An integrated software based analytical model for the signal path efficiency of the HartRAO lunar laser ranger optical system

S C Ndlovu[1, 2]**, L Combrinck**[1, 2, 3]**, N P Nkosi**[1, 2] **and R C Botha**[1]
[1] Space Geodesy Programme, Hartebeesthoek Radio Astronomy Observatory, Krugersdorp, South Africa,
[2] Programme of Land Surveying (Geomatics), School of Engineering, University of KwaZulu-Natal, Durban, South Africa
[3] Department of Geography, Geoinformatics and Meteorology, University of Pretoria, Pretoria, South Africa

E-mail: sphume@hartrao.ac.za

Abstract. The Lunar Laser Ranger (LLR) system under development at HartRAO (25.8900° S, 27.6853° E) will accurately measure the Earth-Moon distance through the use of laser pulses. The complete signal chain needs to be optimally configured to ensure that the transmitted pulses reaches the retroreflectors located on the Moon and a signal is returned to the Earth-fixed receiving telescope. We discuss the hardware components and software used in HartRAO's LLR system to achieve optimal signal path efficiency. This includes thorough descriptions of the laser source, optical components used throughout the coudé path, atmospheric transmission efficiency and retroreflector's optimal reflectance value. The use of the link budget equation in this work estimates the number of photons that are expected to be received; this result has a direct relationship with total system efficiency. An integrated model for HartRAO's LLR is an essential tool to enable optimal signal path (optical and electrical) efficiency and is useful in estimating the expected number of photon returns for given observational parameters.

1. Introduction

The Lunar Laser Ranger (LLR) system under development at HartRAO (25.8900° S, 27.6853° E) will accurately measure the Earth-Moon distance by the use of laser pulses [1-3]. It will accurately measure the time of flight (TOF) for pulses transmitted to the Moon's five arrays of corner cube reflectors and returned back to the Earth fixed receiving telescope. The measured TOF is basically multiplied by the speed of light and divided by two in order to obtain the separation between the surfaces of Earth and Moon. Several other corrections need to be included for high accuracy, these include general relativistic range delay and LLR station position changes due to solid Earth tide corrections. The success of this technique depends highly on a number of parameters which include the quantum efficiency of the detector, characteristics of the transmit and receive optics, transmission properties of the atmosphere and cirrus cloud cover as well as the reflector's effective cross section.

The obtained number of photons resulting from the returned laser pulses is a key technique in lunar science and the General Theory of Relativity (GTR). The LLR data analysis is the most effective technique to study the interior of the Moon and dynamics of the Earth Moon system [4]. It has also contributed to studies of the lunar core, detection of lunar free libration, evaluation of the strong principle of equivalence for massive bodies and time variability of the universal gravitational constant [5-7]. The LLR technique has also provided measurements of the Moon's tidal acceleration, geodetic precision of the lunar orbit and its rotation where it contributes to the determination of the Earth orientation parameters, such as nutation, precision and polar motion [8-10].

A good site for laser ranging is determined by the magnitude of atmospheric transmittance which depends on the behaviour and amount of different particles in the atmosphere (e.g. aerosols, dust, water vapour, thermal and density variations and air mass). The fluctuations in the atmosphere and the presence of cloud cover can cause a propagating laser beam to diverge and thus, adversely affect lunar laser ranging data quantity and quality. Nickola (2012) investigated these fluctuations by determining the astronomical seeing conditions for the HartRAO LLR site selection and characterisation. Once the HartRAO LLR is completed and fully operational, it will be moved to Matjiesfontein in the Karoo (a semi-desert region in South Africa) where the astronomical seeing conditions will be better and less cloud cover will occur for precise laser ranging.

The aim of this work is to discuss the vital components (hardware and software) that will ensure success for HartRAO lunar laser ranging. An integrated software based analytical model will provide estimates of signal path efficiency for the HartRAO lunar laser ranger optical system. After installation at Matjiesfontein the system could be supplemented with collocated space geodesy systems (GNSS, SLR, VGOS and DORIS) to form a node of the IAG Global Geodetic Observing System [1].

2. HartRAO LLR System Description

The HartRAO LLR system consists of two TME_{00} laser sources, one for Lunar Laser Ranging and the other for Satellite Laser Ranging (SLR). Both these laser sources have a wavelength of 532 nm and produce laser pulses of different repetition rates (see Table 1). The standard deviation for the sync out pulse jitter for these lasers is less than 0.1 ns. The output beam diameters of the transmitted lasers are; 1 m for LLR and 0.2 m for SLR. For LLR, the laser beam is transmitted through the transceiver 1 m optical telescope and directed to the retroreflector mirrors mounted on the Moon surface. The laser transmitter for SLR is mounted and properly aligned parallel with the 1 m receiving telescope to ensure that the transmitted laser signals are received through the 1 m aperture.

Table 1: The specifications of the laser installed at HartRAO for Satellite/Lunar Laser Ranging system.

