Four-body structure of tetra-neutron system

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Abstract. A recent experiment on the $^4\mathrm{He}(^8\mathrm{He}, ^8\mathrm{Be})4n$ reaction reported to have a resonant state at $E_R = 0.83\pm0.65\pm1.25$ MeV above the $4n$ breakup threshold and an upper limit of width $\Gamma = 2.6$ MeV [1, 2]. Motivated by the experiment, using a phenomenological $T = 3/2$ three neutron force and a realistic NN interaction, we solve *ab initio* solution of the $4n$ Schrödinger equation with the complex scaling method. We find that in order to generate narrow $4n$ resonant state a remarkably attractive $3N$ force in the $T = 3/2$ channel is necessary.

1. Introduction

Historically, we have been discussing on the possibility of detecting a four-neutron ($4n$) structure of bound or resonant state for the last fifty years in Refs. [3, 4, 5].

For this purpose, there were so many experimental effort to find bound or resonant states of $4n$ systems [6, 8, 7, 1, 2]. However, it was difficult to confirm the state.

Also, theoretically, some of authors tried to find some states in $4n$ system and however, they showed the impossibility to observe a bound $4n$ state [9, 10, 11, 12, 13, 14, 15, 17, 16].

A recent experiment on the $^4\mathrm{He}(^8\mathrm{He}, ^8\mathrm{Be})4n$ reaction generated an excess of $4n$ events with low energy in the final state. This observation has been associated with a possible $4n$ resonance with an estimated energy $E_R = 0.83 \pm 0.65 \pm 1.25$ MeV above the $4n$ breakup threshold and an upper limit of width $\Gamma = 2.6$ MeV [1, 2]. Due to the Low statistics, it however, was not possible to obtain the spin or parity for the corresponding state. It shoule be noted that a further analysis of the experimental results of Ref. [6] concluded that the observed (very few) events were also compatible with a $E_R = 0 - 2$ MeV tetraneutron resonance [8].

Motivated by this recent experimental data, we will focus on the strength of the $3N$ force in the total isospin $T = 3/2$ channel. The main purpose of this work is, thus, to investigate whether a resonant tetraneutron state is compatible with our knowledge of the nuclear interaction, in particular with the $T = 3/2$ $3N$ force. To this aim we will fix the NN force with a realistic interaction and introduce a simple isospin-dependent $3N$ force acting in both isospin channels. Its $T = 1/2$ part will be adjusted to describe some $A = 3$ and $A = 4$ nuclear states and the $T = 3/2$ part will be tuned until a ^4n resonance is manifested. The exploratory character of this study, as well as the final conclusions, justify the simplicity of the phenomenological force adopted here.

2. Method and interaction

We solve the following equation

$$H = T + \sum_{i<j} V_{ij}^{NN} + \sum_{i<j<k} V_{ijk}^{3N}, \tag{1}$$

where T is a four-particle kinetic-energy operator, V_{ij}^{NN} and V_{ijk}^{3N} are respectively two- and three-nucleon potentials. In this work we use the AV8$'$ version [18] of the NN potentials derived by the Argonne group. This model describes well the main properties of the NN system and it is relatively easy to handle. The main properties of this interaction are outlined in the benchmark calculation of the ^{4}He ground state [19].

This AV8 potential leads less bound of three nucleon systems, ^{3}H and ^{3}He. Therefore, we introduce a purely phenomenological $3N$ force which is assumed to be isospin-dependent and given by a sum of two Gaussian terms:

$$V_{ijk}^{3N} = \sum_{T=1/2}^{3/2} \sum_{n=1}^{2} W_n(T) e^{-(r_{ij}^2 + r_{jk}^2 + r_{ki}^2)/b_n^2} \mathcal{P}_{ijk}(T) \ . \tag{2}$$

where $\mathcal{P}_{ijk}(T)$ is a projection operator for the total three-nucleon isospin T state. The parameters of this force – its strength W_n and range b_n – are adjusted to reproduce the phenomenology.

