Short Path Length Energy Loss in the Quark Gluon Plasma

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Abstract. High Energy Particle Physics collider experiments at the Relativistic Heavy Ion Collider (RHIC) in the USA and the Large Hadron Collider (LHC) in Geneva, Switzerland, are probing the most fundamental properties of matter by accelerating a range of particles, from protons to Lead nuclei, to relativistic speeds and causing them to collide. The decay products of these violent collisions can be studied in detail and have revealed that a new state of matter in which the constituents of nucleons, quarks and gluons, exist in a deconfined state, creating what appears to be a perfect fluid called the Quark Gluon Plasma (QGP). The QGP only exists for a few femto seconds and is therefore extremely difficult to characterize. The manner in which a highly energetic particle loses energy as it traverses the QGP has proven to be an effective probe of the QGP, but recent results in smaller colliding systems such as proton-lead (pPb) have brought into question our understanding of perturbative Quantum Chromodynamical (pQCD) descriptions of energy loss, particularly in small systems of QGP. We present a short separation distance correction to the well-known (static scattering centre) DGLV (Djordjevic, Gyulassy, Levai, Vitev) pQCD energy loss calculation, revealing a number of shortcomings and problematic assumptions. We also investigate the feasibility of a similar small system correction in the (dynamical scattering centre) thermal field theory formalism.

1. Introduction

For decades now, particle physicists have explored the fundamental properties of the universe by colliding various nucleii at ever increasing energies in order to probe their constituent particles - quarks and gluons. In recent years, evidence has arisen of the production of a new state of matter, the Quark - Gluon Plasma (QGP), in which the quarks contained within nucleons and other hadrons become deconfined [1]. The QGP offers unique insight into the structure of the most fundamental building blocks of matter as well as an opportunity to study the physics of many body non-abelian gauge theories.

High momentum particles produced along with the QGP can be used as tomographic probes in a phenomenon known as jet-quenching, in which high-momentum particles lose energy as they traverse the QGP. The physics of jet-quenching is complex, but the same phenomenon of energy loss affects all the particles of the QGP, resulting in a reduction of the cross section of charged hadrons seen in nucleus-nucleus (AA) collisions as compared to proton-proton (pp) collisions (if scaled appropriately). Such studies have met with great success in AA collisions, leading to the rise of a number of perturbative quantum chromodynamical (pQCD) energy loss formalisms that have evolved to provide a very detailed description of the energy-loss mechanisms [2, 3, 4, 5] in the QGP.

However, new analyses of experimental data [6, 7, 8, 9, 10, 11] have shown that there is evidence for collective behaviour even in small colliding systems previously thought to be too small to create a QGP. If a QGP does exist in these smaller colliding systems, a clear understanding of the energy loss mechanisms in small systems is of crucial importance, because current energy loss models rely heavily on the assumption that the system is large compared to the debye screened length of the scattering centres. Mathematically, relaxing the assumption that the system is large in the GLV (Gyulassy, Levai, Vitev) formalism amounts to relaxing the assumption that the distance between scattering and radiation is large compared to the inverse debye mass. That is, the present calculation is an *all* separation distance generalization of the DGLV (Djordjevic, GLV) [12] energy loss for a massive quark traversing a static medium.

The large separation distance assumption led to a mathematical simplification through the exponential suppression of certain terms at the amplitude level. We present here the energy loss formula obtained by retaining terms that are exponentially suppressed due to $1/\mu_D \ll \Delta z$. Alarmingly, upon numerical evaluation of the energy loss formula, we find that the correction term dominates at high (~ 100 GeV) parton energies.

2. Setup

For the present calculation we follow precisely the setup of the DGLV calulation [12]. The details of the current calculation can be found in [13], but will not be discussed at length. For clarity, we treat the transverse momentum eikonal parton produced at an initial point $(t_0, z_0, \mathbf{x}_0)$ inside a finite QGP, where we have used $\mathbf{p}$ to mean transverse 2D vectors, $\vec{\mathbf{p}} = (p_z, \mathbf{p})$ for 3D vectors and $p = (p^0, \vec{\mathbf{p}}) = [p^0 + p^z, p^0 - p^z, \mathbf{p}]$ for four vectors in Minkowski and light cone coordinates respectively. As in the DGLV calculation, we consider the target to be a Gyulassy-Wang Debye screened potential [14] with Fourier and color structure given by

$$\begin{aligned} V_n &= V(\vec{\mathbf{q}}_n) e^{-i\vec{q}_n \cdot \vec{x}_n} \\ &= 2\pi\delta(q^0) v(\mathbf{q}_n, q_n^z) e^{-i\vec{q}_n \cdot \vec{x}_n} T_{a_n}(R) \otimes T_{a_n}(n). \end{aligned} \tag{1}$$

