49	236.6	0.0052
RXJ1131-1231	0.45	1.90	982	4.50	305.5	0.0067
Q0047-2808	0.58	1.35	1027	4.30	292.1	0.0064
PMNJ0134-0931	0.37	0.37	2456	1.80	122.1	0.0027
HE0230-2130	0.49	1.03	1231	3.62	243.7	0.0054
CFR503.1077	0.34	1.05	1581	2.80	189.8	0.0042
HST15433+5352	0.52	0.59	1640	2.69	182.9	0.0040
MG1549+3047	0.83	0.85	1081	4.09	277.5	0.0061
PKS1830-211	0.34	0.50	2214	2.00	135.5	0.0030
MG2016+112	0.33	1.76	1192	3.71	251.7	0.0055

Q2237+030	0.94	0.89	993	4.45	302.1	0.0066
HE0435-1223	0.51	1.21	1156	3.82	259.5	0.0057

5. Discussion

The results indicate that the orbital velocity of the constituents of a lensing system can be attributed to a globule of quantized vacuum energy called the nexus graviton of density $\rho = n^2\rho_{DE}$ with a radius of dimensions in the order of a few megaparsecs. When in close proximity to other gravitons, tidal forces arise which transform the nexus graviton from a spherical shape to an ellipsoid. The profile of the ellipsoid is an ellipse and the graviton radius becomes

$$r_n = \frac{R_n}{1 - ecos\varphi} \qquad (23)$$

Where e is the eccentricity.
Hence the orbital velocity becomes

$$v_n = H_0 r_n = \frac{H_0 R_n}{1 - ecos\varphi} \qquad (24)$$

From which we calculate density as

$$\rho = \frac{c^2}{v_n^2}\rho_{DE} = \frac{R_H^2(1 - ecos\varphi)^2\rho_{DE}}{R_n^2} = n^2(1 - ecos\varphi)^2\rho_{DE} \qquad (25)$$

where R_H is the Hubble radius. The deflection of light under these conditions becomes

$$\alpha = (\theta - \beta)D_{ls}/D_s = 4/n^2(1 - ecos\varphi)^2 \qquad (26)$$

Here we notice an increase in vacuum energy density with a decrease in curvature radius. The density profile is thus stratified in quantized concentric radii $r_n = R_H/n$ with a maximum radius at $R_N < R_H$ and the minimum at $R_{min} = R_H/10^{60}$. Spacetime in the inner core of the nexus graviton is therefore flat as described by Eqn.(7) and curved at large radii. In the CDM paradigm, the gravitational lensing at galactic and cosmic scales is an effect arising largely due to the presence of the hypothetical dark matter in the lensing system and so it can be used to constrain the dark matter mass model of lenses as in Fig.2. In the Nexus Paradigm, the gravitational lensing can be used to constrain the value of the quantum state n of spacetime within the lensing system also as in Fig.2. The relationship between the quantum state of space-time and the hypothetical dark matter is expressed in Eqn.(28) if $M(r)$ is considered as the dark matter distribution.

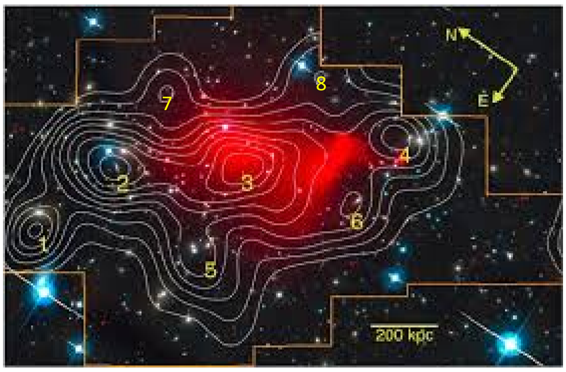

Fig.2 The Abell 520 Cluster
The Abell 520 cluster in Fig.2 shows eight interacting nexus gravitons. The interaction deforms each graviton into an ellipsoid of cross section described by Eqn. (25).

By comparing the quantized metric of Eqn.(7) with Schwarzschild metric we notice that the quantum state of spacetime around baryonic matter increases with distance from the mass

$$\frac{2}{n^2} = \frac{2GM(r)}{c^2 r} \tag{27}$$

Such that

$$n^2 = \frac{c^2 r}{GM(r)} \tag{28}$$

Thus the curvature of spacetime in the inner core of the nexus graviton is only curved by the presence of baryonic matter and not by the increased vacuum energy density. This provides a viable solution to the so called core-cusp problem [22] of astrophysics. If the nexus graviton is surrounded by multiple gravitons tugging on it gravitationally, then the eccentricity becomes a function of the azimuthal angle ψ. Two gravitons of radii $r_n=f(\psi)$ and $r_s=g(\psi)$will intersect when $f(\psi) = g(\psi)$.

6. Conclusion

In this work we have used Einstein rings to reveal the quantized and dynamical states of spacetime in a region of impressed gravitational field as predicted by the Nexus Paradigm of quantum gravity. This endeavour has enabled us to constrain orbital speeds of objects found therein and the radius of curvature of the quantized spacetime, a technique similar to the singular isothermal sphere in the cold dark matter paradigm. The benefit of the nexus graviton formulation is that unlike the singular isothermal sphere, it does not contain singularities or divergent integrals. This provides a viable solution to the core cusp problem.

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