Can Shapiro step subharmonics be "charged"?

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Abstract. Using the capacitively coupled Josephson junctions with diffusion current (CCJJ + DC) model, we performed a precise numerical study of phase dynamics of intrinsic Josephson junctions under external electromagnetic radiation. We survey the different Shapiro step subharmonics found in these systems. We demonstrate the charging of superconducting layers in bias-current interval corresponding to a given Shapiro step subharmonics and the existence of longitudinal plasma wave along the stack of junctions.

1. Introduction

The system of superconducting layers in high temperature superconductors (HTSC) such as $Bi_2Sr_2CaCu_2O_{8+\delta}$ (Bi-2212) represents intrinsic Josephson junctions (IJJs). In such systems, as it is the case in single Josephson junctions, the locking of the Josephson oscillations with frequency ω_J to the frequency ω of external electromagnetic radiation leads to the appearance of Shapiro steps [1, 2] in the current voltage characteristics (IV-characteristics). Many devices in existence exploit this effect, notably voltage standards and terahertz radiation emitters/detectors. Therefore, a detailed study of the Shapiro steps and their subharmonics in the IJJs under different resonance conditions presents important research questions with various potential applications.

An interesting feature of the IJJs is the possibility of longitudinal plasma wave (LPW) propagating along the c-axis [3]. Each superconducting layer is characterized by the order parameter $\Delta_l(t) = |\Delta|\exp(i\theta_l(t))$ with the time dependent phase $\theta_l(t)$. The thickness of superconducting layers (about ~ 3 Å) in an HTSC is comparable with the Debye length r_D of electric charge screening. Therefore, there is no complete screening of the charge in the separate layers, and the electric field induced in each JJ penetrates into the adjacent junctions. Thus, the electric neutrality of superconducting layers is dynamically broken and, in the case of the ac Josephson effect, a capacitive coupling appears between the adjacent junctions [3]. The absence of complete screening of charge in the superconducting layer leads to the formation of a generalized scalar potential Φ_l of the layer, which is related to the charge density Q_l in the superconducting layer as follows [3, 4]: $Q_l = -\frac{1}{4\pi r_D^2}\Phi_l$.

The existence of a relationship between the electric charge Q_l of the l-th layer and the generalized scalar potential Φ_l of this layer reflects a non-equilibrium nature of the ac Josephson effect in layered HTSC [4]. In this case, the diffusion contribution to the quasiparticle current arises due to the generalized scalar potential difference. The latter is taken into account in the capacitively coupled Josephson junction model with diffusion current (CCJJ+DC model)[5]. At $\omega_J = 2\omega_{LPW}$ (ω_J and ω_{LPW} are the Josephson and longitudinal plasma wave frequencies, respectively) the parametric resonance is realized: the Josephson oscillations excite the longitudinal plasma wave. The charge in the superconducting layer at parametric resonance can have complex oscillations depending on the number of junctions in the stack, coupling and dissipation parameters and boundary conditions. Fourier analysis [6] of the temporal dependence of the charge in a superconducting layer shows a spectrum of frequencies, in particular, ω_{LPW}, ω_J, and their combinations.

In this paper, we present the results of the investigation of the effects of electromagnetic radiation on the phase dynamics of the intrinsic JJs and the temporal oscillations of the electric charge in superconducting layers in HTSC. For fixed parameters of JJs and parameters of simulations, we have studied the manifestation of the Shapiro step subharmonics in IV-characteristics and demonstrated their "charging", i.e. the charging of superconducting layers in the corresponding bias current intervals at fixed amplitude of external radiation. To escape the complexity related to the overlapping of the SS subharmonics, we consider here a case of small radiation amplitudes.

2. Model and method

To investigate the phase dynamics of the IJJ, we use the one-dimensional CCJJ+DC model [5] with the gauge-invariant phase differences $\varphi_l(t)$ between S-layers l and $l+1$ in the presence of electromagnetic irradiation described by the system of equations:

$$\begin{cases} \dfrac{\partial \varphi_l}{\partial t} = V_l - \alpha(V_{l+1} + V_{l-1} - 2V_l) \\ \dfrac{\partial V_l}{\partial t} = I - \sin\varphi_l - \beta\dfrac{\partial \varphi_l}{\partial t} + A\sin\omega t + I_{noise} \end{cases} \tag{1}$$

