Effect of atmospheric turbulence on entangled photon field generated by partially coherent pump beam

Stuti Joshi[1], Yaseera Ismail[1] and Francesco Petruccione[1,2]
[1]University of KwaZulu-Natal, Private Bag X54001, Durban 4000, South Africa
[2]National Institute for Theoretical Physics, KwaZulu-Natal, South Africa

E-mail: joshis@ukzn.ac.za

Abstract. The propagation of two-photon fields from down-conversion of a partially coherent Gaussian Schell-model pump beam in turbulent atmosphere is reported. The results show that the spatial coherence of the pump beam affects the detection probability of the photon pair at two different positions. It is also found that the detection probability of the entangled photon is less susceptible to atmospheric turbulence, if a partially coherent pump beam produces the field.

1. Introduction
Entanglement received attention over the years owing to its potential applications in quantum communication [1] and information processing [2]. Spontaneous parametric down-conversion (SPDC) [3] is one of the convenient sources of entangled photon fields. These photons are entangled in position, momentum and polarization. In previous studies, the pump beam was considered to be spatially fully coherent. Recently, Jha and Boyd [4] showed theoretically that the spatial coherence properties of the pump field were entirely transferred to the down-converted two-photon field.

Of late, the effect of atmospheric turbulence on the entangled photon fields produced by the down-conversion of the fully spatially coherent pump has been reported [5, 6]. However, the effect of atmospheric turbulence on the entangled photon fields produced by a partially coherent pump beam has not been reported yet. Recently, a theoretical model for the influence of atmospheric turbulence on entangled photon fields produced by partially coherent dark hollow beam has been reported [7]. It has been shown that the detection probability of the entangled two-photon fields is higher and less susceptible to turbulence if the field is produced by a lower mode of partially coherent pump beam.

In the present paper, we have theoretically studied the influence of atmospheric turbulence on the entangled photon fields produced by spatially partially coherent pump beam. It is well known that the spatially partially coherent light is less sensitive to phase distortion and less affected by atmospheric turbulence than spatially fully coherent light. We show that the photon field produced by spatially partially coherent pump beam (PCPB) is less affected by atmospheric turbulence than the photon field produced by the spatially fully coherent pump beam (FCPB).

2. Theoretical Background
A generic situation to study the effect of atmospheric turbulence on the coincidence counts of the two-photon fields is represented in figure (1). The signal-idler photons produced by SPDC are detected in coincidence by detectors D_1 and D_2 respectively. The two-photon field can be expressed as [8],

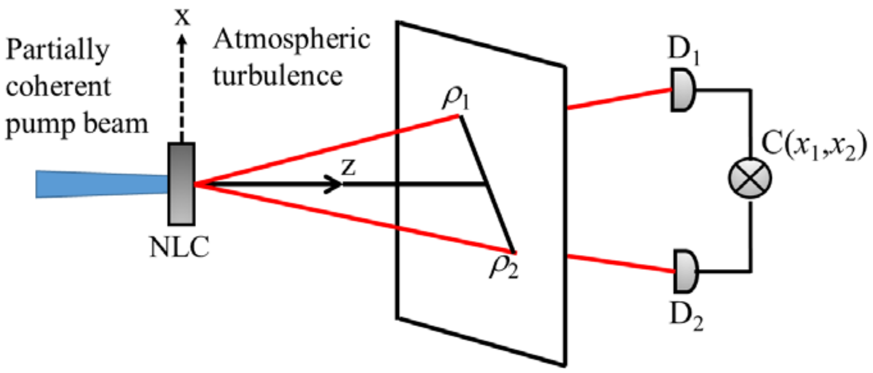

Figure 1. Schematic setup that could be used to study the influence of atmospheric turbulence on the coincidence counts. The bump beam is partially spatially coherent.

