An automated temperature control model for a well-mixed biomass reactor

Felix Pagona, Golden Makaka
Physics Department, University of Fort Hare, Private Bag X1314, Alice 5700, Eastern Cape, South Africa.

felixpagona@gmail.com

Abstract. A control strategy for temperature is developed using a mathematical model. An analysis of energy balance is given in order to establish cooling rate. A coil immersed in a coolant provides sufficient heat removal [4]. The approach shows that by carefully controlling coolant flow, a reactor would operate optimally.

1. Introduction
It is necessary for bioreactors to operate at optimal temperature range since undesirable temperature fluctuations in the operation of a biomass reactor plant can adversely affect performance [1]. Control of temperature in a biomass reactor is crucial to process stability and system performance. To ensure optimized efficiency an analysis of heat transfer rates in relation to energy balance for a continuously stirred tank reactor is given. A mathematical model is presented for control of temperature variations. The model predicts viability of the approach for specific ranges of values of the exponent for coolant flow.

2. Methodology
Consider a reactor of type shown in Figure 1 below.

Figure 1: reactor tank with jacket for cooling.
To reduce temperature, heat must be removed from the reactor system by carefully manipulating coolant flow, \( F_c \), resulting in a possible variation of \( Q \). Equation 1 suggests that for a bioreactor to operate at a uniform and constant temperature, the following condition must be satisfied.

\[
\frac{dT}{dt} = 0 \tag{1}
\]

That is, the rate of change of temperature of the system must be zero. To attain this goal, the following heat energy balance equation was investigated.

\[
\begin{bmatrix}
\text{rate of change of } \\
\text{liquid energy}
\end{bmatrix} =
\begin{bmatrix}
\text{rate of energy flow } \\
\text{into the reactor}
\end{bmatrix} - \begin{bmatrix}
\text{energy flow } \\
\text{out of reactor}
\end{bmatrix} - \begin{bmatrix}
\text{rate of energy } \\
\text{generation by reaction inside reactor}
\end{bmatrix} \tag{2}
\]

This can be written in mathematical form as

\[
\rho C_p V \frac{dT}{dt} = \rho C_p F(T_{in} - T) - Q - kC_A H \tag{3}
\]

where,

\[
Q = \frac{\alpha F_c^{\beta+1}(T - T_c(\text{in}))}{F_c + \frac{\alpha F_c^{\beta}}{2C_p}} \tag{4}
\]

\( Q \) represents the rate of heat removal by a coolant (oil or water circulated in a jacket), \( \rho \) is the density of the reactor liquid, \( V \) is the working volume and \( \beta \) and \( \alpha \) are appropriately chosen constants. \( T \) represents the temperature of the system at a time \( t_i \), \( F_c \) is the coolant flow rate, \( C_p \) is the specific heat for the flowing liquid.

\[
\rho C_p F(T_{in} - T) - kV C_A \Delta H = Q \tag{5}
\]

Further, notice that the expression for \( Q \) can be written as

\[
Q = \frac{2\alpha F_c^{\beta+1}(T_c(\text{in}) - T)\rho C_p}{2F_c\rho C_p + \alpha F_c\beta} \tag{6}
\]

For large values of \( \beta \), the expression for \( Q \) is dominated by the term \( F_c^{\beta+1} \) which allows us to re-write \( Q \) as

\[
Q \approx \alpha F_c^{\beta+1} \tag{7}
\]

Equation 7 implies that the rate of heat removal must increase exponentially if a reactor is to operate at constant and uniform temperature as this allows sufficient heat removal from the system.

### 2.1. Modelling of heat removal from reactor tank

For a reactor to operate at a constant and within a specific temperature level, the system must be in thermal equilibrium. That is, there must be almost no net change in temperature. Hence it is a requirement to satisfy the condition (by rewriting Equation 5)

\[
\frac{dT}{dt} = \frac{FT_1 - F(T - kV^2 C_A H - QV)}{\rho C_p V^2} = 0. \tag{8}
\]
This condition can be achieved if only the rate of heat removal $Q$ is manipulated accordingly since the other parameters can hardly be controlled. Since $Q$ is related to coolant flow rate $F$ (see equation 7), it is practical to manipulate $Q$ by varying $F$ in exponential mode in order to remove heat from the system. Assuming constant density for the coolant and negligible dynamics for the reactor walls, various values of the exponent $\beta$ with $\alpha$ fixed to unity (Equation 7) were used to determine their effect on the rate of heat removal as cooling flow varies exponentially.