In the case of $T = 1/2$ the parameters were fixed in Ref. [?] when studying the $J^\pi = 0^+$ states of ^{4}He nucleus. They are:

$$\begin{aligned} W_1(T=1/2) &= -2.04 \text{ MeV}, & b_1 &= 4.0 \text{ fm}, \\ W_2(T=1/2) &= +35.0 \text{ MeV}, & b_2 &= 0.75 \text{ fm}. \end{aligned} \tag{3}$$

Using this parameter set, in addition to the AV8$'$ and Coulomb interaction, one obtains the following binding energies: ^{3}H=8.41 (8.48) MeV, ^{3}He=7.74 (7.72) MeV, ^{4}He (0_1^+)= 28.44 (28.30) MeV and the excitation energy of ^{4}He(0_2^+)=20.25 (20.21) MeV [?], where the experimental values are shown in parentheses. Furthermore, this parameterization allows one to reproduce the observed transition form factor ^{4}He$(e,e')^4$He(0_2^+) (cf. Fig. 3 of Ref. [?]).

However, the contribution of isospin $T = 3/2$ configurations to the binding energies of the ^{3}H and ^{3}He nuclei are negligible small, then, we do not include this configuration in the calculation. On the other hand, $T = 3/2$ force contributes to the energy of $4n$ system mainly. Therefore, we introduce $T = 3/2$ force the following. We take the same function of $T = 1/2$ force, Namely,

$$\begin{aligned} W_1(T=3/2) &= \text{free}, & b_1 &= 4.0 \text{ fm}, \\ W_2(T=3/2) &= +35.0 \text{ MeV}, & b_2 &= 0.75 \text{ fm}. \end{aligned} \tag{4}$$

Here, attractive part of $T = 3/2$ force, W_1 is free parameter. We tune the W_1 so as to analyze the existence of $4n$ resonance state.

Here, we focus on the possible existence of the narrow resonant states of ^4n, which may enhance significantly ^4n production cross section. We employ the complex scaling method (CSM) in order to calculate resonance positions and widths. The CSM and its application to nuclear physics problems are extensively reviewed in Refs. [20, 21] and references therein. Using the CSM, the resonance energy (its position and width) is obtained as a stable complex eigenvalue of the complex scaled Schrödinger equation:

$$[H(\theta) - E(\theta)]\Psi_{JM,TT_z}(\theta) = 0 \;, \tag{5}$$

where $H(\theta)$ is obtained by making the radial transformation of the four-body Jacobi coordinates (Fig. 1) in H of Eq. (1) with respect to the common complex scaling angle of θ:

$$r_{\rm c} \to r_{\rm c}\, e^{i\theta},\; R_{\rm c} \to R_{\rm c}\, e^{i\theta},\; \rho_{\rm c} \to \rho_{\rm c}\, e^{i\theta} \;\; ({\rm c} = {\rm K}, {\rm H}). \tag{6}$$

According to the ABC theorem [22, 23], the eigenvalues of Eq. (5) may be separated into three groups:

For CSM, we use Gaussian Expansion Method (GEM) [24, 25, 26, 27, 28, 29].

In order to expand the system's wave function $\Psi_{JM,TT_z}(\theta)$ we employ the Gaussian basis functions of the same type as those used in the aforementioned references. An isospin rather than a neutron-proton (particle) basis is used to distinguish between different nuclear charge states ^4n, ^{4}H, ^{4}He and ^{4}Li. In the GEM approach, the four-nucleon wave function is written as a sum of the component functions in the K- and H-type Jacobi coordinates (Fig. **??**), employing the LS coupling scheme:

$$\Psi_{JM,TT_z}(\theta) = \sum_\alpha C_\alpha^{({\rm K})}(\theta)\Phi_\alpha^{({\rm K})} + \sum_\alpha C_\alpha^{(\mathbf{H})}(\theta)\Phi_\alpha^{({\rm H})}, \tag{7}$$