$$|\psi\rangle = \int\int dq_s dq_i \Gamma(q_s,q_i)\hat{b}_s^\dagger(q_s)\hat{b}_i^\dagger(q_i)|0,0\rangle , \tag{1}$$

where, $b_s^\dagger$ and $b_i^\dagger$ are the creation operators for signal (s) and idler (i) with the corresponding transverse wave-vectors q_s and q_i respectively. The vacuum state is denoted by $|0,0\rangle$ and $\Gamma(q_s,q_i)$ describes the phase-matching and perfect energy conservation in the SPDC process,

$$\Gamma(q_s,q_i) = A\int dq_p U(q_p)\delta(q_p - q_s - q_i)\tilde{\varsigma}(q_s,q_i) , \tag{2}$$

where $U(q_p)$ is the pump field and $\zeta(q_s,q_i)$ is defined as,

$$\tilde{\varsigma}(q_s,q_i) = \mathrm{sinc}\left(\frac{\Delta qL}{2}\right)\exp\left(-\mathrm{i}\frac{\Delta qL}{2}\right), \tag{3}$$

and A is the integral constant, L is the crystal length. The positive electric field component of the signal and idler photon at the detection plane after propagation through an arbitrary optical system is given by [8],

$$E_\alpha^+ = \int dq_\alpha H_\alpha(x,q_\alpha)\exp(-\mathrm{i}\omega_\alpha t)\hat{b}_\alpha^-, \qquad \alpha = s,i, \tag{4}$$

where, $H_\alpha(x,q_\alpha)$ is the response of the signal (idler system), ω_α.the frequency and t is the time photons take to reach the detector. The detection probability of signal photon at x_1 and idler photon at x_2 is given by [8],

$$C(x_1,x_2) = \langle\psi|\hat{E}_s^+(x_1)\hat{E}_i^+(x_2)\hat{E}_i^-(x_2)\hat{E}_s^-(x_1)|\psi\rangle . \tag{5}$$

Substituting equations (1)-(4) into equation (5) and considering the crystal is illuminated by a partially coherent pump beam and the two photon field is propagated through a turbulent atmosphere, we have,

$$C(x_1,x_2) = A\int\int\int\int W(x_1',x_1'';x_2',x_2'')\left\langle h_s(x_1,x_1')h_s^*(x_1,x_1'')h_i(x_2,x_2')h_i^*(x_2,x_2'')\right\rangle , \\ dx_1'dx_1''dx_1'dx_2'' \tag{6}$$

where $h_s(x,x')$ is the spatial Fourier transform of $H_s(x,q_s)$ and similarly for the idler system. The Cross-spectral density (CSD) of the photon field is expressed as,

$$\mathrm{W}(x_1',x_1'';x_2',x_2'')=\iint\left\langle U_p(x)U_p^*(x_0)\right\rangle\eta(x+x_1',x+x_2')\eta^*(x_0+x_1'',x_0+x_2'')\mathrm{d}x\mathrm{d}x_0\,, \tag{7}$$

where $\eta(x,x')$ is the Fourier transform of $\tilde{\varsigma}(q_s,q_i)$ and

$$h_\alpha(x,x')=\left(-\mathrm{i}\frac{k_\alpha}{2\pi z}\right)^{1/2}\exp\left[\mathrm{i}\frac{k_\alpha}{2z}(x-x')^2+\phi_\alpha(x,x')\right],\quad \alpha=s,i, \tag{8}$$

where k_α is the wavenumber, z is the distance between nonlinear crystal and detectors and $\phi_\alpha(x,x')$ is the phase turbulence due to scattering for a Kolmogorov atmosphere model and is given by [7],

$$\left\langle\exp[\phi_\alpha^*(x_1,x_1')+\phi_\alpha(x_2,x_2')]\right\rangle=\exp\left[-\frac{(x_1-x_2)^2+(x_1-x_2)(x_1'-x_2')+(x_1'-x_2')^2}{\rho_\alpha^2}\right], \tag{9}$$

where, $\rho_\alpha=(0.55\mathrm{C}_\mathrm{n}^2k_\alpha^2z)^{-3/5}$ (α=s,i). C_n^2, describes the turbulence level. Within the paraxial approximations, we have assumed $k_s\approx k_i\approx k_p/2$ and $\Delta q=\left|q_s-q_i\right|^2/(2k_p)$, k_p is the wavenumber of the pump. Using the approximation $\mathrm{sinc}(\Delta qL/2)\approx\exp[-\gamma L\sqrt{\Delta q}^2/2]$, the CSD is given by,

$$\mathrm{W}(x_1',x_1'';x_2',x_2'')=\frac{4\pi k_p}{L\sqrt{\gamma^2+1}}\left\langle U_p\left(-\frac{x_1'+x_2'}{2}\right)U_p^*\left(-\frac{x_1''+x_2''}{2}\right)\right\rangle$$

$$\times\exp\left[-\frac{(x_1'+x_2')^2k_p}{4L(\gamma+\mathrm{i})}-\frac{(x_1''+x_2'')^2k_p}{4L(\gamma-\mathrm{i})}\right], \tag{10}$$