3. Results
The variation of rate of heat removal $Q$ with $\alpha$ fixed to unity, for various chosen values of $\beta$ is shown in figures 1 to 6 below.

**Figure 1.** Heat removal for flow $1 \leq F \leq 10$ and $0.1 < \beta \leq 1.2$

**Figure 2.** Heat removal for $1 \leq F \leq 10$ and $1 \leq \beta \leq 2.1$

**Figure 3.** Heat removal for $0 \leq F \leq 10$ and $0.01 \leq \beta \leq 0.11$

**Figure 4.** Heat removal for $0 \leq F \leq 2.3$ and $0.1 \leq \beta \leq 2.4$
Figure 5. Heat removal at low flow rate ($0.01 \leq F \leq 0.11$) and for $0.01 \leq \beta \leq 0.11$

Figure 6. Heat removal at constant flow rate ($F=10\text{L/min}$) at higher rates for $0.2 \leq \beta \leq 3.5$

Figure 7. Heat removal at low flow rate ($0.1 \leq F \leq 1.1$) for $1 \leq \beta \leq 9$, $\beta=1$ and $\beta=8$

Figure 8. Heat removal at constantly increasing higher rates ($10 \leq F \leq 60$) followed by a decreased constant flow rate for $1 \leq \beta \leq 11$

4. Analysis and Discussion

Figures 1 and 2 indicate that a possible successful heat removal from a bioreactor. Clearly, heat removal is high for the same flow rate $F$ and large $\beta$. Interestingly, by varying $\beta$ incremental values of $\beta$ in steps of 0.2 (Figure 2) instead of multiples of 0.1, we get ten times the rate of heat removal from the system (compare Figures 1 and 2) for the same flow rate of coolant. However, this may result in excessive heat removal that may eventually cause the microorganisms in the reactor tank to freeze thereby resulting in system halt. However, if the sensors detect excess heat removal, it is possible to reduce the rate of heat removal (as suggested by Figures 5, 6 and 8) until the heat model equation balances (i.e. equates to zero), a condition of zero net temperature change in the system.

By reducing the value of $\beta$ by 10 (incremental variation of $\beta$ in steps of 0.01 instead of 0.1), the rate of heat removal $Q$ increases by a factor of 10 for the same coolant flow rate (Figure 3). (Compare Figure 3 with Figure 1 where $\beta$ was varying in steps of 0.1). On the other hand, increasing the coolant flow rate in steps of 0.3 while maintaining the same range of $\beta$ as used in case 1 and 2 produces about five times heat removal rate as before. This suggests that reducing coolant flow would be a useful means to slowly remove heat from the system, a technique suitable for systems that operate cold temperature environments. Figure 8 sums it all. The nature of the graph suggests that it is possible to
control temperature in any desirable manner. By varying flow rate and choosing the appropriate value of \( \beta \), heat can be removed (or added) accordingly until the system is thermally stable.

5. Conclusion
A comparison of the various graphs indicates that by carefully choosing the value of \( \beta \), a required and necessary rate of heat removal \( Q \) from the system can be achieved. The magnitude of \( \beta \) determines how fast heat can be removed and hence establishing the rate of cooling of the system. Thus, the ability to efficiently remove heat from a reactor is a highly significant aspect in order to control unwanted temperature changes. Arbitrary selection of \( \beta \) in the closed interval \([0, 10]\) proves a success of the approach used in this paper as revealed by figures 1 and 2. Thus, our approach reveals that a careful choice of \( \beta \) can produce required results, that is, the desirable rate of heat removal that subsequently affects the rate of change of temperature in the system, thereby keeping the system in almost thermal equilibrium.

References

[6] Lienhard J H IV and Lienhard J H V 2006 A heat transfer textbook, Department of Mechanical Engineering, Massachutes University, Cambridge, USA