where t is the dimensionless time normalized to the inverse plasma frequency ω_p^{-1}, $\omega_p = \sqrt{2eI_c/\hbar C}$, C is the capacitance of the junctions, $\beta = 1/\sqrt{\beta_c}$, β_c is the McCumber parameter, α gives the coupling between junctions [3], ω and A are the frequency and amplitude of the radiation, respectively. To find the IV-characteristics of the stack of the intrinsic JJ, we solve this system of nonlinear first order differential equations (1) using the fourth order Runge-Kutta method. In our simulations we measure the voltage in units of $V_0 = \hbar\omega_p/(2e)$, the frequency in units of ω_p, the bias current I and the amplitude of radiation A in units of I_c.

To calculate the voltages $V_l(I)$ at each I, we simulate the dynamics of the phases $\varphi_l(t)$ by solving the system of equations (1) using the fourth-order Runge-Kutta method with a step in time T_p (a schematic of the numerical procedure and additional parameters of simulation are shown in figure 1 of Ref. [7]). As a result, we obtain the temporal dependence of the voltages in each junction at a fixed value of bias current. So, we can calculate the temporal dependence of the charge in each layer as well as through the voltage difference in the neighboring junctions (see below). After completing the calculations for bias current value I, the current value is increased or decreased by a small amount of δI (bias current step) and the voltages in all junctions at the next point of the IV-characteristics is calculated. So actually, the time dependence of voltage in each junction or the charge in each layer consists of intervals at each fixed current value. We use the distribution of phases and voltages achieved at the previous point of the IV-characteristics as the initial distribution for the current point. The average of the voltage $\bar{V}_l$ is given by

$$\bar{V}_l = \frac{1}{T_f - T_i}\int_{T_i}^{T_f} V_l dt \tag{2}$$

where T_i and T_f determine the interval for the temporal averaging. In our simulations, we use $T_i = 2^3T$, $T_f = 2^{10}T$. The time step is $\Delta T = T/(2 \times 160)$. Here the factor T is $T = 2\pi/\omega(2/3)(3/5)(4/7)$. This choice is not unique but necessary for the stability of our calculations.

To study time dependence of the electric charge in the S-layers, we use the Maxwell equation $div(\varepsilon\varepsilon_0\vec{E}) = Q$, where ε and ε_0 are relative dielectric and electric constants, respectively. The charge density Q_l (in the following text - charge) in the S-layer l is proportional to the difference between the voltages V_l and V_{l+1} in the neighbor insulating layers

$$Q_l = Q_0\alpha(V_{l+1} - V_l), \tag{3}$$

where $Q_0 = \varepsilon\varepsilon_0 V_0/r_D^2$. An estimate, taking the parameters of the IJJ, gives a value of about $2.4 \times 10^{-16}C$. This value of charge is not significantly high, but it makes for an interesting physics. In our numerical calculations, we have used the coupling parameter $\alpha = 0.05$, the dissipation parameter $\beta = 0.2$ and periodic boundary conditions. We note that the qualitative results are not very sensitive to these parameters and boundary conditions. The details of the model and simulation procedure are presented in Ref. [8].

3. Results and discussion

3.1. Shapiro step subharmonics in IV-characteristic

As mentioned earlier, the Shapiro step subharmonics appear when Josephson junctions are exposed to electromagnetic radiation. This happens when the resonance condition $q\omega_J = p\omega$ is fulfilled. As a result, we expect to find steps on the IV-curves at voltages $V_{p,q}$ such that,

$$V_{p,q} = N\omega_J = (\frac{p}{q})N\omega, \tag{4}$$

Integers ratio p/q correspond to the main Shapiro steps, whereas non-integer ratio p/q give the subharmonics.

