The light bulb effect: University students' problem solving cognitive processes in a physics problem solving skills test

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Abstract. A diagrammatic cognitive process representation, called the 'Light Bulb effect', with the ultimate intended purpose of teaching students problem solving skills through empowering them metacognitively, is used here for probing the question of what makes a problem difficult. The 'light bulb effect' is used, in this study, as a way to analyse students' problem solving skills on entering a first year Physics major course, at the University of the Witwatersrand. Analysis of student written answers to a test designed to probe some essential problem solving skills revealed that students had difficulty with the algebraic manipulation of equations, understanding displacement and taking logical steps that are unfamiliar or not immediately obvious. In this representation, these are called 'light bulbs'. The more linked 'light bulbs' are required in reaching a solution to a problem, the more difficulty the students have with solving that problem. This diagrammatical representation thus suggests a way of determining the level of difficulty of any physics problem, through the determination of the number of cognitive skill links and the number of not obvious logical steps, required to solve a specific problem.

1. Introduction

A problem is a situation facing an individual for which there is no obvious path to the solution so the problem solver has to create a path [1]. Thus problem solving is both a part of everyday life and an important aspect of learning physics. In physics, it is used as a learning strategy through which students get to practice the application of principles and concepts they are learning [2]. However, teaching students problem solving has met with very little success as this is usually done by a teacher, to whom the problem is not new, who presents the problem and the solution in a very clear and logical manner. The student who learns the solution will then apply it when he recognizes a similar problem. But because most teachers do not mention that a new or real problem usually needs many attempts at solving it [3], the student gives up when he faces a problem that he does not recognize [1]. However, successfully solving a problem that is new or unrecognized is dependent on metacognitive monitoring and students can improve their problem solving performance through metacognitive training [4, 5].

Metacognition is both knowledge of one's cognitive processes and regulation of those processes [6]. The specific skills used in a problem solving process are the essence of metacognition [7]. In order to be able to help students gain metacognitive tools and thus help them with problem solving it is important to first identify the different cognitive processes involved in problem solving and then determine some way of putting them together so that their interaction is clear. It is the aim of this study to identify the cognitive processes involved in physics problem solving and to determine which of these processes make a problem more or less difficult for students. Since 'one picture is worth one thousand words', in this study, a diagrammatic cognitive process representation is designed and used to analyse problem solving cognitive processes.

There are several cognitive skills that play a role in solving physics problems. These skills can be divided into several basic categories:

1. Language skills of several forms needed to understand the question or problem description. These include the use of normal English words, terms specific to physics, pictorial representations and mathematical expressions.
2. Mathematical manipulations involving for example: Algebra, Arithmetic, Calculus, or Geometry.
3. Knowledge of physical principles or laws, for example: Newton's second law of motion.
4. Writing answers to the problem in some mathematical form or through the use of a definition, or a relationship.

A large number of problems encountered by both students and physicists at the forefront of physics require only the combination of several skills from the four above categories, in some logical sequence, for the successful invention of a path to the final solution. But there are also problems where a logical step or the 'figuring out of something' that is not immediately obvious is also required. This is when a problem solver looks at the several 'strings' of given information and at the various 'strings' of manipulations he can do, and suddenly realizes how to put it together in order to arrive at the answer. This is the *realization moment*. This is the 'light bulb' moment. This is the moment when it may seem as if a light bulb goes off inside his mind. It will be demonstrated in this study that students find these types of problems, where a 'light bulb' moment is required, much more difficult to deal with than the problems where only the linking of several skill categories leads to the answer.

2. Method and Analysis

A test was set to probe students' problem solving skills and in the process of setting and writing solutions for the test, the different cognitive processes in use in each of the questions was identified. The different processes were then represented as either a string or a bulb and linked in a diagram that attempts to represent the complete problem solving process with a diagram containing various diagrammatic elements. These elements are either a string, represented by a straight or a curved line, or a bulb, represented by a circle. The two bulb elements and an example of a string element are shown in Figure 1. Each of the string elements are accompanied by a code comprising of a letter and a subscript. All the string cognitive processes required in the test are shown in Table 1.