The color exchanges are handled using the applicable $SU(N_c)$ generator $T_a(n)$ in the d_n dimensional representation of the target or $T_a(R)$ in the d_R dimensional representation of the p_T parent parton.

In light cone coordinates the momenta of the emitted gluon, the final p_T parton, and the exchanged medium Debye quasiparticle are

$$\begin{aligned} k &= \left[xP^+, \frac{m_g^2 + \mathbf{k}^2}{xP^+}, \mathbf{k} \right], \\ p &= \left[(1-x)P^+, \frac{M^2 + \mathbf{k}^2}{(1-x)P^+}, -\mathbf{k} \right], \\ q &= [q^+, q^-, \mathbf{q}], \end{aligned} \tag{2}$$

where the initially produced p_T particle of mass M has large momentum $E^+ = P^+ = 2E$ and negligible other momentum components. Notice that we include the Ter-Mikayelian plasmon effect with an effective emitted gluon mass m_g [12, 15]. See Fig. 1 for a visualization of these momenta.

A shorthand for energy ratios will prove useful notationally. Following [12] we define $\omega \approx xE^+/2 = xP^+/2$, $\omega_0 \equiv \mathbf{k}^2/2\omega$, $\omega_i \equiv (\mathbf{k} - \mathbf{q}_i)^2/2\omega$, $\omega_{(ij)} \equiv (\mathbf{k} - \mathbf{q}_i - \mathbf{q}_j)^2/2\omega$, and $\tilde{\omega}_m \equiv \left(m_g^2 + M^2x^2\right)/2\omega$.

We will also make the following assumptions: 1) the eikonal, or high energy, approximation, for which E^+ is the largest energy scale of the problem; 2) the soft (radiation) approximation

$x \ll 1$; 3) collinearity, $k^+ \gg k^-$; 4) that the impact parameter varies over a large transverse area; and, most crucially for this letter, 5) the large formation time assumption $\omega_i \ll \mu_i$, where $\mu_i^2 \equiv \mu^2 + \mathbf{q}_i^2$.

Note that the above approximations, in addition to allowing us to systematically drop terms that are small, permit us to 1) (eikonal) ignore the spin of the p_T parton; 2) (soft) assume the source current for the parent parton varies slowly with momentum $J(p-q+k) \approx J(p+k) \approx J(p)$; 3) (collinearity) complete a separation of energy scales

$$E^+ \gg k^+ \gg k^- \equiv \omega_0 \sim \omega_{(i\ldots j)} \gg \frac{(\mathbf{p}+\mathbf{k})^2}{P^+}; \tag{3}$$

and 4) take the ensemble average over the phase factors, which become $\langle e^{-i(\mathbf{q}-\mathbf{q}')\cdot\mathbf{b}}\rangle = \frac{(2\pi)^2}{A_\perp}\delta^2(\mathbf{q}-\mathbf{q}')$.

In the original DGLV calculations [12], the large formation time played only a minor role. However, when considering short separation distances between the scattering centers, the large formation time assumption naturally increases in importance.

With the above approximations, we reevaluated the 10 diagrams contributing to the $N = 1$ in opacity energy loss amplitude [12] without the additional simplification of the large separation distance $\Delta z \gg 1/\mu$ assumption.

