Quark-gluon plasma physics from string theory

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Abstract. The goal of high-energy nuclear physics is to create and study quark-gluon plasma (QGP), the predicted deconfined state of QCD matter at energy densities greater than 1 GeV/fm^3 that permeated the universe a microsecond after the Big Bang. Contrary to original expectations, the properties of the QGP seem best described by the strong-coupling, phenomenological string theory methods of the AdS/CFT correspondence instead of the usual weak-coupling, Feynman diagram methods of perturbative QCD (pQCD). In particular, the AdS/CFT paradigm predicts a very small value for the viscosity to entropy ratio of the QGP, in remarkable agreement with data collected from the Relativistic Heavy Ion Collider (RHIC) and the Large Hadron Collider (LHC). In search of a consistent description for all observables related to QGP, we extend the AdS/CFT theory to that of high-momentum probes of the plasma and compare our results to data from LHC.

1. Introduction

Recent experiments performed at the Relativistic Heavy Ion Collider (RHIC) and the Large Hadron Collider (LHC) has provided spectacular evidence that suggests that a deconfined state of hadronic matter has been formed which is called quark gluon plasma (QGP) [1–4] with a small ratio of shear viscosity to entropy density [5; 6]. The experimental discovery that QGP is a strongly coupled plasma with low viscosity, poses a challenge to theorists. While lattice QCD is the proper tool for understanding the static equilibrium thermodynamics of such strongly coupled plasma, it does not allow us to calculate its dynamics evolution on heavy-ion collision.

Recently, a novel tool called "the AdS/CFT correspondence" [7–11] provide valuable insight into the strongly coupled plasma. In according to the original conjecture of Maldacena, the $\mathcal{N} = 4$ SYM theory in the large N_c and large 't Hooft coupling is dual to classical supergravity on ten-dimensional $AdS_5 \times S^5$ geometry [7]. In order to study the theory at finite temperature, one can add black hole (BH) to the geometry [8] which yields to the AdS-Sch metric. Fundamental quarks are described by open strings moving in the 10d geometry. In the large λ limit, the quantum fluctuation of string world sheet are suppressed and the dynamics of string are described by the classical string theory.

Jets are produced within the expanding fireball and probe the QGP. Analyzes the energy loss of these energetic partons as they travel throw QGP may reveal extremely valuable information about the dynamics of the plasma and exhibit distinctive properties such as jet-quenching which can clearly be observed at RHIC [1–4] and more recently LHC [12–14].

In this paper, we study the light quark jet energy loss in both gravitational dual to static and expanding plasma. We propose a new prescription of jets in string theory based on the separation of hard and soft sectors. We demonstrate that the light quark jet energy loss reveal the "Bragg peak" at late times. Then we compute the nuclear modification factor of jets R_{AA}^{jet}, renormalise the quantity, and compare the results with the preliminary CMS data [15].

2. Light quark jet energy loss

According to the AdS/CFT correspondence [10], the $\mathcal{N} = 4$ SYM theory at finite temperature is dual to a $10d$ black hole geometry with the AdS-Schwarzschild (AdS-Sch) metric as follows,

$$ds^2 = \frac{L^2}{u^2}\left[-f(u)\,dt^2 + d\mathbf{x}^2 + \frac{du^2}{f(u)}\right], \tag{1}$$

where $f(u) \equiv 1 - (u/u_h)^4$ is the blackening factor and L is the AdS curvature radius. Four dimensional Minkowski coordinates are denoted by x_μ and the coordinate u is an inverse radial coordinate. So, the boundary of the AdS-Sch spacetime is at $u = 0$ and the event horizon is located at $u = u_h$. The temperature of the equilibrium SYM plasma relates to the event horizon as $T \equiv \frac{1}{(\pi u_h)}$. World sheet coordinates are as σ^a where $\tau \equiv \sigma^0$ is denoted as the timelike world sheet coordinate, while the spatial coordinate is $\sigma \equiv \sigma^1$.

