Reproducing observed solar radiation characteristics in tropical regions using stochastic theoretical models

Marie Chantal Cyulinyana and Hartmut Winkler

Department of Physics, University of Johannesburg, PO Box 524, 2006 Auckland Park, Johannesburg, South Africa

E-mail: kamirwa@gmail.com

Abstract. This paper discusses the framework for developing a theoretical model that characterizes solar radiation at ground level and evaluates the model using Monte Carlo simulations. It presents an overview of the methodology and calculations required for such a study, including the effect aerosols have on irradiance. It explores the results of previous studies that have provided broad band solar irradiance measurements at specific locations and that have developed empirical relationships between a series of solar parameters appropriate for African conditions. It seeks to reproduce these relationships using a theoretical modelling approach that incorporates radiative transfer processes, as well as Rayleigh and Mie scattering. It presents wavelength-dependent techniques that characterize the various components of solar radiation using theoretical models, and evaluates their suitability in this context. The application of Monte Carlo based atmospheric radiative transfer models is also briefly discussed.

1. Introduction

Solar radiation is energy distributed over a wide continuous spectrum ranging from ultraviolet to the infrared. The transmittance of the atmosphere to solar light is wavelength dependent, as is the efficiency of the solar device, which is a function of the technology employed.

The intensity of the Sun’s rays reaching the outer surface of the Earth’s atmosphere is approximately 1360 W/m^2. The solar beam intensity at the Earth’s surface is reduced by scattering and absorption due to molecules, aerosols, water vapour and naturally clouds. The beam of sunlight incident at the Earth’s surface from the direction of the Sun is called direct solar radiation, while scattered sunlight incident from other directions, i.e. sky light, is referred as diffuse radiation [1]. Despite their significance, direct and diffuse solar radiation measurements are not abundantly available due to the cost and difficult maintenance of measuring equipment such as pyranometers.

In past decades, the characterization of solar radiation has been based on satellite derived data as well as empirical models based on, amongst other things, meteorological records. This includes a variety of studies of solar irradiance in African and other tropical localities [2-5]. These approaches rely on simplifying assumptions to deal with the mathematical complexities inherent in analytical determinations of the irradiance.

An alternative means of treating this problem is through stochastic computation, commonly referred to as a Monte Carlo simulation. This involves launching photons from the top of atmosphere as if they come from the Sun. Each photon is tracked through a sequence of interactions until it is absorbed in the atmosphere, observed at the surface, or scattered out of the atmosphere. The outcome of each interaction is governed by the application of randomly generated numbers to the physics of the process and the probability of achieving a particular outcome [6]. The true solar irradiance profile is then sufficiently approximated through multiple computational repetition of the photon generation process.

The study presented here is a precursor to a wider PhD study that aims to determine the solar irradiance and solar energy potential of Rwanda. The methodology developed is however equally suited to other localities, when atmospheric parameters are suitably adjusted. In the case of Rwanda, the model to be eventually developed will be required to match not only the measured local annual average solar irradiation (4.3-5.2 kWh/m^2/day, depending on the exact locality [2,5]), but also established cloud cover patterns and atmospheric turbidity (with seasonal trends).

In addition to the above, the Monte Carlo simulations will also be providing important details about the angular and spectral distribution of the scattered skylight. This is critical for the determination of the efficiency of solar power generating devices, and to determine their optimal configuration (orientation). This is particularly significant in view of the high equipment and maintenance costs that would be required to secure this information through experimental means.

2. The stochastic model employed

The equation of radiative transfer mathematically describes a light beam‘s energy losses to absorption, gains by emission, and redistribution by scattering. Radiative transfer modelling is the numerical prediction of quantities related to the radiative energy transfer in a medium such as the Earth’s atmosphere. The approach here involves stochastic modelling of the scattering and absorption of sunlight in complex atmospheric conditions [6-8].

As solar light penetrates the atmosphere, the solar radiation is attenuated by the atmosphere through scattering, absorption and reflection. The attenuation is proportional to the density of the medium, the incoming beam, and the path length or optical path length, and is described by the Beer-Lambert equation.

$$I_\lambda = I_{0,\lambda} \exp(-\tau_\lambda m) \tag{1}$$

Where I_λ and $I_{0,\lambda}$ are the solar beam intensity per unit wavelength at ground level and at the top of the atmosphere respectively, τ_λ is the (wavelength dependent) optical depth and m is the airmass, i.e. the relative beam path through the atmosphere measured in units of the vertical atmospheric depth.

Scatterers in the atmosphere are usually divided into two main types - aerosols and molecules. The scattering mechanism largely depends on the scattering particle size. Rayleigh scattering occurs for particle sizes smaller than the wavelength (as is the case with air molecules), while Mie Scattering happens if the particle size is of the order of the wavelength (i.e. for a large fraction of aerosols).

