**Magnetic vector charges in the realization of non-zero magnetic work**

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**Abstract.** Up to this day the traditional notion is that magnetic forces act on moving electric scalar charges and the hitherto undetected Dirac’s magnetic (scalar) charges, and that work done by magnetic forces is always zero, that is, such forces do no work. Any possible evidence to the contrary is vehemently denied in order to justify the above long held myth. In this paper, the principles of mechanical generation of electricity and operation of simple electric motors, as well as related phenomena, are described in terms of our recently developed and validated concept of magnetic vector charges as sources of magnetic fields. An immediate sequel is that magnetic forces, like gravitational and electric forces, act on objects with corresponding physical attributes, and do nonzero work. Numerous other supporting examples and technological applications that can be cited include the jumping ring experiment, Gouy magnetic balance, Hall magnetic field probe, vibrating sample magnetometer, magnetic levitation and magnetic separation of materials. This simple realization may have far-reaching implications on our overall understanding of magnetism and its eventual effective utilization.

**1. Introduction**  
Currently there are numerous discussions on the internet on whether or not magnetic work can be nonzero. Traditionally it is claimed that a magnetic force does no work [1–3]) because its direction is perpendicular to the displacement of the electric charge carrier due to the cross product between the carrier velocity and the magnetic field given by the magnetic term in the Lorentz force equation. Mosca’s [3] argument is incorrect because of an assumed pre-existing motion of charge carriers tangential to the transversely moving conductor. This makes the Lorentz force no longer parallel to the conductor. In further denial, the magnetic work done on an electric scalar charge carrier per cycle and divided by the magnitude of the scalar charge is conveniently referred to as motional electromotive force [1, 4–5]. Often the mechanical generation of electricity and the operation of an electric motor is described in terms of changing magnetic flux threading a loop, but there are situations where this Faraday flux rule fails. Gardner’s [6] argument for nonzero magnetic work is on changes in the stored magnetic field energy.

In this article our recently developed and validated concept of sources of magnetic fields as magnetic vector charges [7–8] is applied in describing the principles of mechanical generation of electricity and operation of simple electric motors, as well as related phenomena. This was partly prompted by its application in distinguishing electric and magnetic dipole moments [9–10] and showing that the net magnetic flux from any dipolar or non-dipolar source of magnetic field is always
zero [11]. Here it is verified that work done by a magnetic force on an object can be nonzero. This shatters the fallacy that such work is always zero.

2. A harmonized view: physical objects versus their physical attributes

According to the new theory [7], at any point \( r \) the elemental gravitational, electric and magnetic fields given as

\[
\text{dg} = \frac{dm'}{4\pi\xi_0 R^2}, \quad \text{dE} = \frac{dq'}{4\pi\epsilon_0 R^2}, \quad \text{dH} = \frac{dQ' \times \hat{R}}{4\pi\mu_0 R^2}
\]  

(1)

are due to the corresponding physical attributes of elemental mass \( dm' \), electric scalar charge \( dq' \) and magnetic vector charge \( dQ' \) which may simultaneously belong to the same physical object at a source point \( r' \) (figure 1). In free space the field constants of gravitativity \( \xi_0 \), permittivity \( \epsilon_0 \) and permeability \( \mu_0 \) are related by \( \xi_0 = 4\pi G = -1 \) and \( \epsilon_0 \mu_0 \epsilon_0 = +1 \) to the universal gravitational constant \( G \) and speed of light \( c_0 \). Then the interaction forces of the fields with the respective elemental entities \( dm, dq \) and \( dQ \) of another object at the field point \( r \) are

![Figure 1](image_url)

Figure 1. Displacement \( \mathbf{R} \) for (a) scalar charges \( dq, dq' \) and (b) vector charges \( dQ, dQ' \).

\[
\text{dF}_g = (dm) \text{dg} \quad \text{dF}_e = (dq) \text{dE} \quad \text{dF}_m = dQ \times \text{dH}
\]

(2)