Fundamental representation quarks added to the $\mathcal{N}=4$ SYM theory are dual to open strings moving in the $10d$ geometry. Addition of a $\mathcal{N}=2$ hypermultiplet to the $\mathcal{N}=4$ SYM theory is performed by adding D7 branes to the $10d$ geometry [16]. These branes extend along the radial coordinate from the boundary at $u = 0$ down to maximal coordinate at $u = u_m$ as well as fill the whole 4d Minkowski space. Also, they wrap on S^3 from the S^5 sphere. The bare mass M of quark is proportional to $1/u_m$ [17], so for massless quarks the D7 brane should fill the whole radial direction. Open strings that are attached to the D7 brane are dual to the quark-anti quark pairs on the field theory side. In the $5d$ geometry these strings can fall unimpeded toward the event horizon until their end points reach the radial coordinate u_m where the D7 brane ends. Since for sufficiently light or massless quarks $u_m > u_h$, open string end points can fall into the horizon.

We are interested in studying the back-to-back jets so we consider the configurations in which the two endpoints of string move away from each other as the total spatial momentum of the string vanishes. By choosing the appropriate frame, one half of the string has a large spatial momentum in x direction, and the other half of the string carries a large negative spatial momentum. We will limit our attention to strings which move in one direction in the R^3 space, x direction, so the embedding function of string $X^\mu(\tau,\sigma)$ will be a map to $(t(\tau,\sigma), x(\tau,\sigma), u(\tau,\sigma))$. So, the profile of an open string which is created at a point in space at time $t = t_c$ is given by

$$t(0,\sigma) = t_c\,, \;\; x(0,\sigma) = 0\,, \;\; u(0,\sigma) = u_c\,. \tag{2}$$

By these conditions, the string created at time t_c and by time evolution, the string evolves from a point into an extended object and the string endpoints fall toward the horizon. Polyakov action for the string has the form

$$S_P = -\frac{T_0}{2}\int d^2\sigma\, \sqrt{-\eta}\,\eta^{ab}\,\partial_a X^\mu \partial_b X^\nu\, G_{\mu\nu}\,. \tag{3}$$

Variation of the Polyakov action with respect to the embedding functions X^μ lead to the equation of motion as

$$\partial_a\left[\sqrt{-\eta}\,\Pi^a_\mu\right] = \frac{1}{2}\sqrt{-\eta}\,\eta^{ab}\frac{\partial G_{\nu\rho}}{\partial X^\mu}\,\partial_a X^\nu \partial_b X^\rho, \tag{4}$$

where Π^a_μ are the canonical momentum densities associated to the string obtained from varying the action with respect to the derivatives of the embedding functions.

One then has the problem of finding the proper object in the dual string theory that corresponds to a jet, a slippery object even in field theory; jets are truly only defined by the algorithm used to measure them. Presumably the ideal way to compute jet observables in the dual theory is to compute the energy momentum tensor associated with a high-momentum probe and "run" a jet finding algorithm on the result. However, one can define a jet prescription in the AdS/CFT and calculate the rate of energy loss from the string profile itself.

The authors of [18] are motivated by the localization of the baryon density on the boundary which is of scale of order $\Delta x \sim 1/\pi T$ and defined jet as a part of string which is in the Δx spatial distance from the endpoint. We called this as the "$\Delta x - prescription$" of jet [18].

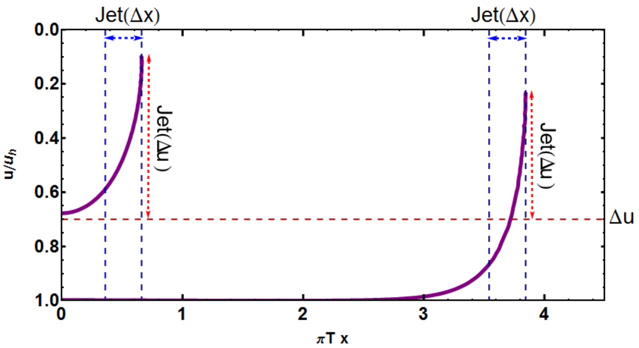


Figure 1. Illustration of the Δx and Δu prescriptions of a jet in the string theory; see text for details.

In this paper, motivated by the separation of energy scales in, e.g., thermal field theory, we propose rather a Δu prescription which we believe will ultimately provide a closer approximation to the result of a more complete calculation, figure 1. Since the radial coordinate in the string theory sets an energy scale in the field theory, in our Δu prescription the portion of the string above some cutoff $u = u^*$ in the radial direction is considered part of the jet; the portion of the string below the cutoff is considered part of the thermalized medium. By choosing any value of u above the black hole horizon as the cutoff, we regain the natural result that a jet that is thermalized no longer has detectable energy or momentum.