where the antisymmetrized four-body basis functions $\Phi_\alpha^{({\rm K})}$ and $\Phi_\alpha^{({\rm H})}$ (whose suffix JM, TT_z are dropped for simplicity) are described by

$$\begin{aligned}
\Phi_\alpha^{({\rm K})} \;&=\; \mathcal{A}\Big\{ \Big[[[\phi_{nl}^{({\rm K})}(\mathbf{r}_{\rm K})\varphi_{\nu\lambda}^{({\rm K})}(\boldsymbol{\rho}_{\rm K})]_\Lambda\, \psi_{NL}^{({\rm K})}(\mathbf{R}_{\rm K})]_I \\
&\quad \times [[\chi_s(12)\chi_{1/2}(3)]_{s'}\chi_{1/2}(4)]_S \Big]_{JM} \\
&\quad \times\, [[\eta_t(12)\eta_{1/2}(3)]_{t'}\eta_{1/2}(4)]_{TT_z} \Big\} \;, \qquad (8) \\
\Phi_\alpha^{({\rm H})} \;&=\; \mathcal{A}\Big\{ \Big[[[\phi_{nl}^{({\rm H})}(\mathbf{r}_{\rm H})\varphi_{\nu\lambda}^{({\rm H})}(\boldsymbol{\rho}_{\rm H})]_\Lambda\, \psi_{NL}^{({\rm H})}(\mathbf{R}_{\rm H})]_I \\
&\quad \times [\chi_s(12)\chi_{s'}(34)]_S \Big]_{JM} \\
&\quad \times\, [\eta_t(12)\eta_{t'}(34)]_{TT_z} \Big\} \;, \qquad (9)
\end{aligned}$$

with $\alpha \equiv \{nl, \nu\lambda, \Lambda, NL, I, s, s', S, t, t'\}$. $\mathcal{A}$ is the four-nucleon antisymmetrizer. The parity of the wave function is given by $\pi = (-)^{l+\lambda+L}$. The χ's and η's are the spin and isospin functions, respectively. The spatial basis functions $\phi_{nl}(\mathbf{r})$, $\varphi_{\nu\lambda}(\boldsymbol{\rho})$ and $\psi_{NL}(\mathbf{R})$ are taken to be Gaussians multiplied by spherical harmonics:

$$\begin{aligned}
&\phi_{nlm}(\mathbf{r}) = N_{nl}\, r^l\, e^{-(r/r_n)^2}\, Y_{lm}(\widehat{\mathbf{r}}) \;, \\
&\varphi_{\nu\lambda\mu}(\boldsymbol{\rho}) = N_{\nu\lambda}\, \rho^\lambda\, e^{-(\rho/\rho_\nu)^2}\, Y_{\lambda\mu}(\widehat{\boldsymbol{\rho}}) \;, \\
&\psi_{NLM}(\mathbf{R}) = N_{NL}\, R^L\, e^{-(R/R_N)^2}\, Y_{LM}(\widehat{\mathbf{R}}) \;.
\end{aligned} \tag{10}$$

Here, we take the Gaussian ranges lie in geometric progression:

$$\begin{aligned}
&r_n = r_1\, a^{n-1} &&(n = 1 - n_{\rm max}) \;, \\
&\rho_\nu = \rho_1\, \alpha^{\nu-1} &&(\nu = 1 - \nu_{\rm max}) \;, \\
&R_N = R_1\, A^{N-1} &&(N = 1 - N_{\rm max}) \;.
\end{aligned} \tag{11}$$

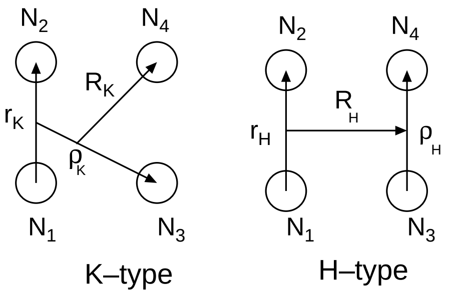

Figure 1. Four-nucleon Jacobi coordinates of K-type and H-type configurations.