Since an object can have more than one physical attribute, cross interactions (advocated by traditional theory) are superfluous and not allowed here. The behaviour of the object at \( r \) is determined by the presence or absence of fields which may interact with it by virtue of its own attributes. In each case the work done on object along a given path, where is an, are similarly given by

Now if two objects at points \( r \) and \( r' \) interact through their elemental properties such that the object at \( r \) moves along a path with elemental displacement \( \text{d}l \), then from (1) and (2) the gravitational, electric and magnetic works done on it are

\[
\delta W_g = \text{dF}_g \cdot \text{d}l = dm \frac{dm'}{4\pi\xi_0 R^2} \hat{R} \cdot \text{d}l = \text{dF}_g \hat{R} \cdot \text{d}l
\]

(3a)

\[
\delta W_e = \text{dF}_e \cdot \text{d}l = dq \frac{dq'}{4\pi\epsilon_0 R^2} \hat{R} \cdot \text{d}l = \text{dF}_e \hat{R} \cdot \text{d}l
\]

(3b)
\[ \delta W_m = dF_m \cdot dl = \left[ dQ \times \left( \frac{dQ' \times \hat{R}}{4\pi\mu_0 R^2} \right) \right] \cdot dl = \left[ \frac{dQ \cdot \hat{R}}{4\pi\mu_0 R^2} dQ' - \frac{dQ \cdot dQ' \hat{R}}{4\pi\mu_0 R^2} \right] \cdot dl \quad (3c) \]

From (3a) and (3b), it is obvious that gravitational and electric work will be nonzero if the two objects move towards or away from each other, as the forces are collinear with \( \hat{R} \). From (3c), magnetic attraction or repulsion is possible when \( dQ' \) and \( dQ \) are jointly normal to \( \hat{R} \). Thus under these conditions magnetic work may be nonzero. In the following, we apply equations (2) for \( dF_m \) and (3c) for \( \delta W_m \) to verify the reality of \( W_m \neq 0 \).

3. Magnetic vector charges, magnetic work and magnetic torque

3.1. Mechanically induced magnetic vector charges: electric generators

In figure 2, a rectangular conducting loop ABPCDSA of width \( 2b \), length \( 2\ell \), and centred at the origin O, rotates with angular frequency \( \omega \) about its central longitudinal axis POS on the \( z \)-axis. At time \( t \) the loop’s plane inclines at an angle \( \phi = \omega t \) to the \( \hat{x} \) direction, with axial sides AB and CD at radial positions \( \rho = \pm \hat{\rho}b \) The lower radial side DSA is at \( z = -\hat{z}\ell \) and the upper one BPC is at \( z = \hat{z}\ell \).

![Figure 2. Conducting loop ABPCDSA rotates about axis POS in a uniform magnetic field \( H \).](image)

The mechanically induced elemental magnetic vector charges at any position \( r = \pm \hat{\rho}\rho + \hat{z}\zeta \) on loop ABPCDSA are

\[ dQ_\phi = \pm \nu \mu_0 dq = \pm \hat{\nu} \mu_0 \omega \rho dq \quad (4) \]

For an electron, \( dq \) is replaced by the electric scalar charge \( -e \) so that (4) becomes

\[ Q_\phi = \pm \nu \mu_0 e = \pm \hat{\nu} \mu_0 \omega \rho \quad (5) \]
That is, an electron acquires these magnetic attributes by virtue of its relative motion. Then according to equation (2), interaction of $Q_e$ with the magnetic field (see figure 2)

$$
\mathbf{H} = \hat{y}H = \hat{p}H \sin \phi + \hat{\phi}H \cos \phi
$$

(6)
yield the axial magnetic forces

$$
\mathbf{F}_a = \hat{z}F_z = Q_e \times \mathbf{H} = \pm \hat{z}\mu_0e\cos H \sin \phi = \pm \hat{z}\mu_0e\cos H \sin \omega t
$$