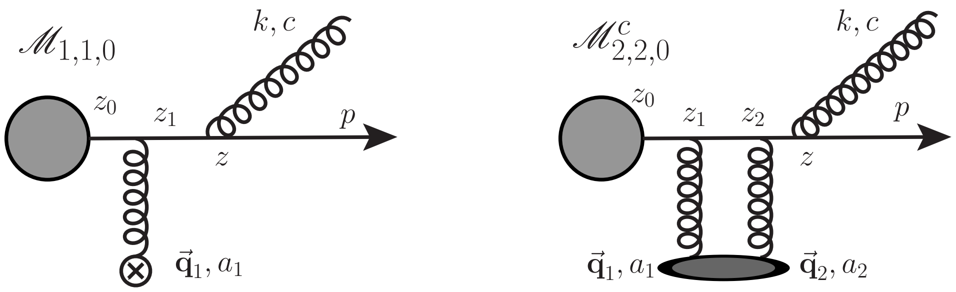


Figure 1: $\mathscr{M}_{1,1,0}$ (left hand panel) and $\mathscr{M}^c_{2,2,0}$ (right hand panel) are the only two diagrams that have non-zero short separation distance corrections in the large formation time limit. $\mathscr{M}^c_{2,2,0}$ is the double Born contact diagram, corresponding to the second term in the Dyson series in which two gluons are exchanged with the single scattering center.

3. Calculation and Results

The $N = 1$ in opacity energy loss derivation that was originally performed by DGLV evaluated 10 diagrams and utilized the large separation distance approximation $\Delta z \gg 1/\mu$ to neglect terms proportional to $\exp(-\mu\Delta z)$. Although we retained such terms in our reevaluation of the 10 diagrams in question, the large radiated gluon formation time approximation, $\omega_i \ll \mu_i$, allowed for a further simplification. As a result, only 2 of the 18 new small distance correction pole contributions are suppressed. We show the two diagrams with non-zero contributions at the amplitude level $\mathscr{M}_{1,1,0}$ and $\mathscr{M}^c_{2,2,0}$ in the large formation time approximation in Fig. 1.

The full result for these two amplitudes under our approximation scheme is then

$$\begin{aligned}
\mathscr{M}_{1,1,0} &\approx -J(p)e^{ipx_0}2gT_{a_1}ca_1\int\frac{d^2\mathbf{q}_1}{(2\pi)^2}v(0,\mathbf{q}_1)e^{-i\mathbf{q}_1\cdot\mathbf{b}_1}\\
&\times\frac{\mathbf{k}\cdot\boldsymbol{\epsilon}}{\mathbf{k}^2+m_g^2+x^2M^2}\left[e^{i(\omega_0+\tilde{\omega}_m)(z_1-z_0)}-\frac{1}{2}e^{-\mu_1(z_1-z_0)}\right] \qquad (4)\\
\mathscr{M}^{c}_{2,2,0} &\approx J(p)e^{i(p+k)x_0}\int\frac{d^2\mathbf{q}_1}{(2\pi)^2}\int\frac{d^2\mathbf{q}_2}{(2\pi)^2}e^{-i(\mathbf{q}_1+\mathbf{q}_2)\cdot\mathbf{b}_1}\\
&\times igT_{a_2}T_{a_1}ca_2a_1v(0,\mathbf{q}_1)v(0,\mathbf{q}_1)\frac{\mathbf{k}\cdot\boldsymbol{\epsilon}}{\mathbf{k}^2+m_g^2+x^2M^2}\\
&\times\left[e^{i(\omega_0+\tilde{\omega}_m)(z_1-z_0)}+e^{-\mu_1(z_1-z_0)}\left(1-\frac{\mu_1e^{-\mu_2(z_1-z_0)}}{2(\mu_1+\mu_2)}\right)\right]. \qquad (5)
\end{aligned}$$

The double differential single inclusive gluon emission distribution is given by [12]

$$d^3N_g^{(1)}d^3N_J=\frac{d^3\vec{\mathbf{p}}}{(2\pi)^32p^0}\frac{d^3\vec{\mathbf{k}}}{(2\pi)^32\omega}\times\left(\frac{1}{d_T}\,\mathrm{Tr}\langle|\mathscr{M}_1|^2\rangle+\frac{2}{d_T}\Re\,\mathrm{Tr}\langle\mathscr{M}_0^*\mathscr{M}_2\rangle\right), \qquad (6)$$