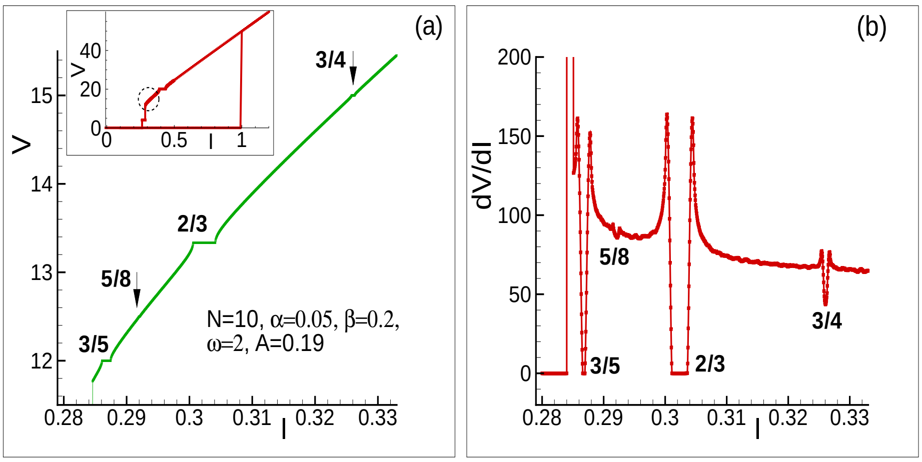

Figure 1. (Color online) (a) Demonstration of subharmonics in IV-characteristic of the stack of 10 IJJs with $\beta = 0.2$, $\alpha = 0.05$ under external radiation at frequency $\omega = 2$ and amplitude $A = 0.19$. Inset shows the total IV-curve with dashed circle indicating the enlarged part of IV-characteristic in main figure; (b) Manifestation of the subharmonics on the dependence of the differential resistance dV/dI versus bias current I for the same stack.

The inset to figure 1(a) shows the total IV-characteristic of the stack of 10 IJJs with $\beta = 0.2$, $\alpha = 0.05$ under external radiation at frequency $\omega = 2$ and amplitude $A = 0.19$. We see the

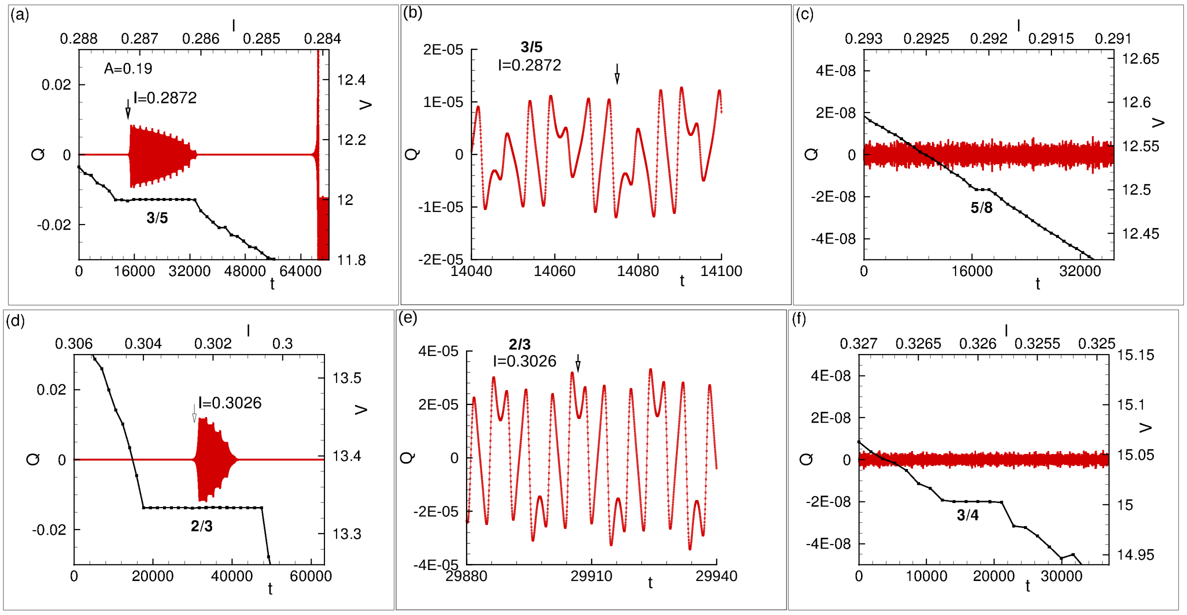

Figure 2. (Color online) Charge-time dependence (top curve) and corresponding parts of IV-characteristics (bottom curve/dotted lines) around Shapiro step subharmonics: (a) 3/5, (c) 5/8, (d) 2/3 and (f) 3/4. Panels (b) and (e) enlarge the parts of charge-time curves indicated by arrows in (a) and (d). They demonstrate the character of charge oscillations in the superconducting layers at chosen values of bias current.