(7)
Then integrating anticlockwise along the loop, the magnetic work done on an electron in the section SABP, due to its magnetic vector charge $Q_+$, is

$$
W_+ = \int \hat{z}F_z \cdot \hat{r}d\rho = \int_{\rho=0}^{\rho=b} \hat{z}F_z \cdot \hat{p}d\rho + \int_{z=-\ell}^{z=0} \hat{z}F_z \cdot \hat{z}dz + \int_{\rho=b}^{\rho=+b} \hat{z}F_z \cdot \hat{p}d\rho = \int F_zdz = 2\mu_0e\omega/\ell H \sin \omega t
$$

(8a)

Similarly, the magnetic work done an electron with vector charge $Q_-$ in section PCDS is

$$
W_- = \int \hat{z}F_z \cdot \hat{r}d\rho = \int_{\rho=0}^{\rho=-b} \hat{z}F_z \cdot \hat{p}d\rho + \int_{z=-\ell}^{z=0} \hat{z}F_z \cdot \hat{z}dz + \int_{\rho=-b}^{\rho=-b} \hat{z}F_z \cdot \hat{p}d\rho = \int F_zdz = 2\mu_0e\omega/\ell H \sin \omega t
$$

(8b)
Finally the net nonzero magnetic work done on an electron traversing the loop is

$$
W_m = W_+ + W_- = 4\mu_0e\omega/\ell H \sin \omega t
$$

(8c)

It is clear that the magnetic field isn’t always perpendicular to the direction of motion. In both cases only the axial sides at $\rho = \pm b$ contribute to the nonzero magnetic work done on the electron, as the electron moves parallel to the prevailing magnetic forces. Thus, it is not even necessary to place the radial sides in the region of the magnetic field. For example when a length $dz$ of an axial conductor moves with velocity $v = \hat{x}v_z$, normal to $dz$, the magnetic vector charge induced on an electron is

$$
Q_e = -\mu_0e = \left(\hat{p}\cos \phi - \hat{\phi}\sin \phi\right)\mu_0e\hat{x}
$$

(9)
This interacts with the magnetic field (6) to create the longitudinal magnetic force

$$
\mathbf{F}_m = Q_e \times \mathbf{H} = \hat{z}\mu_0e\hat{x}v_zH
$$

(10)

Figure 3. Cross sectional view of loop area $\pm A 4/\ell$ with its radial edge $\pm \hat{p}b$ vectors in $xy$-plane.
exerted on the electron in the conductor. Then the work done on the electron, as it traverses the whole length $\ell$ of the conductor is

$$W_m = \mu_0 e v_x \ell H$$  \hspace{1cm} (11)

A traditional treatment [1, 4, 5] would be equivalent to dividing equations (8) to (12) by $-e$. Then the results are the so called motional electric field and induced electromotive force emf. This deceptively masks the nonzero work done by magnetic forces on an electron. Also the traditional Faraday flux rule derivation applied to the loop of area vector $A = \hat{\phi} 4/\ell b$ aligned at angle $\phi = \omega t$ to $H$ (see cross sectional view in figure 3) yields the potential rise as:

$$V = -\frac{\partial \Phi_m}{\partial t} = -\frac{\partial}{\partial t} (\mu_0 \mathbf{H}) = -\frac{\partial}{\partial t} (\mu_0 A H \cos \omega t) = 4 \mu_0 \omega \ell b H \sin \omega t$$  \hspace{1cm} (12)

This further enhances the deception as it does not show that $V$ is the sum of two potential differences developed independently over two separated portions of the loop.