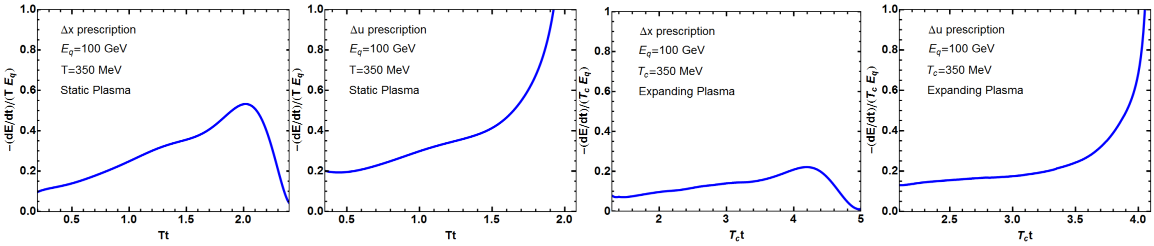
We evaluate the energy loss rate of jet in the radial $\sigma = u$ parametrization for both prescriptions of jet and plot in figure 2(a,b). In order to define jet using the $\Delta x - prescription$, we choose $\sigma_\kappa(t)$ as $\Delta x = 0.3/\pi T$. Our result on the energy loss rate of light quark using the $\Delta u - prescription$ shows a Bragg peak at late times which means the explosive transfer of quark energy to the plasma at late times and is consistence with the previous works [18].

Since the quark-gluon plasma produced at ultra-relativistic heavy ion collision is an expanding and cooling medium, we study the light quark jet energy loss in a time dependent gravity dual to the boost invariant flow [19]. This geometry is similar to the static black hole geometry, but the location of the horizon moves in the bulk as $\tau^{1/3}$ where τ is the proper time. Also the temperature of plasma cools as $T(\tau)\ \tau^{-1/3}$.

We calculate the energy loss rate of light quark using both $\Delta x - prescription$ and $\Delta u - prescription$ of jet. We consider the initial temperature of plasma in JP metric T_c the same as the temperature of plasma in AdS-Sch metric. The result is shown in figure 2(c,d) which demonstrates that the behavior of light quark energy loss in the JP metric is same as the AdS-Sch metric, but the distance that quark traveled before thermalizing increases.

3. Jet Nuclear Modification Factor

To compare our toy model with experimental data, we calculate an approximation of the nuclear modification factor R_{AA} for jet in the following way. We consider the contributions of both quark and gluon jets. We assume that the produced parton with initial energy p_T^i loses a fraction of its



(a) (b) (c) (d)

Figure 2. The instantaneous energy loss of a light quark jet as a function of time in the AdS-Sch (a,b) and JP metrics (c,d) in the Δx prescription and Δu prescription. The normalization constant $E_q = 100$ GeV is the initial energy of the jet, which has a virtuality of 175 GeV^2, and $T = 350$ MeV is the temperature of the plasma.

energy ϵ with probability $P\left(\epsilon|p_T^i, L, T\right)$ as the final energy of parton is given by $p_T^f = (1-\epsilon)p_T^i$. Also, we suppose that the the AdS energy loss is approximately independent of the initial energy [20], and the gluons loss their energy by the factor of 2 in the large N_c limit respect to the quarks.

We suppose that the production spectrum can be approximately by a power law [20], with slowly varying with respect to p_T, then we may find a simple equation for the jet nuclear modification factor as follows,

$$R_{AA}^{R\to jet}(p_T) = \left\langle \int d\epsilon\, P(\epsilon|p_T, L, T)\,\left(1-\epsilon^R\right)^{n_R(p_T)-1}\right\rangle. \tag{5}$$

For an absolutely uniform nucleus that is a 1D line, the geometric average is carried out as an integral over a line of production points with a parton that propagates through the line. In this case, $R_{AA}^{R\to jet}(p_T)$ is obtained from the below line integral [20]

$$R_{AA}^{R\to jet}(p_T) = \int_0^{L_{max}} \frac{dl}{L_{max}}\,\left(1-\epsilon^R(p_T, l, T)\right)^{n_R(p_T)-1}. \tag{6}$$