Shapiro step at $V = 20$ and some inner branches before transition to the zero voltage state. Part of the IV-curve marked by circle is enlarged in the main figure. It demonstrates the Shapiro step subharmonics 2/3 and 3/5 observed and the corresponding values of voltage. We note that these steps belongs to the third level of continued fractions $N - 1/(n + (1/m))$ with $N = 1$ and $n = 2$ [9]. We show also in figure 1(b) the differential resistance dV/dI versus bias current I to demonstrate the subharmonics 3/4 and 5/8 which are not clearly visible in the IV-characteristics. Their positions in IV-characteristic are indicated by arrows. They belong to the continued fractions $N - 1/(n \pm (1/m))$ with $N = 1$, $n = 3$ and sign plus and minus, respectively.

3.2. "Charging" of Shapiro steps subharmonics

A charging of superconducting layers in the bias current interval corresponding to the Shapiro step at double resonance conditions was previously demonstrated [10]. A question concerning the "charging" of Shapiro step subharmonics was not investigated. Here, we attempt this question.

For the current intervals corresponding to the Shapiro steps subharmonics in figure 1, we compute the electric charge in each superconducting layer using Eq. 3. The charge versus time graphs for these current intervals and corresponding parts of the IV-characteristics (black curves) can be found in figure 2.

We observe that the stack of IJJ in the current interval corresponding to the SS subharmonics 3/5 and 2/3 has noticeable charge oscillations. The character of charge oscillations is demonstrated in the panels (b) and (e) for the parts of charge-time curves indicated by arrows in (a) and (d). We see the difference in the dynamics of charge oscillations which reflects the different wavelength of realized longitudinal plasma wave in both cases. The SS subharmonics 5/8 and 3/4 do not demonstrate "charging": the amplitude of charge oscillations is in the order of 10^{-8} (i.e. on the noise level of our simulations).

3.3. Longitudinal plasma waves

The effect of the increase of the external radiation amplitude on the LPW created at the radiation related parametric resonance was investigated in Ref. [10]. A remarkable change in the longitudinal plasma wavelength was demonstrated. Here we show that LPWs with different wavelength can be realized in the stack in the bias current intervals corresponding to the Shapiro step subharmonics.

Figure 3 demonstrates the longitudinal plasma waves created in the stack of 10 IJJs in current intervals corresponding to the "charged" regions of 3/5 and 2/3 steps. We see the realization of LPW with wavelength $\lambda = 5$ in case of Shapiro step subharmonic 3/5 and $\lambda = 2$ in case of 2/3. The effect of the increase of radiation amplitude on the wavelength of the LPW will be studied somewhere else.

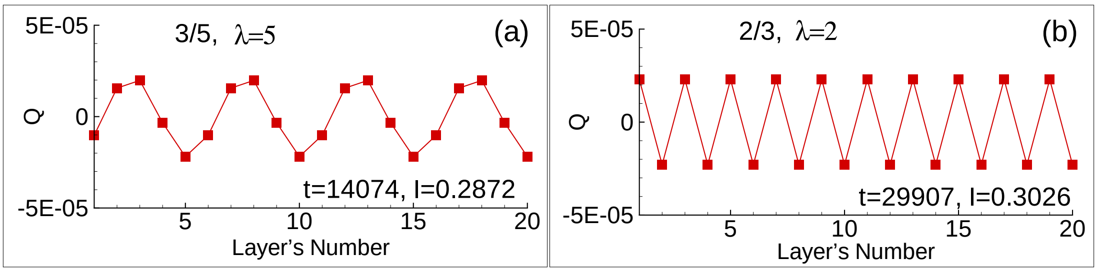

Figure 3. (Color online) Demonstration of the longitudinal plasma waves created in the stack of 10 IJJ: (a) Distribution of charge along the stack at fixed time in the current interval of step 3/5; (b) The same for the step 2/3.

3.4. Conclusions

We have demonstrated the charging of superconducting layers in bias current intervals corresponding to Shapiro step subharmonics. The existence of the longitudinal plasma wave along the stack of junctions in these intervals was illustrated. We found an interesting feature: this "charging" of subharmonics is not regular. Particularly, the step 5/8 which is placed between the "charged" steps 3/5 and 2/3, is not "charged". This phenomenon requires additional investigation.

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