For Rayleigh scattering, the relationship between the relative intensity of scattered light and the angle of deflection ξ is described by the phase function,

$$p_R(\xi) = \frac{3}{4}\left(1 + \cos^2 \xi\right), \tag{2}$$

which we note reaches a minimum for scattering at 90° to the original beam direction [9].

For Mie scattering, the phase function is far more complex, as it depends on aerosol shape, size and composition. A good way of approximating aerosol scattering is by means of the Henyey-Greenstein phase function

$$p_{aer}(\xi) = \frac{1 - g_\lambda^2}{\left(1 + g_\lambda^2 - 2g_\lambda \cos \xi\right)^{3/2}} \tag{3}$$

where g is a parameter referred to as the asymmetry factor [10].

3. Application of the stochastic computation model to specific scenarios

A random number generator is used to provide random numbers between 0 and 1 which are used to calculate the photon step size (i.e. the distance covered by the photon until it is absorbed or scattered), the nature of the interaction (absorption or scattering) and the scattering angle (deflection angle ξ and azimuth). In addition, through application of the spectral distribution of the solar beam above the atmosphere, one can even use random numbers to determine the photon wavelength. Where a scattering event has taken place, the process is then repeated until the photon reaches the ground, is scattered back into space or becomes absorbed. Repeating this procedure to a very large sample of photons yields a good approximation of the solar irradiance observed on the ground. Through considering the zenith angle and azimuth from which the scattered photons strike the ground, one is able to model the angular light distribution of diffuse light, and even to estimate its spectral properties. Even cloud cover and weather patterns can be determined stochastically, though none of that is attempted here. The formalism for applying random numbers to the two phase functions described earlier has been previously established. Prahl et al used a generated random number (*r*) to calculate the step size, Δs (see equation 4):

$$\Delta s = \frac{-\ln r}{u_t} \tag{4}$$

(where u_t is the total attenuation coefficient).

They also used generated random numbers to calculate the azimuthal angle, $\psi = 2\pi r$ and the scattering angle, ξ [11], which are the key elements to get the new direction of a moving photon:

$$\cos\xi = \frac{1}{2g}\left\{1+g^2-\left[\frac{1-g^2}{1-g+2gr}\right]^2\right\} \quad . \tag{5}$$

For the simulation that formed the basis to this paper, we made the simplifying assumption that the Sun is overhead. We computationally launched 21000 photons from the top of the atmosphere at three specific wavelengths and tracked the progress of each one through the atmosphere. We chose an asymmetry factor of *g* = 0.2, typical of many aerosols.

4. Results

The following results represent Rayleigh and aerosol scattering, brightness and the colour characteristic of the sky at different wavelengths as determined by our simulations.

4.1 Rayleigh and aerosol scattering

Figure 1 compares the deflection angle for the Rayleigh and Mie scattering processes. The preferred forward direction for scattering of aerosols as well as the ‘dumbbell’ shape of the Rayleigh phase function is clearly visible in the histogram.

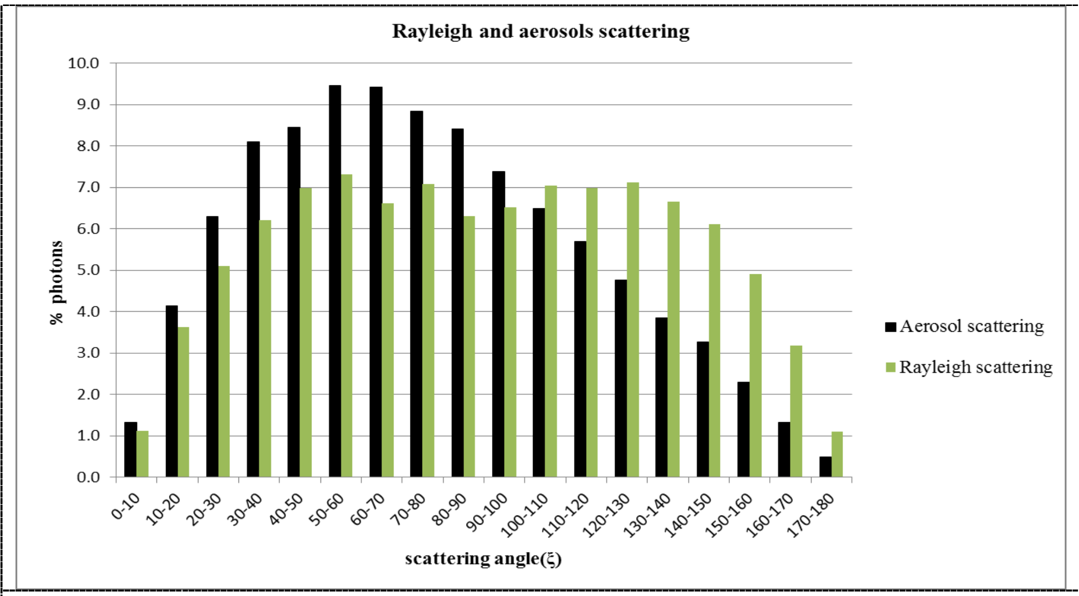

Figure 1. Comparison between Rayleigh and aerosol scattering. The graph displays the percentage of photons scattered during atmospheric transit as a function of ξ (in degrees)