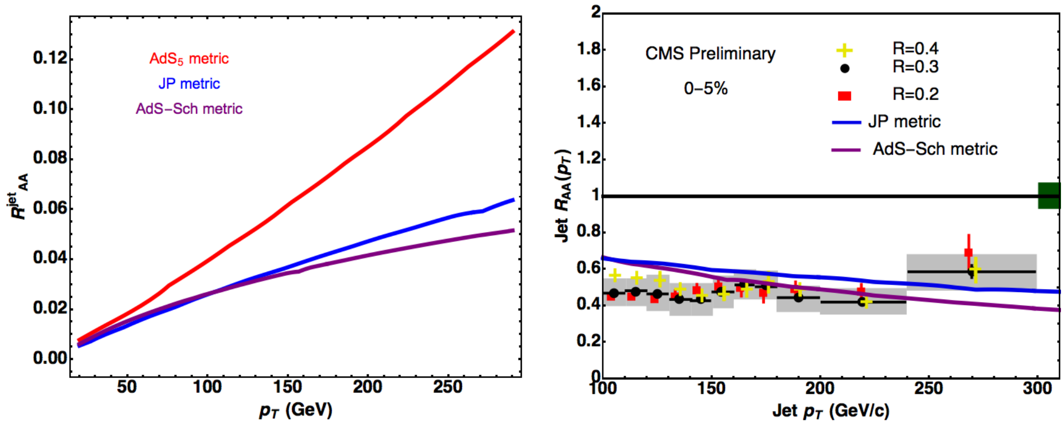
We calculate R_{AA} of jet by using our results of jet energy loss in both AdS-Sch and JP metric. Our results are shown in figure 3. The purple curve shows the R_{AA} obtained from the AdS-Sch metric, while the blue curve is the R_{AA} obtained from the JP metric. The AdS/CFT results of jet energy loss show an over suppression of jets in both static and expanding plasma.

The point-like initial condition falling string that we consider here is dual to creation of a pair of quark-antiquark which fly away each other in the strongly coupled plasma, interact and loss their energy. We expect that jets produced in the pp collision do not loss their energy. So, we consider the falling string with the same initial conditions in AdS_5 metric. Our results show that the string falls in the empty AdS_5. So, jet loses its energy in the vacuum! We calculate the R_{AA} for a falling string in AdS_5 metric and show in figure 3 (a) in red curve.

In order to compare our results with the experimental data, we define a renormalized R_{AA} in AdS/CFT as

$$R_{AA}^{renorm} = \frac{R_{AA}^{medium}}{R_{AA}^{AdS_5}} \tag{7}$$

We plot the renormalized R_{AA}^{renorm} for jets in both AdS-Sch and JP metric in figure 3 (b) and compare with the CMS preliminary data for the most central Pb-Pb collision at $\sqrt{s_{NN}} = 2.76\,GeV$ [15]. A suppression factor of 0.5 for high p_T jets is observed in central Pb-Pb collision in comparison to the pp collisions. Our results also show surprisingly agreement with CMS preliminary results on jet R_{AA}.



(a) (b)

Figure 3. (a) Jet R_{AA} as a function of p_T in the most central Pb-Pb collision obtained via AdS/CFT in AdS_5 (red), JP (blue) and AdS-Sch (purple) metrics. (b) AdS/CFT jet R_{AA} as a function of p_T compared with the preliminary CMS data in different effective cone sizes for anti-k_T jets using the Bayesian unfolding method for most central Pb-Pb collision at the LHC with $\sqrt{s} = 2.76$ TeV per nucleon [15].

4. Conclusions

In this paper we have purposed a novel prescription of jet in the context of string theory and AdS/CFT correspondence. We have defined jet as a part of a falling string which lies above the radial coordinate $u^\star$ near the event horizon. Our motivation is defining the jet as hard partons. The instantaneous energy loss of light quark is identified with the energy flux from the point at $u(\tau, \sigma_\kappa) = u^\star$. We have shown that using the $\Delta u - prescription$ of jet, the light quark energy loss exhibit the Bragg peak again at late times in both static and expanding plasmas. This late time behavior of jet energy lost implies that after traveling substantial distances through the plasma, the thermalization of light quark ends with a large amount of energy transferring to the plasma which is similar to the energy loss rate of a fast charge particle moving through ordinary matter.