For the special case of partially coherent pump field of Gaussian-Shell model type the correlation of the field is represented as [9],

$$\left\langle U(x_1')U^*(x_2')\right\rangle=S_0\exp\left[-\frac{x_1'^2+x_2'^2}{4\sigma^2}\right]\exp\left[-\frac{\left(x_2'-x_1'\right)^2}{2\delta^2}\right], \tag{11}$$

where, S_0 is a constant, σ is the beam width and δ is the spatial coherence length of the pump beam. Substituting equations (7)-(11) into equation (6) we get,

$$\mathrm{C}(x_1,\mathrm{x}_2)=\left(\frac{4\pi k_p}{L(\gamma^2+1)}\right)\mathrm{S}_0\left(\frac{k_p}{4\pi z}\right)^2\iiiint\exp\left[-\frac{(x_1'+x_2')^2+(x_1''+x_2'')^2}{16\sigma^2}\right]\exp\left[-\frac{\left(x_1''+x_2''-x_2'-x_1'\right)^2}{2\delta^2}\right]$$

$$\times\exp\left[-\frac{(x_1'+x_2')^2k_p}{4L(\gamma+\mathrm{i})}-\frac{(x_1''+x_2'')^2k_p}{4L(\gamma-\mathrm{i})}\right]\exp\left[-\frac{\mathrm{i}k_p}{4z}\left((x_1-x_1'')^2-(x_1-x_1')^2+(x_2-x_2'')^2-(x_2-x_2')^2\right)\right]$$

$$\times\exp\left[-\frac{\left(x_1'-x_1''\right)^2+\left(x_2'-x_2''\right)^2}{\rho^2}\right]\mathrm{d}x_1'\mathrm{d}x_2'\mathrm{d}x_1''\mathrm{d}x_2''\,. \tag{12}$$

3. Results and Discussion

The theoretical results are shown in figures (2)-(4). The effect of atmospheric turbulence ($\mathrm{C_n}^2$) on the coincidence count rate (in arbitrary units), when one detector was fixed (x_2=0) and other was moved (x_1) in the transverse direction, was plotted using equation (12) for FCPB and PCPB. The wavelength of the pump beam was assumed to be 405 nm, L=0.7 mm and σ=10 mm. Figures (2a), (3a) and (4a) show the influence of weak ($C_n^{\ 2} = 10^{-14}\,m^{-2/3}$) and strong turbulence ($C_n^{\ 2} = 5\times10^{-14}\,m^{-2/3}$) on the coincidence count rate when the pump beam is considered to be partially coherent ($\delta = 0.5$ mm). While figures (2b), (3b) and (4b) are plotted when the pump beam was considered to be fully coherent ($\delta = \infty$).

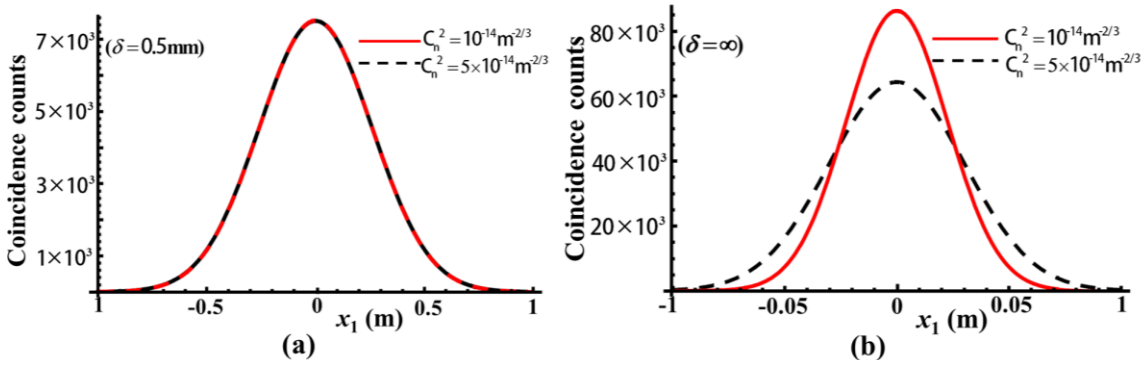

Figure 2. The effect of varying turbulence on the coincidence counts rate of two-photon entangled field at z=1 km. (a) partially coherent pump beam (b) fully coherent pump beam.

