The EEG forward problem describes the scalp potentials sourced by brain activity. The primary current vector describes the current sourced by the brain. The primary currents occur due to movements of ions within the dendrites of pyramidal cells in the active regions of the brain. A stimulus will excite many excitable synapses of a whole pattern of neurons which leads to a negative current just under the brain surface and a positive current quite close but underneath. The source is thus a “dipole current” modelled as:

\[ \vec{J} = \sigma \hat{\vec{E}} \times \vec{x} \quad \text{or} \quad \sigma \hat{\vec{E}} = \vec{J} / \vec{x} \]

The physiological capacitance is modeled as a linear dielectric medium:

\[ \sigma \epsilon \hat{\vec{E}} = \vec{J} / \vec{x} \]

where \( \sigma \) is the conductivity and \( \epsilon \) is the permittivity.

The EEG forward problem is described by these two equations, and the solution is to find the potential \( \phi \) and current density \( \vec{J} \) given the source \( \vec{J} \) and boundary conditions. This is a typical boundary value problem which can be formulated as:

\[ \nabla \cdot \sigma \epsilon \hat{\vec{E}} = \vec{J} / \vec{x} \quad \text{in} \quad \Omega \]

\[ \nabla \cdot \sigma \epsilon \hat{\vec{E}} = 0 \quad \text{on} \quad \partial \Omega \]

where \( \Omega \) is the domain and \( \partial \Omega \) is the boundary.

The solution to this problem is known to be singular at the source, and the potential and current density are infinite. However, for practical purposes, we can approximate the solution by using a boundary integral equation method. This method involves discretizing the boundary \( \partial \Omega \) into a set of surface elements and approximating the potential and current density on these elements. The integral equation can then be solved numerically to obtain the potential and current density at any point in the domain.

\[ \int_{\partial \Omega} (\sigma \epsilon \hat{\vec{E}} \cdot \hat{n}) dS = \int_V \vec{J} dV \]

where \( \hat{n} \) is the outward normal vector to the boundary.

The singular integral equation is then regularized using a suitable boundary approximation, such as a piecewise linear or quadratic approximation. The regularized integral equation can then be solved using a iterative method, such as the conjugate gradient method.

The EEG forward problem is a classic example of a boundary value problem in electromagnetism, and it is a fundamental problem in the field of bioelectromagnetics. The solution to this problem is crucial for understanding brain activity and for developing brain imaging techniques such as EEG and MEG.