We considered a brick of plasma and calculated the nuclear modification factor of jet in both AdS-Sch and JP metric. We assumed that the temperature of plasma around 350 MeV at AdS-Sch metric and at initial time in JP metric. Our results show an aver suppression of jet of order of ten respect to the data. We investigated that it is because of the falling string setup at AdS space. In fact, R_{AA} of jet using the falling string in AdS_5 which is dual to jets in vacuum is not one, even though it is less than one. We introduced a renormalized R_{AA}^{renorm} by dividing the R_{AA} in the medium to the R_{AA} in the vacuum. Surprisingly, our ratio shows good agreement with the experimental data on the R_{AA}^{jet} of most central Pb-Pb collision at LHC figure 3 (b).

On the other hand, the light quark energy loss is highly depends on the initial conditions of falling string. The only way to determine the energy loss of a jet precisely in strongly coupled regime is solving the gravitational bulk-to-boundary problem. One can solve Einstein's equations for the perturbation in the $5d$ geometry due to the presence of the string and according to the bulk to boundary map interpret the near boundary behavior of the metric perturbation as the perturbation in the SYM energy-momentum tensor by the presence of jet which will be left for the future work.

References

[1] Adams J *et al.* (STAR Collaboration) 2005 *Nucl.Phys.* **A757** 102–183 (*Preprint* nucl-ex/0501009)

[2] Adcox K *et al.* (PHENIX Collaboration) 2005 *Nucl.Phys.* **A757** 184–283 (*Preprint* nucl-ex/0410003)

[3] Arsene I *et al.* (BRAHMS Collaboration) 2005 *Nucl.Phys.* **A757** 1–27 (*Preprint* nucl-ex/0410020)

[4] Back B, Baker M, Ballintijn M, Barton D, Becker B *et al.* 2005 *Nucl.Phys.* **A757** 28–101 (*Preprint* nucl-ex/0410022)

[5] Policastro G, Son D T and Starinets A O 2001 *Phys.Rev.Lett.* **87** 081601 (*Preprint* hep-th/0104066)

[6] Kovtun P, Son D T and Starinets A O 2005 *Phys.Rev.Lett.* **94** 111601 (*Preprint* hep-th/0405231)

[7] Maldacena J M 1998 *Adv.Theor.Math.Phys.* **2** 231–252 (*Preprint* hep-th/9711200)

[8] Witten E 1998 *Adv.Theor.Math.Phys.* **2** 253–291 (*Preprint* hep-th/9802150)

[9] Gubser S, Klebanov I R and Polyakov A M 1998 *Phys.Lett.* **B428** 105–114 (*Preprint* hep-th/9802109)

[10] Aharony O, Gubser S S, Maldacena J M, Ooguri H and Oz Y 2000 *Phys.Rept.* **323** 183–386 (*Preprint* hep-th/9905111)

[11] Casalderrey-Solana J, Liu H, Mateos D, Rajagopal K and Wiedemann U A 2011 (*Preprint* 1101.0618)

[12] Yin Z B (ALICE Collaboration) 2013 *Acta Phys.Polon.Supp.* **6** 479–484

[13] Aad G *et al.* (ATLAS Collaboration) 2010 *Phys.Rev.Lett.* **105** 252303 (*Preprint* 1011.6182)

[14] Chatrchyan S *et al.* (CMS Collaboration) 2011 *Phys.Rev.* **C84** 024906 (*Preprint* 1102.1957)

[15] Collaboration C (CMS Collaboration) 2012

[16] Karch A and Katz E 2002 *JHEP* **0206** 043 (*Preprint* hep-th/0205236)

[17] Herzog C, Karch A, Kovtun P, Kozcaz C and Yaffe L 2006 *JHEP* **0607** 013 (*Preprint* hep-th/0605158)

[18] Chesler P M, Jensen K, Karch A and Yaffe L G 2009 *Phys.Rev.* **D79** 125015 (*Preprint* 0810.1985)

[19] Janik R A and Peschanski R B 2006 *Phys.Rev.* **D73** 045013 (*Preprint* hep-th/0512162)

[20] Horowitz W A 2010 (*Preprint* 1011.4316)