# Simulaser, a graphical laser simulator based on Matlab Simulink

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**Abstract.** We present a single-element plane-wave laser rate equation model and its implementation as a graphical laser simulation library using Matlab Simulink. Simulink's graphical interface and vector capabilities provide a unique layer of abstraction for numerical integration particularly well suited for laser simulation. Numerical integration of the coupled laser rate equations in Simulink enables simulation of laser dynamics for a range of intricate and novel laser cavity designs. In particular, simulation and real-world results will be compared for a dual crystal Ho:YLF oscillator with one of its two birefringent crystals rotated by 90° about the optical axis for efficient absorption of the incident unpolarised pump light.

## 1. Introduction

Solid-state laser gain dynamics can be fairly accurately modelled using coupled rate equations for inversion population and cavity photon density which is used extensively in the design and optimisation of solid-state lasers. The basic theory and derivation of both three and four level laser rate equations have been covered in detail by Siegman [1] and Koechner [2], with several expanded models presented in literature [3, 4]. As general analytic solutions don't exist, numerical integration and simulation are typically done using script-based computer programming like Matlab [5] or LAS-CAD [6].

In previous work [7] we presented a single-element plane-wave laser rate equation model with numerical simulations done using Matlab's ordinary differential equation solvers. Customising these Matlab scripts for multiple laser systems were complicated and time-consuming, and limited in the complexity of the laser cavity design. This same set of laser rate equations have been adapted and implemented in Matlab Simulink, providing a graphical programming environment with vector support to enable simulation of a variety of complicated laser system designs. In this proceeding we present the adapted rate equation model, it's implementation as a Simulink laser library and a comparison between simulated and measured results obtained from a dual crystal Ho:YLF oscillator.

## 2. Laser rate equation model

Both four and three level solid-state laser systems can be approximated by a simplified two energy level scheme, given that the non-radiative transitions between different energy levels occur much faster than the primary laser transition, and that effective absorption and emission cross-sections are used to account for stark level splitting and level degeneracy. The upper laser level population density $N_2(t)$ and the cavity photon density $\Omega(t)$ as functions of time are then represented by the following coupled rate equations:

$$\frac{d\Omega}{dt} = \frac{cl_{crystal}}{l_o}(\Omega+1)\left(\sigma_{laser}^{em}N_2(t) - \sigma_{laser}^{abs}N_1(t)\right) - \frac{\Omega}{\tau_c} \qquad (1)$$

$$\frac{dN_2}{dt} = R_P - \frac{c\Omega(t)l_{crystal}}{V_{crystal}l_0}\left(\sigma_{laser}^{em}N_2 - \sigma_{laser}^{abs}N_1\right) - \frac{N_2}{\tau_2} \qquad (2)$$

$$\text{with} \qquad R_P = \frac{\eta_{pump}P_{inc}(1 - e^{-\left(\sigma_{pump}^{abs}N_1 - \sigma_{pump}^{em}N_2\right)l_{crystal}})}{V_{crystal}h\nu_{pump}} \qquad (3)$$

where $R_p$ is the pump rate, $N_1$ is the ground manifold population density, $\sigma^{em}$ and $\sigma^{abs}$ are the effective emission and absorption cross-sections (at either the pump or laser wavelength depending on the subscript), $\tau_2$ is the upper laser level lifetime, $\eta_p$ the pump efficiency and $V_{crystal}$, $l_{crystal}$ and $l_0$ are the crystal volume, crystal length and corrected optical cavity length respectively. Furthermore, define $c$ as the speed of light, $\tau_c$ as the cavity lifetime (equal to $2l_0/c\delta$ with $\delta$ the logarithmic round-trip loss including the output coupler) and $\nu_{pump}$ as the frequency of the pump light.

The coupled differential equations were carefully implemented in Simulink as shown in detailed block diagram form in Figure 1 and Figure 2. Simulink provides basic building blocks, such as a numerical integrator block to enable the construction of mathematical models. For readers unfamiliar with Simulink it helps to start at the integrator block and follow the signal wires to realise the block form version of a differential equation. The two differential equation block diagrams were each encapsulated in a Simulink subsystem (i.e. one gain medium block and one cavity mode block, as shown in Figure 3) with signals for simulated and spontaneous emission, and cavity photons connecting between them. Factors and terms were carefully grouped in order to confine input variables to either of the two subsystems (i.e. cavity length is only referenced in the "cavity mode" block). This was done in order to achieve logical separation and allow easy construction of novel resonator cavities containing multiple gain media blocks and cavity modes. Additionally, Simulink's intrinsic vector-based signals were exploited to further this goal, essentially implementing multiple-coupled differential equations in the form:

$$\frac{d\Omega_i}{dt} = \frac{cl_{crystal,m}}{l_{oi}}(\Omega_i+1)\left(\sigma_{laser,i}^{em}N_{2,m}(t) - \sigma_{laser,i}^{abs}N_{1,m}(t)\right) - \frac{\Omega_i}{\tau_{ci}} \qquad (4)$$

$$\frac{dN_{2,m}}{dt} = R_{P,m} - \sum_j \frac{c\Omega_j(t)l_{crystal,m}}{V_{crystal,m}l_{0j}}\left(\sigma_{laser,j}^{em}N_{2,m} - \sigma_{laser,j}^{abs}N_{1,m}\right) - \frac{N_{2,m}}{\tau_{2,m}} \qquad (5)$$