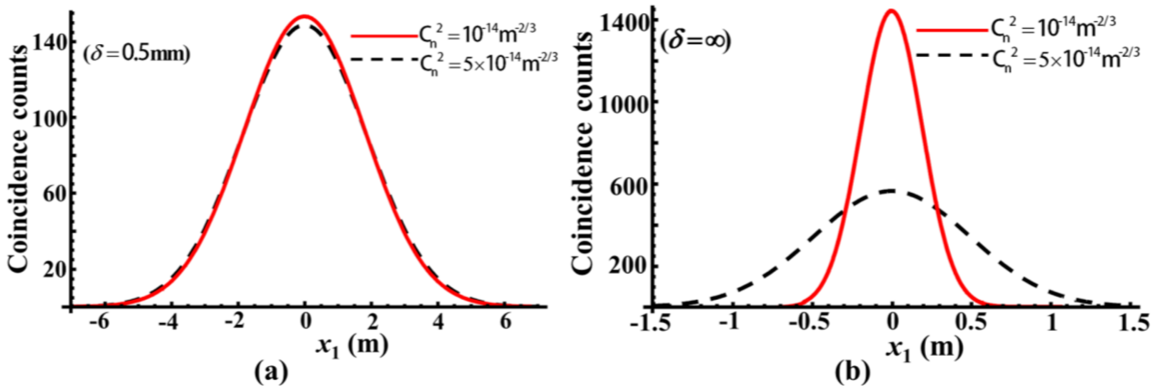

Figure 3. The effect of varying turbulence on the coincidence counts rate of two-photon entangled field at z =7 km. (a) partially coherent pump beam (b) fully coherent pump beam.

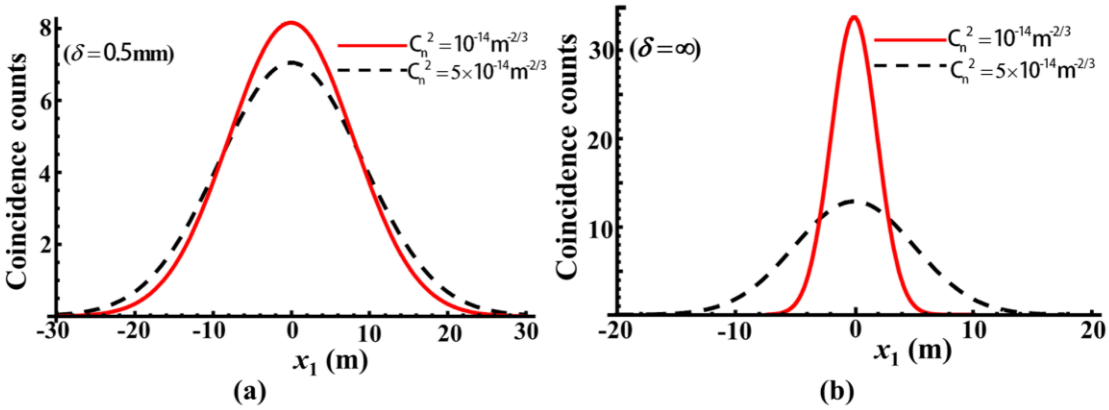

Figure 4. The effect of varying turbulence on the coincidence counts rate of two-photon entangled field at z=30 km. (a) partially coherent pump beam (b) fully coherent pump beam.

When the propagation distance was small, z=1 km, the detection probabilities for both the values of C_n^2 was almost the same when the entangled field was generated by PCPB (figure (2a)). This means the effect of atmospheric turbulence on the detection probabilities was negligible at this distance. On the other hand, there was a difference in the detection probabilities for two different values of C_n^2 when the crystal was illuminate by a FCPB (figure (2b)). As the propagation distance increased (figure (3a) and figure (4a)), the difference in the detection probabilities for two different values of C_n^2 was very small for PCPB. On the other hand, when we considered FCPB the difference of the detection probabilities for two different values of C_n^2 increased with the increase in the propagation distance (figures (3b) and figure (4b)). It can therefore be concluded that the detection is more stable when the entangled photons are generated by the PCPB.