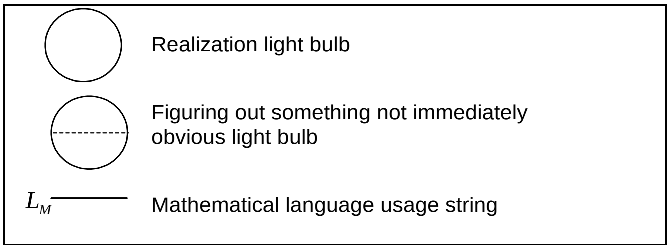


Figure 1: Three diagrammatic problem solving cognitive process representation elements

The number of students who wrote the test was 310 and a sample of 100 student test answers were analysed. The initial diagrammatic representation was modified slightly if the analysed answers showed that there were some possible additional links. The bulbs represent a cognitive process that requires greater mental effort and so are viewed in the analysis as representing a higher level process than the processes that involve only strings. Thus levels were assigned to the diagrams according to the number of bulbs in them. If a problem solving process did not contain any bulbs it was assigned a problem solving process level of 0 (L0). If there was one bulb it was assigned level 1 (L1), if it contained five bulbs, as the last question in the test, it was assigned level 5 (L5).

Table 1: String cognitive process elements involved in the problem solving cognitive processes relevant to the test given to the first year Physics major students, at the beginning of 2015

String code	Cognitive process description
L_E	Reading and understanding English language such as the language used in the wording of a question and usually includes terms specific to physics
L_M	Usage of mathematical language, usually used in reading and understanding given mathematical expressions
L_P	Reading and understanding information in a pictorial or graphical format
M_{Ar}	Use of arithmetic
M_{Al}	Algebraic mathematical manipulation
M_{Cal}	Mathematical manipulation using calculus
M_G	Mathematical process using Geometry
R_P	Remembered physical concept, law or principle
A_{DL_E}	Answer in the form of a definition using English words and terms specific to physics
A_{RL_E}	Answer in the form of a relationship using English words and terms specific to physics
A_M	Answer in mathematical language, may be a number and may contain variables. Needs to be simplified as far as possible

The first question in the Problem Solving Skills Test is shown in Figure 2. The diagrammatic representation, of the cognitive processes, required in solving this problem is shown next to the question. This question requires the use of the English language and physics terms (L_E), mathematical language (L_M) to read and understand the given mathematical expression, and the use of algebra (M_{Al}). These three strings need to be linked in order to arrive at the answer, which is in mathematical format (A_M). No realization or 'light bulb' process is required for this question so it is a level 0 (L0) process diagram. Notice that the progression of time is represented in going from left to right, with the answer appearing on the right or at the end of the time progression. But since the string processes can be combined in any order, the initial time point is the point where the three strings link.

The focal length, *f*, of a lens is given by the following equation:

$$\frac{1}{2a}+\frac{3}{5a}=\frac{1}{f},$$

where *a* is length and is measured in centimetres (cm). Solve for *f* in terms of *a*.

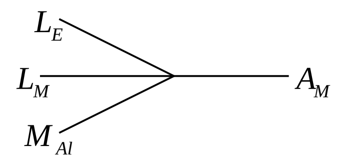


Figure 2: Question 1 in the Problem Solving skills test with the corresponding cognitive process diagram appearing on the right. This is a 4 string L0 diagram.

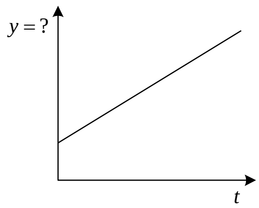
Question 2, shown in Figure 3, has five parts, so each is treated as a separate problem and a separate diagram is drawn for each part. There are two possible diagrams for parts (b) and (c) of question 2 because it was discovered after examining the students' test answers that some students knew and used calculus instead of going through the 'light bulb' process or realization, they would use if calculus was unknown to them. The realization required was that the given expression matched the given equations so that the initial speed and acceleration could be identified. The R_P string was added to the diagram, for part (e), after it was determined, from looking at the answers the students gave, that quite a few students were not familiar with the term: displacement. This process was initially included in the L_E string as a normal physics term but it became obvious that it would be more appropriate to include it as a separate string process, namely as a 'remembered physical concept'. Parts (b), (c) and (f) contain L1 diagrams. The realization required in part (f) was that a straight line would be obtained from the given information only if *x/t* or *v* was plotted along the *y*-axis.

The motion of a particle, moving along a straight line is described by the following equation:

$$x = (2.3\ \text{m.s}^{-1})t + (4.8\ \text{m.s}^{-2})t^2$$

where *x* refers to the particle's position, which is measured in metres, and *t* refers to time, which is measured in seconds.