2.1. Acknowledgments

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3. Results and Discussion

The recent experiment, providing evidence of the possible existence of a resonant tetraneutron, reported some structure at $E = 0.83 \pm 0.65$(stat.)± 1.25(sys.) MeV, measured with respect to the $4n$ breakup threshold with an estimated upper limit width $\Gamma = 2.6$ MeV [1, 2].

However, the experiment data did not report any spin-parity. Therefore, to indicate the spin-parity, e calculate a critical strength of the attractive $3N$ force $W_1(T = 3/2)$, defined by Eq. (2), to make different $4n$ states bound at $E = -1.07$ MeV. This energy corresponds to the lowest value compatible with the RIKEN data [2]. The calculated results, denoted as $W_1^{(0)}(T = 3/2)$, are given in Table 1.

Table 1. Critical strength $W_1^{(0)}(T = 3/2)$ (MeV) of the phenomenological $T = 3/2$ $3N$ force required to bind the $4n$ system at $E = -1.07$ MeV, the lower bound of the experimental value [2], for different states as well as the probability (%) of their four-body partial waves. The table is taken Ref.[30]

J^π	0^+	1^+	2^+	0^-	1^-	2^-
$W_1^{(0)}(T = \frac{3}{2})$	-36.14	-45.33	-38.05	-64.37	-61.74	-58.37
S-wave	93.8	0.42	0.04	0.07	0.08	0.08
P-wave	5.84	98.4	17.7	99.6	97.8	89.9
D-wave	0.30	1.08	82.1	0.33	2.07	9.23
F-wave	0.0	0.05	0.07	0.0	0.10	0.74

As one can see from this table, the smallest critical strength is $W_1^{(0)}(T = 3/2) = -36.14$ MeV and corresponds to the $J = 0^+$ state. It is consistent with a result reported in Ref. [16], where the tetraneutron binding was forced using an artificial four-body force in conjunction with the Reid93 nn potential. The next most favorable configuration is established to be a 2^+ state, which is bound by 1.07 MeV for a 3NF strength of $W_1^{(0)}(T = 3/2)$. The calculated level ordering is $J^\pi = 0^+, 2^+, 1^+, 2^-, 1^-, 0^-$. The level ordering calculated in Ref. [16] is $J^\pi = 0^+, 1^+, 1^-, 2^-, 0^-, 2^+$. These differences are related to the different binding mechanism of the four-nucleon force used in Ref. [16].

It should be noted that, in comparison with $W_1(T=1/2) = -2.04$ MeV established for the $T = 1/2$ $3N$ force, we need extremely strong $T = 3/2$ attractive term to make the $4n$ system weakly bound; when the $J = 0^+$ state is at $E = -1.07$ MeV with $W_1(T=3/2) = -36.14$ MeV, the expectation values of the kinetic energy, NN and $3N$ forces are $+67.0, -38.6$ and -29.5 MeV, respectively. We see that the expectation value of the $3N$ potential is almost as large as that of NN potential. The validity of this strongly attractive $T = 3/2$ $3N$ force will be discussed after presenting results for $4n$ resonant states.

After determining critical strength of $W_1(T = 3/2)$ required to bind the tetraneutron we gradually release this parameter letting the $4n$ system to move into the continuum. In this way we follow complex-energy trajectory of the ^4n resonances for $J = 0^+, 2^+$ and 2^- states. We remind the readers that these trajectories are controlled by a single parameter $W_1(T = 3/2)$, whereas other parameters remain fixed at the values given in Eq.(3) and Eq.(4).