Laser Specifications	SLR	LLR
Output Energy, mJ	0.5	120
Repetition rate, Hz	1000	20
Beam Diameter, mm	~3	~12
Pointing stability, µrad	<30	<50
Beam Diameter, m	0.2	1.0

Each laser pulse is transmitted via the same coudé path which forms the laser ranging optical train, starting from the optical table and endings at the transmitting telescope. The optical train includes the optical components that will play a key role in ensuring that the transmitted laser is directed to the distant retroreflector mirrors and laser signals are reflected to the Earth “fixed” receiving telescope. It consists of an average transmit and receive optics efficiency of greater than 90% which maximises the success of laser ranging. The complete optical layout (see Figure 1) consists of a laser sources (SLR/LLR) mounted inside the two boxes, respectively, to prevent dust contamination and reduce thermal variations.

The 1 metre cassegrain optical telescope mount configuration is of Azimuth-Elevation configuration and consists of transceiver (transmitting and receiving) optical mirrors. A hyperbolic 0.3 m secondary mirror mounted on a metal spider structure to the front of the telescope tube directs the returned signal to the detector mounted at the back of the telescope. The telescope is equipped with servo drives and a high accuracy steering and pointing software that maximises high precision tracking capabilities [11]. This optical system will be integrated with a solid state photon detector, event timer, precipitation and visibility sensors, range gate generator, start diode and programmable pulse repetition frequency (PRF) system.

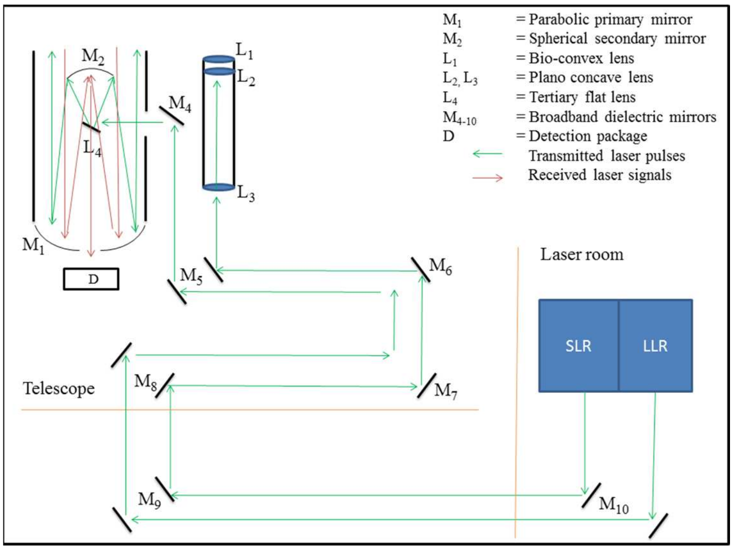

Figure 1: The schematic diagram of the HartRAO LLR system. Laser pulses for both SLR and LLR propagate through the same coudé path. Each laser has a 532 nm wavelength.

3. Laser signal path optimisation

There are many detrimental factors that affect propagating laser beams [12-16]. Thermal and density fluctuations in the atmosphere are some of the limiting factors whenever operating laser ranging. Analysis of these atmospheric fluctuations and development of an integrated model and system that enables optimal efficiency of a lunar laser ranger signal path will improve the chances of effectively targeting the cube retroreflectors mounted on the lunar surface.

One of the crucial considerations that can enable optical signal path efficiency is to develop a sensitive laser pulse detection system which operates with Single Photon Avalanche Diodes (SPADs). The SPADs help to register the arrival of a single photon and the detection of this photon is time-tagged. Using a single photon detection system is ideal for the superior performance of detectors at the single photon level [17-19].

4. Simulations, results and analysis

The estimated number of returned photons per minute for HartRAO LLR was obtained by developing a mathematical tool which estimates the relationship between the returned number of photons and the varying and fixed link budget equation parameters. The mathematical tool is used to indicate "worse and optimal" parameter values which influence the return signal, presented as an estimate of expected number of returned photons for the HartRAO station. The link budget equation is expressed as, [20]

$$n_p = \eta_q \left(E_T \frac{\lambda}{hc} \right) \eta_t G_t \sigma \left(\frac{1}{4\pi R^2} \right)^2 A_r \eta_r T_a^2 T_c^2 \qquad (1.1)$$

where η_q is the quantum detector efficiency, E_T the total laser pulse energy, η_t the transmit optics efficiency, G_t the transmitter gain, σ the satellite/Moon reflector optical cross-sectional area, R the slant range, A_r the effective area of the receiving telescope aperture, η_r the receive optics efficiency, T_a is the one-way atmospheric transmission and T_c is the cirrus cloud cover.

An LLR photon estimator computer program, written in C++, is under development at HartRAO. It uses known and estimated parameters to determine the expected number of photon returns under various scenarios (see Table 2). This is all done to achieve signal path optimal efficiency to yield an improvement in the return-energy of the laser for accurate ranges to the corner cube retroreflectors.

Table 2: The relationship between varying parameters and the number of photons reflected from Apollo 11 Laser Ranging Retroreflectors.