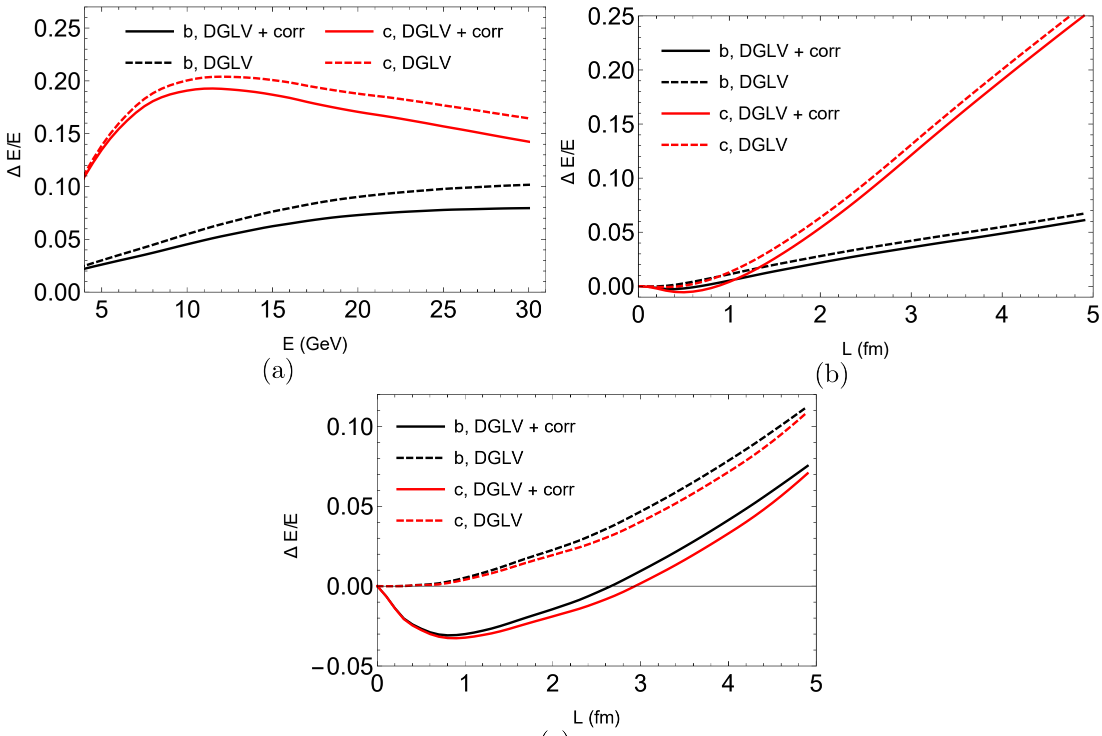
from which the energy loss, given by the energy-weighted integral over the gluon emission distribution $\Delta E = E\int dx\, xdN_g/dx$, can be calculated from the amplitudes.

The main analytic result of our letter is then the $N=1$ first order in opacity small distance generalization of the DGLV induced energy loss of a high-p_T parton in a QGP:

$$\begin{aligned}
\Delta E^{(1)}_{ind} &= \frac{C_R\alpha_sLE}{\pi\lambda_g}\int dx\int\frac{d^2\mathbf{q}_1}{\pi}\frac{\mu^2}{(\mu^2+\mathbf{q}_1^2)^2}\int\frac{d^2\mathbf{k}}{\pi}\\
&\times\int d\Delta z\bar{\rho}(\Delta z)\Bigg[-\frac{2\big(1-\cos\{(\omega_1+\tilde{\omega}_m)\Delta z\}\big)}{(\mathbf{k}-\mathbf{q}_1)^2+m_g^2+x^2M^2}\left(\frac{(\mathbf{k}-\mathbf{q}_1)\cdot\mathbf{k}}{\mathbf{k}^2+m_g^2+x^2M^2}-\frac{(\mathbf{k}-\mathbf{q}_1)^2}{(\mathbf{k}-\mathbf{q}_1)^2+m_g^2+x^2M^2}\right)\\
&+\frac{1}{2}e^{-\mu_1\Delta z}\Bigg\{\left(\frac{\mathbf{k}}{\mathbf{k}^2+m_g^2+x^2M^2}\right)^2\left(1-\frac{2C_R}{C_A}\right)\Big(1-\cos\{(\omega_0+\tilde{\omega}_m)\Delta z\}\Big)\\
&+\frac{\mathbf{k}\cdot(\mathbf{k}-\mathbf{q}_1)}{\big(\mathbf{k}^2+m_g^2+x^2M^2\big)\big((\mathbf{k}-\mathbf{q}_1)^2+m_g^2+x^2M^2\big)}\big(\cos\{(\omega_0+\tilde{\omega}_m)\Delta z\}-\cos\{(\omega_0-\omega_1)\Delta z\}\big)\Bigg\}\Bigg]. \qquad (7)
\end{aligned}$$