(a) What is the particle's initial position at time $t = 0$?
(b) What is the particle's initial speed?
(c) What is the particle's acceleration at time $t = 1$ s?
(d) What is the particle's position at $t = 3$ s?
(e) What is the particle's displacement between $t = 3$ s and $t = 1$ s?
(f) If you were to plot the above equation in order to obtain a straight line graph and *t* had to be on the *x*-axis, what would you plot on the *y*-axis?



Given: $s = v_o t + \frac{1}{2}at^2$ $\qquad v = v_o + at$

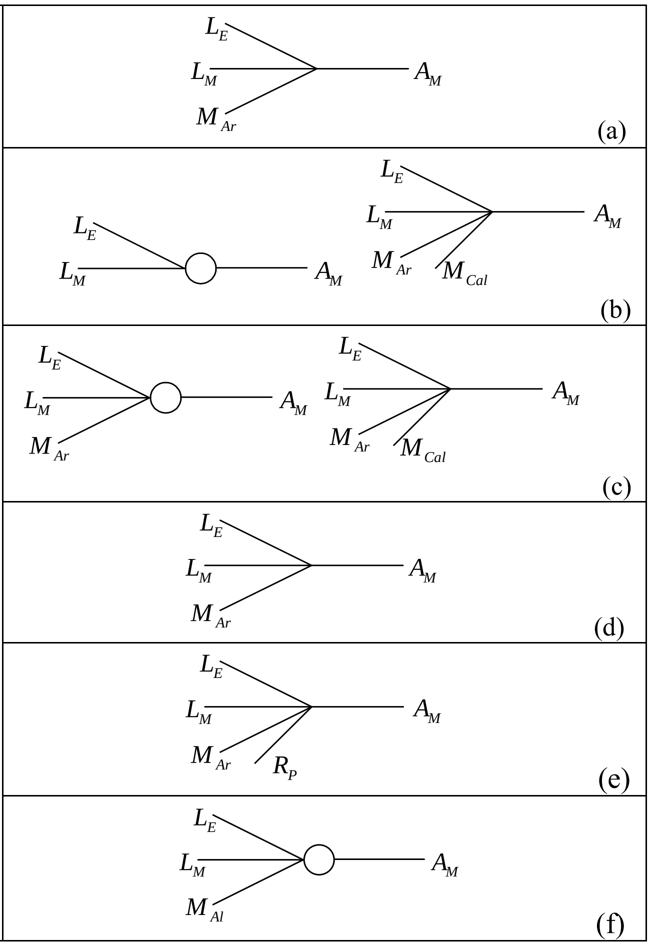
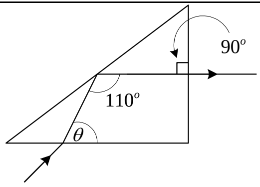


Figure 3: Question 2 in the Problem Solving skills test with the corresponding cognitive process diagrams, for the different parts of the question, appearing on the right.

Question 3, shown in Figure 4, requires a realization that the diagram in the question, shows a quadrilateral or a straight line intersecting two parallel lines. This realization is then combined with mathematical geometric and arithmetic cognitive processes, after which the solution emerges. The other processes required the understanding of English and physics terms (L_E), and the understanding of pictorial language (L_P). The relevant cognitive process diagram, on the right in Figure 4, shows all these processes at a glance. It is L1 diagram with 5 string processes.

Figure 5 shows the final question, question 4, in the Problem Solving Skills Test. This is the only question where the 'figuring out' bulbs appear. Also it is the first time that bulbs are used in the answer part of the process. The strings going through the three middle bulbs are viewed as a single string each, so the diagram has 5 bulbs and 7 strings. The diagram suggests that this is the most complicated problem solving process the students face in the test.

A ray of light enters a right-angled prism through one of its faces, undergoes internal reflection on another face of the prism and exits through the third face, as shown in the figure below. Find the angle θ. Give reasons for your answer.



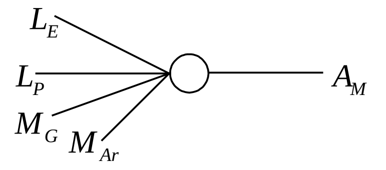


Figure 4: Question 3 in the Problem Solving skills Test with the corresponding cognitive process diagram appearing on the right. The diagram represents a L1 process

Write down an explanation of Newton's second law of motion that you would give to your little 6 year old sister who has not yet done any Physics at school. Make sure you define all the physics terms you use in a way that she can understand.