It can be seen from figures (2)-(4) that the spreading of coincidence counts is almost the same for $C_n^2 = 10^{-14} m^{-2/3}$ and $C_n^2 = 5\times10^{-14} m^{-2/3}$ when PCPB was considered while the spreading of coincidence counts for $C_n^2 = 10^{-14} m^{-2/3}$ and $C_n^2 = 5\times10^{-14} m^{-2/3}$ are different for FCPB. In other words, the diffraction broadening (λ/δ) dominates over broadening caused by atmospheric turbulence for PCPB. On the other hand, in the case of FCPB the broadening in coincidence counts was caused by the atmospheric turbulence. The decrease in the coherence therefore results in lowering the normalized variance of the intensity or the scintillation. In addition, the wandering effect becomes smaller just due to broadening.

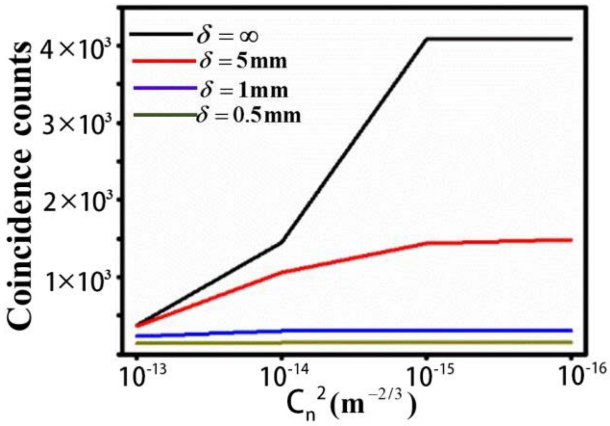

Figure 5. Change in the coincidence counts at a fixed points (x_1=0 and x_2=0) versus refractive-index structure parameter. σ=1 cm, z=7 km.

In figure (5), we illustrated the effect of pump coherence on the coincidence counts where coincidence counts at two fixed points (x_1=0 and x_2=0) were shown as a function of atmospheric turbulence. For the less coherent pump beam, the coincidence counts remain almost constant with the decrease in the atmospheric turbulence strength. As the coherence length increases to δ=5 mm it maintains an almost constant value of coincidence counts until turbulence strength increases to a point where atmospheric turbulence effect dominates over the coherence effect. For weak turbulence, FCPB $(\delta = \infty)$ has constant coincidence counts, which decrease with the increase in the turbulence strength.

4. Conclusion

We have obtained an expression for the coincidence count rate for entangled photon fields generated by the process of spontaneous parametric down-conversion. The effect of atmospheric turbulence on the detection probabilities depends on the spatial coherence property of the pump beam. It is concluded that the photon fields generated by PCPB is more robust for the change in atmospheric turbulence. Present work provides new insights into the nature of SPDC emission by considering pump beam partially coherent and have application in free-space quantum communication.

Acknowledgment
This work is based on research supported by the South African Research Chair Initiative of the Department of Science and Technology and National Research Foundation.

References

[1] Gisin N, Ribordy G, Tittel W, and Zbinden H 2002 *Rev. Mod. Phys.* **74** 145
[2] O'Brien J L 2007 *Science* **318 1567**
[3] Burnham D C, and Weinberg D L 1970 *Phys. Rev. Lett.* **25** 84
[4] Jha A K, and Boyd R W 2010 *Phys. Rev. A* **81** 013828
[5] Gopaul C, and Andrews R 2007 *New J. Phys.* **9** 94
[6] Semenov A, and Vogel W 2010 *Phys. Rev. A* **81** 023835
[7] Qiu Y, and She W 2012 *Appl. Phys. B* **108** 683
[8] Saleh B E, Abouraddy A F, Sergienko A V, and Teich M C 2000 *Phys. Rev.A* **62** 043816
[9] Wolf E 2007 *Introduction to the Theory of Coherence and Polarization of Light* (Cambridge University Press)