Finally, the pump rate $R_p$ was also vectorised to allow orthogonal polarisation components to be propagated throughout the subsystem blocks.
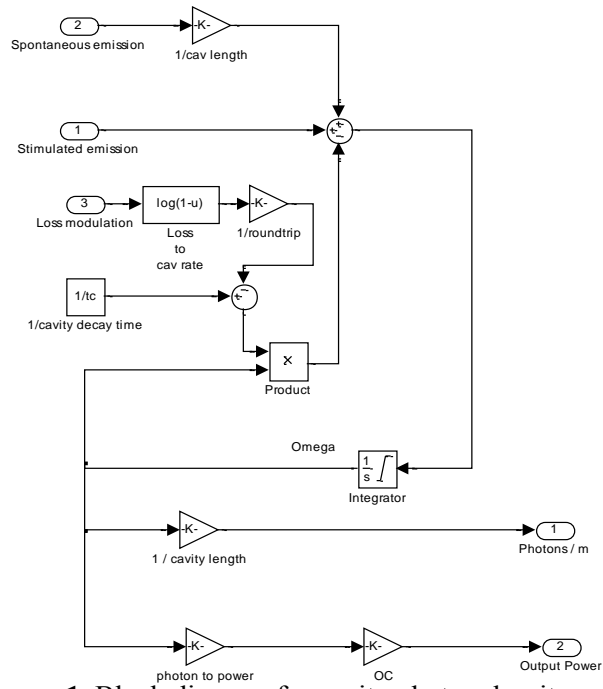
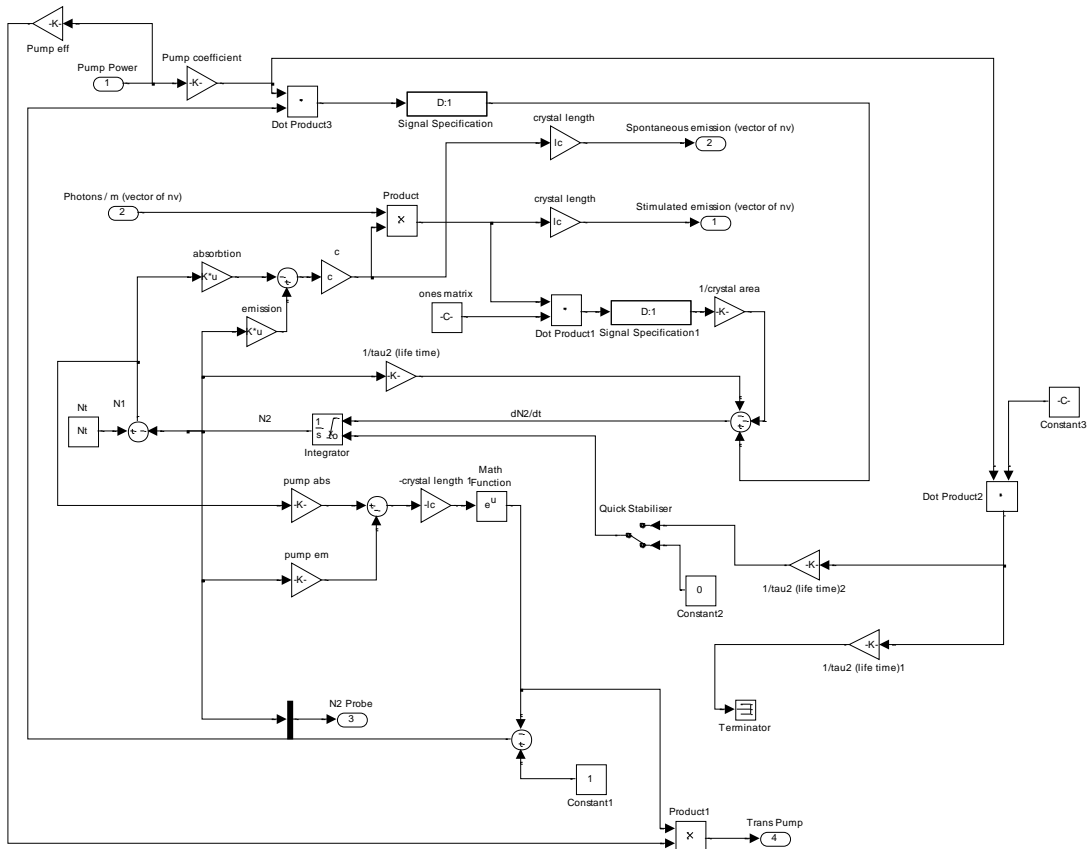**Figure 1.** Block diagram for cavity photon density as per equation (4).