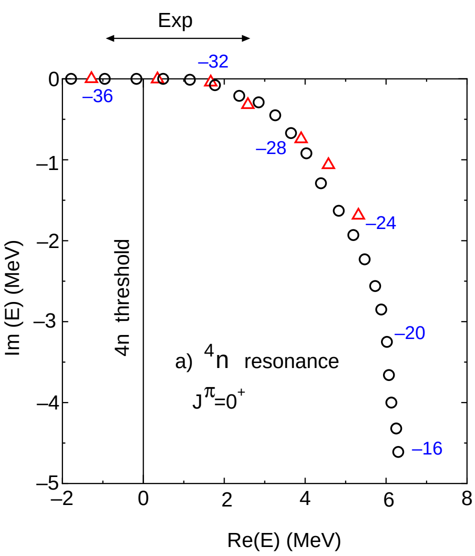

Figure 2. Tetraneutron resonance trajectory for the $J^\pi = 0^+$ state. The circles correspond to resonance positions for the AV8′ and the triangles INOY04'(is-m) potential [31]. Parameter $W_1(T = 3/2)$ of the additional 3NF was changed from -37 to -16 MeV in steps of 1 MeV for calculations based on AV8′ and from -36 to -24 MeV in steps of 2 MeV for INOY04'(is-m). To guide the eye the resonance region suggested by the measurement [2] is indicated by the arrow at the top. The figure is taken from Ref.[30]

As was expected, based on our experience from previous studies on multineutron systems [15, 17], tetraneutron trajectory turns out to be independent of the NN interaction model, provided this model reproduces well the NN scattering data. To illustrate this feature we have calculated the ^4n resonance trajectory for $J = 0^+$state using the INOY04(is-m) NN model [31]. This realistic interaction strongly differs from the other ones in that it contains a fully phenomenological and a strongly non-local short range part in addition to the typical local long range part based on one pion-exchange. Furthermore, this model reproduce the triton and alpha-particle binding energies without any 3NF contribution. Finally, P-waves of this interaction are slightly modified in order to match better the low energy scattering observables in the $3N$ system. Regardless of the mentioned qualitative differences of the INOY04(is-m) interaction with respect to the AV8′ one, the results for the ^4n resonance trajectory are qualitatively the same and demonstrate only minor quantitative differences. These results are displayed in Fig. 3a.

In order to prove or disprove the possible existence of the tetraneutron resonances, one sould consider the validity of the strongly attractive $3N$ force in the isospin $T = 3/2$ channel.

Here, it should be noted that $W_1(T = 1/2) = -2.04$ MeV to reproduce the energies of A=3 and 4 nuclei. As shown in Fig.2, in order to have resonant state, we need $W_1(T3/2) = -16$ MeV, which is still larger than $W_1(T = 1/2) = -2.04$ MeV.

Therefore, we investigate the consequences of a strongly attractive 3NF component in the isospin $T = 3/2$ channel. It is clear that such a force will have the most dramatic effect on nuclei with a large isospin number, i.e. neutron (or proton) rich ones as well as on infinite

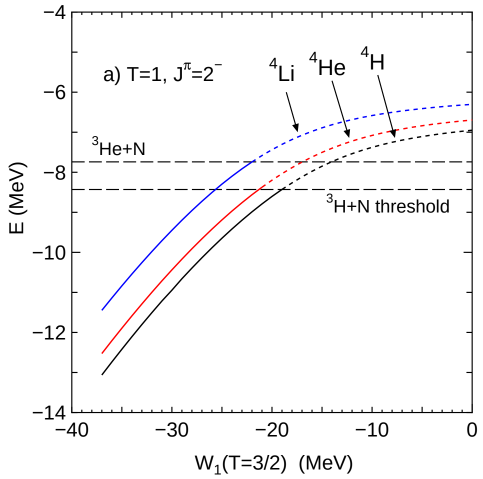

Figure 3. (color online) a) Calculated energies of the lowest $T = 1, J^\pi = 2^-$ states in ^{4}H, ^{4}He and ^{4}Li with respect to the strength of $T = 3/2$ $3N$ force, $W_1(T = 3/2)$. The horizontal dashed lines show the $^3\text{He}+N$ and $^3\text{H}+N$ thresholds. The solid curve below the corresponding threshold indicates the bound state, while the dotted curve above the threshold stands approximately for the resonance obtained by the diagonalization of $H(\theta = 0)$ with the L^2 basis functions. The figure is taken from Ref.[30].

