An integrated software based analytical model for the signal path efficiency of the HartRAO lunar laser ranger optical system

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Introduction

What is HartRAO-LLR?

• Hartbeesthoek Radio Astronomy Observatory’s Lunar Laser Ranger.

• Lunar Laser Ranging (LLR) is a geodetic technique.

• This is currently the only LLR in the Southern Hemisphere.
Overview of the HartRAO-LLR system

Measurement challenge

- The HartRAO Lunar Laser Ranger (LLR) system requires a state-of-the-art software tool.

- The existing link budget equation estimates the number of returned photons by calculating the mean number of detected returned photons as,

\[ n_p = \eta_q \left( E_T \frac{\lambda}{hc} \right) \eta_t G_t \sigma \left( \frac{1}{4\pi R^2} \right)^2 A_r \eta_r T_a^2 T_c^2. \]  

Equation 1 becomes,

\[ n_p = C_{\text{system}} \left( \frac{T_a T_c}{R^2} \right)^2, C_{\text{system}} = \eta_q \left( E_T \frac{\lambda}{hc} \right) \eta_t G_t \sigma \left( \frac{1}{4\pi} \right)^2 A_r \eta_r. \]
Returned photons estimation

Generating data
Simulations and analysis

<table>
<thead>
<tr>
<th>Laser Specifications</th>
<th>SLR</th>
<th>LLR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output Energy, mJ</td>
<td>0.5</td>
<td>120</td>
</tr>
<tr>
<td>Repetition rate, Hz</td>
<td>1000</td>
<td>20</td>
</tr>
<tr>
<td>Beam Diameter, mm</td>
<td>~3</td>
<td>~12</td>
</tr>
<tr>
<td>Pointing stability, µrad</td>
<td>&lt;30</td>
<td>&lt;50</td>
</tr>
<tr>
<td>Beam Diameter, m</td>
<td>0.2</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Slant range

![Diagram of slant range](image)
Retroreflectors

The number of returned photons linearly varies with the lunar reflector cross section. A study on the effective area of the corner cube has revealed that, at arbitrary incident angle, the area is reduced by the factor,

\[
\eta(\theta_{inc}) = \frac{2}{\pi} \left( \sin^{-1} \mu - \sqrt{2} \tan \theta_{ref} \right) \cos \theta_{inc}
\]

where \( \mu = \sqrt{1 - \tan^2 \theta_{ref}} \)

The peak optical cross-section in the centre of the reflected lobe decreases as the incident angle increases,

\[
\sigma_{eff}(\theta_{inc}) = \eta^2(\theta_{inc}) \frac{\pi^3 \rho D^4}{4 \lambda^2}
\]
Results and discussion

\[ n_p \propto \left( \frac{T_a T_c}{R^2} \right)^2 \]
## Returned photons

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transmit optics efficiency</td>
<td>0.4</td>
<td>0.9</td>
</tr>
<tr>
<td>Slant range (km)</td>
<td>399929</td>
<td>347929</td>
</tr>
<tr>
<td>Detector quantum efficiency</td>
<td>0.4</td>
<td>0.7</td>
</tr>
<tr>
<td>Receive optics efficiency</td>
<td>0.4</td>
<td>0.9</td>
</tr>
<tr>
<td>Atmospheric transmission</td>
<td>0.02</td>
<td>0.81</td>
</tr>
<tr>
<td>Cirrus transmission (Cloud cover)</td>
<td>0.1</td>
<td>1</td>
</tr>
<tr>
<td>Returned photons/minute</td>
<td>0.003</td>
<td>12</td>
</tr>
</tbody>
</table>
Conclusion

• We have successfully developed an integrated software model that will enable optimal signal path efficiency for the HartRAO’s LLR system.
• Our estimated signal return rate is a true reflection of the LLR photons returns.
Siyabonga (Thank you).