The small separation distance correction shown in the last four lines of Eq. 7 has the properties we expect: 1) the correction goes to zero as the separation distance becomes large, $\Delta z\to\infty$ (or, equivalently, as the Debye screening length goes to 0, $\mu\to\infty$) and 2) the correction term vanishes as the separation distance vanishes, $\Delta z\to 0$, due to the destructive interference of the LPM effect.

We investigated the importance of the short separation distance correction term in Eq. 7 numerically to produce Figs. 2a, 2b and 2c. The numerical results use the same values as [12]: $\mu = 0.5$ GeV, $\lambda_{mfp} = 1$ fm, $C_R = 4/3$, $C_A = 3$, $\alpha_s = 0.3$, $m_{charm} = 1.3$ GeV and $m_{bottom} = 4.75$ GeV, and the QCD analogue of the Ter-Mikayelian plasmon effect was taken into account by setting $m_{gluon} = \mu/\sqrt{2}$. As in [15], kinematic upper limits were used for the momentum integrals such that $0 \le k \le 2x(1-x)E$ and $0 \le q \le \sqrt{3E\mu}$. This choice of k_{max} guarantees that the final momentum of the parent parton is collinear to the initial momentum of the parent parton and that the momentum of the emitted gluon is collinear to the momentum of the parent parton.



(a) (b) (c)

Figure 2: Fractional energy loss of charm and bottom quarks in a QGP with $\mu = 0.5$ GeV and $\lambda_{mfp} = 1$ fm for (a) fixed path length $L = 4$ fm, (b) fixed energy $E = 10$ GeV, and (c) fixed energy $E = 100$ GeV. In the figures, "DGLV" dashed curves are computed from the original $N = 1$ in opacity large separation distance DGLV formula while "DGLV + corr" solid lines are from our all separation distance generalization of the $N = 1$ DGLV result, Eq. 7.

The fraction of momentum carried away by the radiated gluon, x, was integrated over from 0 to 1. The distribution of scattering centers, although originally assumed to be exponential in [12] in order to account for the rapidly expanding medium, was here assumed to have the form of a unit step function, since an exponential distribution biases towards short separation distance scattering, lending excessive weight to contributions from short separation distance terms. The choice of a step function distribution reduces the effect of the correction terms by $\sim 10\%$ at low (~ 10 GeV) parton energies and $\sim 50\%$ at higher (~ 100 GeV) energies as compared to results using an exponential distribution.

In Fig. 2a we show the fractional energy loss of charm and bottom quarks of varying energy propagating through a 4 fm long static QGP brick. We note that the present calculation amounts to an energy *gain* compared to the DGLV result.

In Fig. 2b we plot the fractional energy loss of charm and bottom quarks of energy $E = 10$ GeV for path lengths up to 5 fm. The integration over all separation distances (even in the DGLV calculation) results in the non-zero effect seen here even for large path lengths and is therefore not surprising, albeit unexpected.

Our most important result is shown in Fig. 2c, which presents the fractional energy loss of 100 GeV charm and bottom quarks propagating up to 5 fm through a QGP. The small distance "correction" term dominates over the leading DGLV result for the first ~ 3 fm of the path.

4. Conclusions

An asymptotic analysis of Eq. 7, following [16], reveals that the correction term dominates because it scales as $E \log E$ while the large separation distance term grows much slower, as $\log E$. This dominance is difficult to reconcile with experimental data and suggests that the large formation time assumption is invalid in the DGLV approach

Since all energy loss formalisms, GLV, BDMPS-Z-ASW, AMY, and HT (see [1] and references therein) exploit the large formation time approximation, we are faced with a need to reassess the applicability of the large formation time assumption in any description of energy loss. Deriving expressions in the other formalisms that do not rely on either the collinear or large formation time approximations might also be formidable. Lastly, the factorization of the production of the hard parton from the scattering will demand careful consideration in finite media, particularly since the behaviour of a Debye screened scattering centre near the edge of a thermalized medium is unclear.

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