neutron matter. Nevertheless this includes mostly nuclei with $A > 4$, not within our current scope. Still we will investigate the effect on other well known states of $A = 4$ nuclei, namely negative parity, isospin $T = 1$ states of ^{4}H, ^{4}He and ^{4}Li. These structures represent broad resonances [32] (see Table 2) established in nuclear collision experiments. Calculated energies of those states are shown in Fig. 3 with respect to increasing $W_1(T = 3/2)$ from -37 to 0 MeV. The solid curve below the corresponding threshold indicates a bound state, whereas the dotted curve above the threshold stands approximately for the resonant state obtained within a bound state approximation.

As demonstrated in Fig. 3, values of an attractive 3NF term in the range of $W_1(T=3/2) \simeq -36$ to -30 MeV, which is compatible with a reported ^4n resonance region in Ref. [2], gives rise to the appearance of bound $J = 2^-$ state in ^{4}H, ^{4}He($T = 1$) and ^{4}Li nuclei. Unlike observed in the collision experiments, these states become stable with respect to the ^{3}H (^{3}He) + N decay channels. This means that the present phenomenological $W_1(T = 3/2)$ is too attractive to reproduce low-lying states of ^{4}H, ^{4}He ($T = 1$) and ^{4}Li.

In contrast, it is interesting to see the energy of $4n$ system when we have just unbound states for ^{4}H, ^{4}He ($T = 1$) and ^{4}Li in Fig. 2. Use of $W_1(T = 3/2) = -19$ MeV gives rise to an unbound state with $J = 2^-$ in ^{4}H with respect to disintegration into $^3\text{H}+N$. However, using this strength of $W_1(T = 3/2)$, we have already a very broad ^4n resonant state at $\text{Re}(E_{\text{res}}) = 6$ MeV with $\Gamma = 7.5$ MeV, see Fig. 3a, which is inconsistent with the recent experimental claim [2] of a resonant ^4n. Moreover the value of $W_1(T = 3/2)$ that reproduces the observed broad resonance

Table 2. Observed energies E_R and widths Γ (in MeV) of the $J^\pi = 2^-_1$ and 1^-_1 states in ^{4}H, ^{4}He ($T = 1$) and ^{4}Li, E_R being measured from the $^3\text{H}+n$, $^3\text{H}+p$ and $^3\text{He}+p$ thresholds, respectively [32].

J^π	^{4}H $E_R\,(\Gamma)$	^{4}He ($T = 1$) $E_R\,(\Gamma)$	^{4}Li $E_R\,(\Gamma)$
2^-_1	3.19 (5.42)	3.52 (5.01)	4.07 (6.03)
1^-_1	3.50 (6.73)	3.83 (6.20)	4.39 (7.35)

data for the 2^- state in ^{4}H should be much less attractive than -19 MeV.

4. Conclusions

Motivated by the recent experimental claim regarding the possible existence of observable tetraneutron ^4n [1, 2] states, we have investigated the possibility that the $4n$ system exhibits a near-threshold bound or narrow resonant state compatible with the reported data.

When studying the tetraneutron sensitivity to the ingredients of the nuclear interaction, we have concluded that this system is not very sensitive to "experimentally allowed" modifications in NN interaction. The most natural way to enhance a tetraneutron system near the threshold is through an additional attractive isospin $T = 3/2$ term in the three-body force. We have examined the consistency of the nuclear Hamiltonian modifications, required to produce observable tetraneutron states, with other four-nucleon observables, like the low-lying $T = 1$ states in ^{4}H, ^{4}He and ^{4}Li.

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