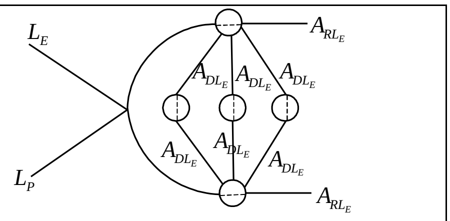


Figure 5: Question 4 in the Problem Solving skills Test with the corresponding cognitive process diagram appearing on the right. Lines without an assigned code are just linking different parts of the process.

3. Results

The results obtained from the analysis of the Problem Solving Test, given to first year Physics major students at the beginning of the academic year, in 2015, appears in Table 2. The first row in the table shows the question number. The second row shows the level of the question according to the 'light bulb' process representation, and the third row shows the maximum possible marks assigned to each question. The total number of marks in the paper was 25 and the number of student test answers analysed was 100. The number of students who obtained a perfect score for each question is shown in the fourth row. The fifth and sixth rows show the main reason why students did not obtain full marks for the question.

Table 2: Student performance in the Problem Solving Test at the beginning of 2015.

Question number	**1**	**2a**	**2b**	**2c**	**2d**	**2e**	**2f**	**3**	**4**
Question level	L0	L0	L1	L1	L0	L0	L1	L1	L5
Marks assigned	5	1	2	2	1	2	3	4	5
Number with perfect score	**72**	**99**	**69**	**21**	**73**	**65**	**36**	**63**	**8**
Main difficulty	M_{Al}	A_M	L1	L1	M_{Ar}	R_P	L1	L1	L1
No. with this difficulty	23	1	28	76	27	21	56	20	35

The main difficulty encountered by the students in answering question 1 was dealing with the required algebraic manipulation (M_{Al}). In question 2a, only 1 student had difficulty, and it was because he/she did not understand the expected way of writing the answer (A_M). He wrote in answer: '*At the start*'. In comparing the diagrams for questions 1 and 2a we see that both diagrams contain 4 strings but the diagram for 2a contains an Algebra string instead of an Arithmetic string. Algebra is a more abstract and therefore a more complex process making the question more difficult for the students. The main difficulty with question 2d was arithmetic (M_{Ar}) and in question 2e, it was remembering and understanding the displacement concept (R_P). In all the questions where a light bulb or realization cognitive process was required, the greatest difficulty was achieving realization (L1). In question 4, where 5 light bulbs, were required, 35 students demonstrated they could not achieve a single bulb process and 18 were able to mentally construct one of the required bulbs. One of these 18 students wrote: '*When you push something it will move in the direction you are pushing*'. This student was able to define force, in words a six year old child could understand, by using the word 'push' and therefore achieved one bulb cognitive process.

Student performance in the four L0 questions, namely 1, 2a, 2d and 2e, was better than in the L1 questions, namely 2b, 2c, 2f and 3. The best score was obtained in question 2a, where students only needed to use very simple arithmetic (M_{Ar}) in combination with understanding the wording (L_E) and the given expression (L_M), in the question, as seen from the top process diagram in Figure 3. The L1 question with the lowest number of students obtaining a perfect score is 2c, where the process seems at first similar to that of 2b but a possible realization is using the units ($m.s^{-1}$ in 2b) rather than comparing equations. This realization works in 2b but not in 2c. There was also possibility of answering using calculus, which changes the cognitive process from a L1 to a L0, with the addition of the M_{Cal} string. Very few (5) students used calculus. Other L1 questions, 2f and 3, also demonstrate that students find L1 questions more challenging then L0 questions as the number of students obtaining a perfect score was 36 and 63, respectively, which is below the numbers obtained for L0 questions. The question with the lowest number of perfect scores was question 4, a L5 question, where only 8 students obtained a perfect score.

4. Discussion and Conclusion

The analysis of the Problem Solving Test at the beginning of 2015 summarized in Table 2, shows that students perform the worst on questions that require a higher level cognitive process as determined by the Light Bulb diagrammatic representation. This representation gives, at a glance, information on the cognitive processes required by different questions so that it is easy to highlight differences between questions. For example: When a comparison is made between the process diagrams for questions 1 and 2d, we see that the difference is that only arithmetic is required in 2d but question 1 requires some algebraic manipulation (M_{Al}). Also, the very low student performance in question 4 is not surprising, when one draws a process diagram for it, which requires 5 bulb processes, and therefore a corresponding greater cognitive effort from the students. Thus, the 'light bulb' effect representation can be used to determine why students find one question more difficult than another, and one of its more powerful applications may turn out to be in teaching metacognitive skills which will hopefully lead to better problem solving performance.

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