Parameter	Worst value	Optimal value
Transmit optics efficiency	0.4	0.9
Slant range (km)	399929	347929
Detector quantum efficiency	0.4	0.7
Receive optics efficiency	0.4	0.9
Atmospheric transmission	0.02	0.81
Cirrus transmission (Cloud cover)	0.1	1
Returned photons/minute	0.003	12

The influence of the beam divergence on a transmitted laser beam reduces the number of photons that illuminate the corner cube reflectors and reflected back to the Earth ‘fixed’ receiving telescope. There is a two-way (uplink and downlink) spread on the laser beam as it propagates through the atmosphere under various scenarios. The variations on the returned number of photons depend on the initial beam diameter transmitted through the telescope, slant range, atmospheric transmission, cirrus cloud cover transmittance and the effectiveness of the lunar reflectors (see Figure 2 and Figure 3). The optimal values of atmospheric and cirrus cloud cover transmissions (see Table 2) are referred to, in this article, as favourable weather conditions (see Figure 3). These values, together with the other optimal link budget equation parameter values, yield a maximum number of returned photons.

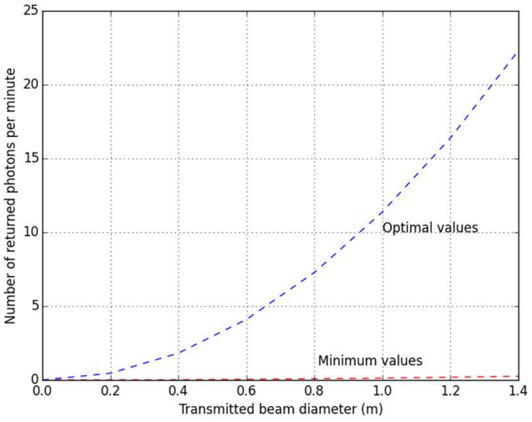

Figure 2: The dependence of the number of returned photons per minute on the output beam diameter and variable link budget equation parameters. The blue dotted line represents optimal values while the red dotted line represents minimum values.

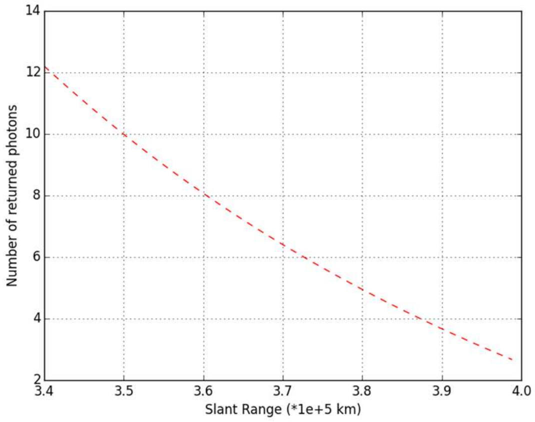

Figure 3: The expected number of returned photons per minute as calculated for the HartRAO LLR system under favourable weather conditions.

The number of returned photons linearly varies with the lunar reflector cross section. A study on the effective area of the corner cube has revealed that, at arbitrary incident angle, θ_{inc}, the area is reduced by the factor, [21]

$$\eta(\theta_{inc}) = \frac{2}{\pi}\left(\sin^{-1}\mu - \sqrt{2}\tan\theta_{ref}\right)\cos\theta_{inc}, \tag{1.2}$$

where the quantity $\mu = \sqrt{1-\tan^2\theta_{ref}}$, θ_{ref} is the internal refracted angle as determined by Snell's law. In this case, the refractive index for retroreflector's optical material type is $n = 1.45$ for fused silica. Thus, the peak optical cross-section in the centre of the reflected lobe decreases as the incident angle increases (see Figure 4) [21], i.e.

$$\sigma_{eff}\left(\theta_{inc}\right) = \eta^2\left(\theta_{inc}\right)\frac{\pi^3\rho D^4}{4\lambda^2}, \tag{1.3}$$

where ρ is the reflectivity of the retroreflector which is typically equal to 0.78 for aluminium-coated back faces and 0.93 for uncoated Total Internal Reflection (TIR) surfaces, D is the retroreflector diameter and λ is the wavelength.

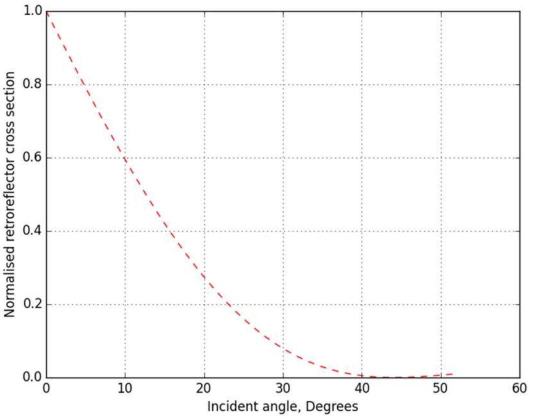

Figure 4: The normalised cross-section as a function of incident angle for fused silica retroreflectors.

5. Conclusion

We have successfully developed an integrated software based model that will enable optimal signal path efficiency for the HartRAO's LLR system [22, 23]. An integrated software based analytical model for the signal path efficiency of the HartRAO lunar laser ranger optical system is an essential tool that can improve and ensure photon returns when ranging to the Moon. Our estimated signal return rate is a true reflection of the LLR photons returns.

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