4.2. Scattered photons at different wavelength

The simulations in this paper were performed for the following wavelengths and optical depths:

Table 1. Wavelength range and optical depth

λ	τ **(low)**	τ **(high)**
415 nm	0.26	0.44
501 nm	0.14	0.26
615 nm	0.09	0.18

The first of these optical depths are typical of clear conditions at a rural site at altitude of ~1200 m above sea level, deduced from previous studies carried out over southern Africa [12]. The other optical depths represent an example of an atmosphere containing some aerosols generated by biomass burning. The wavelengths chosen correspond to standard sunphotometer bands.

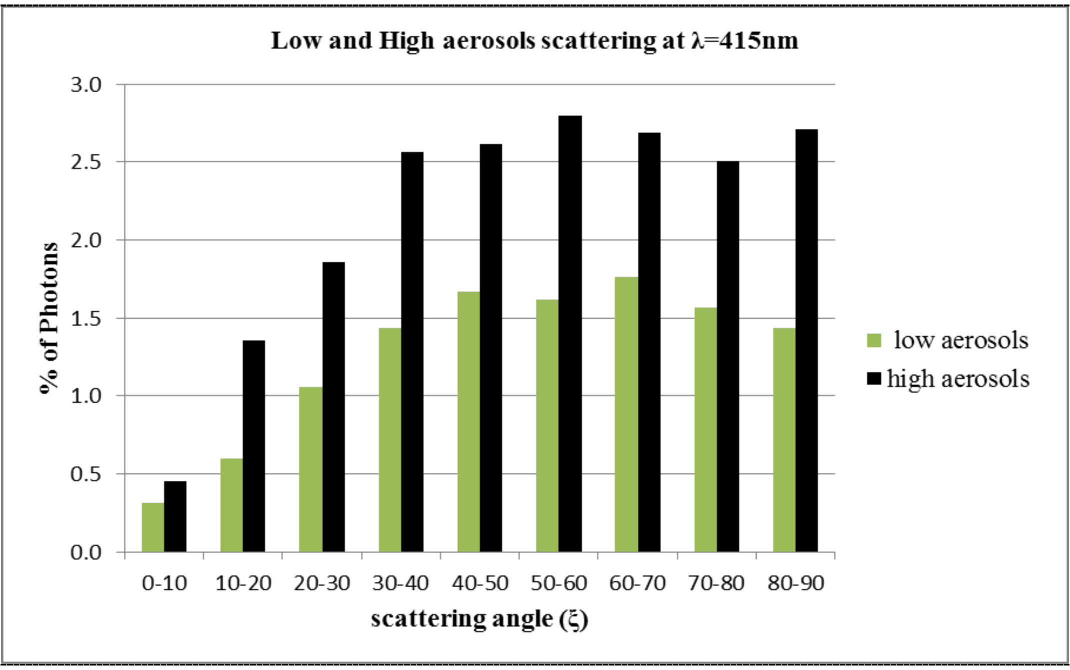

Figure 2. Comparison solar radiation scattering at λ = 415nm in the presence of low and high aerosol concentrations in the atmosphere.

4.3 Brightness of the sky

Figure 3 displays the actual sky brightness, and differs from the distribution in Fig. 2 due to the fact that the solid angle corresponding to any particular deflection angle range is actually proportional to cos ξ. As in the figure before that, the larger aerosol concentration leads to a greater sky brightness, as is expected in view of the known aureole around the Sun in hazy conditions. Here the photon ratio represents the fraction of photons scattered in a given range of scattering angle per total number of launched photons.