3.2. Electrically induced magnetic vector charges: electric motors

The reverse of the electric generator is the electric motor, where in a magnetic field $\mathbf{H}$, the longitudinal scalar charge carriers (that is an electric current with linear density of magnitude $I$ [10]) in a conductor produces transverse motion of the conductor. Then the electrically induced elemental magnetic vector charges in the axial sides $AB$ and $CD$ at radial positions $\pm \hat{p} b$ are

$$dQ_{zb} = \mu_0 I_z d\ell \pm \hat{\mathbf{z}} \mu_0 I d\ell$$  \hspace{1cm} (13)

These interact with the magnetic field (8) to generate the coupled forces

$$dF_{zb} = dQ_{zb} \times \mathbf{H} = \mp \hat{\mathbf{p}} \mu_0 \mathbf{H} d\ell$$  \hspace{1cm} (14)

which are non-collinear except when $\phi = 0$. As their points of application are $(\hat{p} b + \hat{z} z)$ and $(-\hat{p} b + \hat{z} z)$, the total torque on sides $AB$ and $CD$ about $O$ is [8]:

$$\tau_{xs} = \int_{z=-\ell}^{\ell} (\hat{p} b + \hat{z} z) \times dF_{zb} + \int_{z=-\ell}^{\ell} (-\hat{p} b + \hat{z} z) \times dF_{zb} = \hat{z} 4 \mu_0 I \ell b H \sin \phi \equiv \mu_0 I A \times \mathbf{H}$$  \hspace{1cm} (15)

This gives a counter-clockwise rotation about the $z$-axis.

Along the radial sides $DSA$ and $BPC$ at $z = \mp \hat{z} \ell$, the electrically induced elemental magnetic vector charges

$$dQ_{xz} = \mu_0 I_z d\rho \equiv \mp \hat{\rho} \mu_0 I d\rho$$  \hspace{1cm} (16)

also interact with the field $\mathbf{H}$ (6) to generate the coupled magnetic forces

$$dF_{xz} = dQ_{xz} \times \mathbf{H} = \pm \hat{\mathbf{z}} \mu_0 I H \cos \phi d\rho$$  \hspace{1cm} (17)

Then the coupled magnetic torque on the radial sides becomes

$$\tau_{rs} = \int_{\rho=-b}^{b} [(\hat{\rho} \rho + \hat{z} \ell) \times dF_{xz} + (\hat{\rho} \rho - \hat{z} \ell) \times dF_{xz}] = \int_{\rho=-b}^{b} [\hat{\rho} \rho \times \hat{\mathbf{z}} (dF_{xz} + dF_{xz}) + \hat{\mathbf{z}} \ell \times \hat{\mathbf{z}} (dF_{xz} + dF_{xz})] = 0$$  \hspace{1cm} (18)

From (15) and (18) the overall coupled torque, $\tau = \tau_{xs} + \tau_{rs}$, is simply that $\tau_{xs}$ on the axial sides. Obviously this does not imply absence of magnetic forces on (nor of magnetic vector charges in) the radial sides [8]. Electric motors operate on the principle of magnetic torque created as above.

4. Discussion

From the above description, mechanical generation of electricity is a result of the interaction between mechanically induced magnetic vector charges and a magnetic field. The developed electric potential is simply the nonzero magnetic work done on an electron divided by the magnitude of its electric scalar charge. The electric motor effect is the development of nonzero magnetic torque due to the
magnetic force on electrically induced magnetic vector charges. In both effects it is not necessary to have the imaginary magnetic flux lines threading a conducting loop.

Even when a conductor is stationary, the motion of an elemental electric scalar charge-carrier within it gives the carrier the extra attribute of an electrically induced magnetic vector charge which interacts with a magnetic field. An instance of this is the Hall effect where the magnetic work done on an electron is \( eV_H \) is related to the Hall voltage \( V_H \).

5. Conclusion

Interactions of magnetic vector charges with magnetic fields have successfully been used to describe the principles of electric generators and electric motors and other magnetic phenomena. It has also been demonstrated that magnetic forces do nonzero work and have thus shuttered the centuries old myth of “zero magnetic work”. The theoretical basis has also been given. This was possible partly due to the realization of the distinction between physical objects and their physical attributes, and that a given object, such as an electron, may simultaneously possess more than one physical attribute and thus be acted upon by differently generated forces. In realistic descriptions, physical attributes should not be isolated from physical objects.

References