Implementation of an offset-dipole magnetic field in a geometric pulsar emission code

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Introduction

- The first neutron star (NS) was discovered in 1967 by Jocelyn Bell (Hewish et al. 1968).

- Pulsars: compact, highly magnetized NSs, rotating at tremendous rates.

- Emits radiation across the electromagnetic spectrum.

- Focus on γ-ray pulsars (Vela); dominant emission mechanism curvature radiation (CR).
Geometric $\gamma$-ray pulsar models

OG (Cheng et al. 1986; Romani 1996)

$\Omega \cdot B = 0$

Light cylinder

Open magnetic field lines

Radio/Polar cap beam

Slot gap

Gamma rays

PC/PSPC (Daugherty & Harding 1996; HUM 2005)

SG (Arons 1983; Muslimov & Harding 2003/4; see Dyks 2003 for TPC)

Magnetic axis

Outer gap
Magnetic fields

Offset-dipole $B$-field

(Harding&Muslimov2011)

- E.g. due to retardation and asymmetric currents.
- In spherical coordinates, in the $(\hat{Z}' \parallel \mu)$ frame (Symmetric case):

$$B'(r', \theta', \phi') = \frac{\mu}{r'^3} \left[ \cos \theta' \hat{r}' + \frac{1}{2} (1 + a) \sin \theta' \hat{\theta}' - \epsilon \sin \theta' \cos \theta' \sin(\phi' - \phi_0) \hat{\phi}' \right]$$

- Offset characterized by parameter $\epsilon$:

$$a = \epsilon \cos(\phi' - \phi_0)$$

- A special case of the static-dipole $B$-field by setting $\epsilon = 0$. 
Problem statement

• We studied the effect of implementing an offset-dipole $B$-field on $\gamma$-ray LCs.

• Using a TPC emission geometry.

• Solve the particle equation of motion using the offset-dipole SG $E$-field.

• As an application, we compared our model LCs with Fermi LAT data for the bright Vela pulsar.
Implementation

• First transform the $B$-field from spherical to Cartesian coordinate system in $(\hat{z}' \parallel \mu)$ frame:

Transformation:

\[
\begin{align*}
B'(r', \theta', \phi') &= B'_r(r', \theta', \phi')\hat{r}' + B'_\theta(r', \theta', \phi')\hat{\theta}' + B'_\phi(r', \theta', \phi')\hat{\phi}' \\
B'(x', y', z') &= B'_x(x', y', z')\hat{x}' + B'_y(x', y', z')\hat{y}' + B'_z(x', y', z')\hat{z}'
\end{align*}
\]

• Rotate the Cartesian coordinate system from the $(\hat{z}' \parallel \mu)$ to the $(\hat{z} \parallel \Omega)$ frame through an angle $-\alpha$:

Rotation:

\[
B(x, y, z) = B_x(x, y, z)\hat{x} + B_y(x, y, z)\hat{y} + B_z(x, y, z)\hat{z}
\]
Finding the PC rim

- Expanding parameter space.

\[ \Delta r_{PC} \simeq R_{ns} \theta_0 \left[ 1 - \theta_0^\epsilon \right] \]
**E-field of the SG offset-dipole**

- **High-altitude** $E$-field (static-dipole + SG case ($x^\alpha$)) (Muslimov & Harding 04)

$$E_{\parallel,\text{high}} \approx -\frac{3}{8} \left(\frac{\Omega R}{c}\right)^3 \frac{B_0}{f(1)} \nu_{SG} x^\alpha \left\{ \left[ 1 + \frac{1}{3} \kappa \left( 5 - \frac{8}{\eta_c^3} \right) + 2 \frac{\eta}{\eta_{LC}} \right] \times \cos \alpha + \frac{3}{2} \theta_0 H(1) \sin \alpha \cos \phi' \right\} \left(1 - \xi^2_x \right)$$

- **Low-altitude** $E$-field (offset-dipole + SG case):

$$E_{\parallel,\text{low}} \approx -3 \varepsilon_0 \nu_{SG} x^\alpha \left\{ \frac{\kappa}{\eta^4} e_{1A} \cos \alpha + \frac{1}{4} \frac{\theta_0^{1+a}}{\eta} \times \left[ e_{2A} \cos \phi' + \frac{1}{4} \mu \kappa e_{3A} (2 \cos \phi_0 - \cos(2\phi' - \phi_0)) \right] \sin \alpha \right\} \left(1 - \xi^2_x \right)$$

- **Total** $E$-field for SG:

$$E_{\parallel,SG} \approx E_{\parallel,\text{low}} \exp \left( \frac{-(\eta - 1)}{(\eta_c - 1)} \right) + E_{\parallel,\text{high}}$$

- Need the $E$-field to solve $\gamma$ and the CR loss rate to correct the emissivity.
Particle transport equation

$$\eta_c = 1.4; \phi' = 1.60; \xi_* = 0$$

$$\dot{\gamma} = \dot{\gamma}_{\text{gain}} + \dot{\gamma}_{\text{loss}} = \frac{e E_{||}}{m c} - \frac{2e^2 \gamma^4}{3 \rho_{\text{curv}}^2 m c}$$
Phaseplots and LCs: Comparison of the static-dipole and offset-dipole fields

Used \((\alpha, \zeta, \epsilon) = (60^\circ, 85^\circ, 0.2)\). (Representative solutions)
Top panels: Constant emissivity for static-dipole
Bottom panels: Constant emissivity for offset-dipole
Phaseplots and LCs for the offset-dipole \( B \)-field

Used \( (\alpha, \zeta, \epsilon) = (60^\circ, 85^\circ, 0.2) \). (Representative solutions)

Top panels: Constant emissivity
Bottom panels: Solved for \( \gamma \)
Conclusions

• We observed an **offset PC** in comparison with the static dipole $B$-field (assuming constant emissivity).

• The phaseplot is **qualitatively different** when we include an $E$-field and solve $\gamma$, given the fact that $\gamma$ only becomes large enough to yield significant curvature radiation at large altitudes.

• Particles DO NOT attain the **radiation-reaction limit**, due to the low $E$-field.
Future work

• Extend the range of the **offset parameter** $\varepsilon$ for which we are able to solve the PC rim.

• **Solve** $\eta_c$ on each field line, instead of using a constant value where we match the $E$-field solutions.

• Need to **improve** our handling of the field lines where $E_\parallel$ changes sign with altitude.

• Lastly, search for **best-fit LCs**, thereby constraining Vela’s magnetic field and emission geometry ($\alpha, \zeta, \varepsilon$).
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“God’s love and kindness will shine upon us like the sun that rises in the sky.”

Luke 1:73