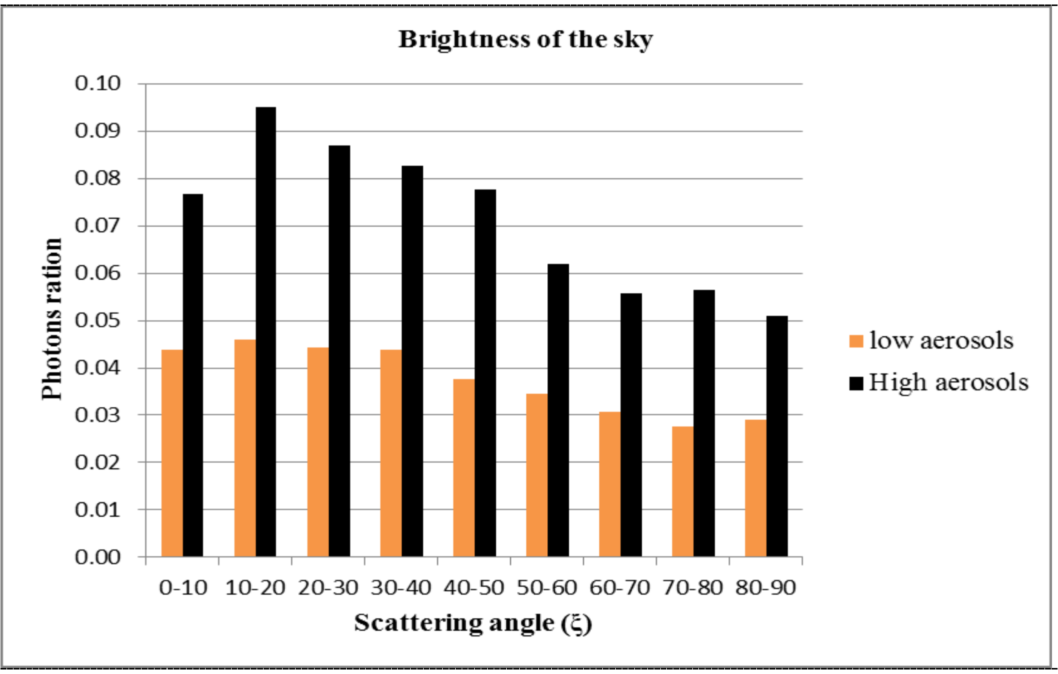

Figure 3. Brightness of the sky at λ = 501nm for high and low aerosol atmospheric concentrations.

4.4 Colour of the sky

Finally, we test whether our simulation leads to colour differences between different scattering angle directions. In Fig. 4, photon ratio refers to the fraction of photons scattered at two different wavelengths (615nm and 415nm respectively) in the presence of low and high aerosol concentrations. The results of this test, shown in Fig. 4 below, are inconclusive. A much larger photon sample is required to detect any dependence on scattering angle or optical depth.

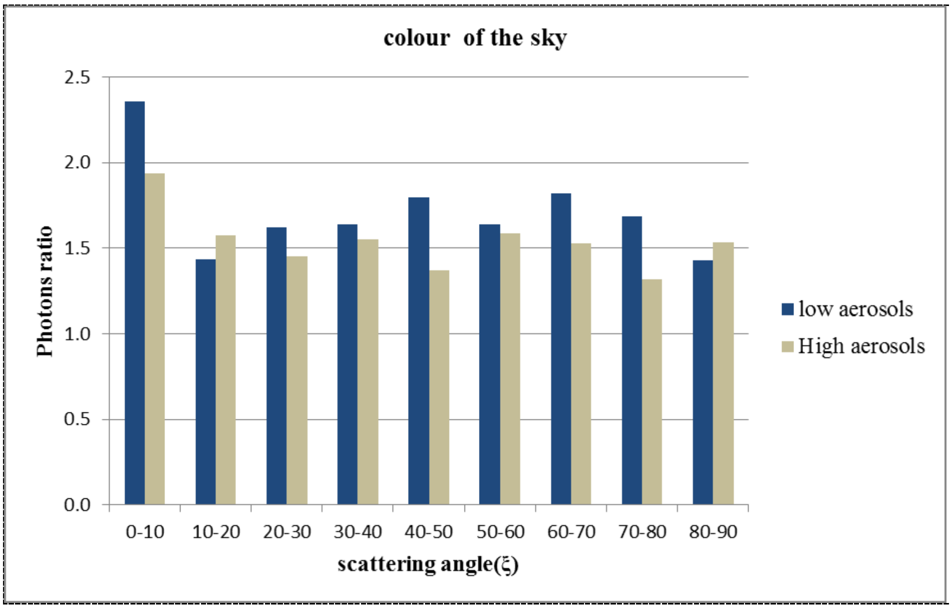

Figure 4. Ratio of the photons scattered at 415 nm as opposed to 615 nm, for low and high aerosol concentrations. The higher this ratio, the bluer the sky.

5. Further work

The results of the preliminary stochastic simulations that form the basis of this paper show the potential of applying such simulations to determine both direct and diffuse irradiance values, especially as these can be expanded to a whole spectrum of wavelengths. Accuracy can be improved through drastically increasing the photon number in the computations. The models are also easily adapted to different solar beam inclination angles, meaning that an irradiation time sequence can be developed. The possibility of refining the aerosol scattering characteristics to match local aerosol types make this a promising approach to the modelling of ground level solar irradiance, and hence determining the solar energy potential at a site.

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