**Figure 2.** Block diagram for upper laser level population density as per equation (5).

## 3. Model validation

The combined subsystem blocks have been organised into a laser simulation Simulink library ("Simulaser") as shown in Figure 3, together with additional blocks for diagnostics and Q-switching. The Simulaser library has been used to simulate several different laser designs at the CSIR National Laser Centre, though mostly classical single crystal, single cavity designs. For validation of the vectorised laser rate equation model, a recently developed dual-crystal Ho:YLF pulsed laser system [8] was simulated and used to determine the optimum mode size in the crystals.
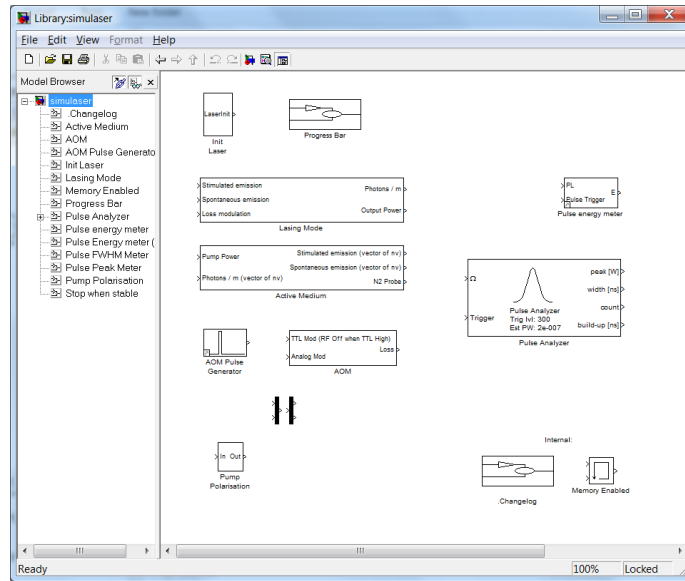


**Figure 3.** *Simulaser* block diagram library implemented in Matlab Simulink.

The Ho:YLF based laser was designed for high efficiency and required optimal absorption of the 84W unpolarised Tm-fibre laser pump light. Since Ho:YLF crystals are naturally birefringent (c-axis cut perpendicular to the optical axis), strong polarisation dependant absorption occurs which effectively polarises the transmitted pump beam after absorption in a single crystal. A second crystal with its c-axis rotated by 90° with respect to the first crystal strongly absorbs the remaining pump light making efficient use of the unpolarised pump. The Simulaser block diagram implementation of this dual crystal laser is shown in Figure 4. The 90° rotation of the gain media was achieved by rotating both absorption and emission cross-sections vectors supplied in the properties of the second crystal.
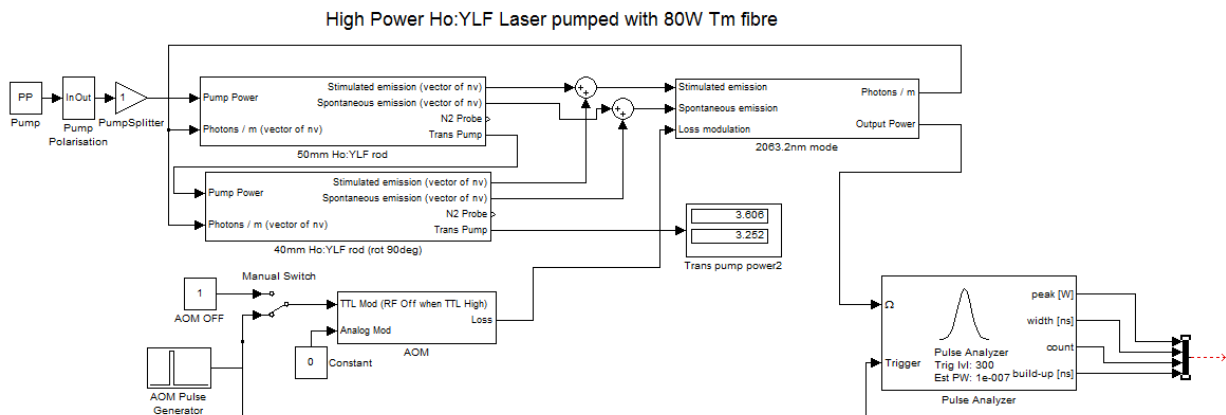


**Figure 4.** Dual-crystal Ho:YLF pulsed laser design implemented in Simulaser

## 4. Results and discussion

The simulation results for laser output power as a function of (a) mode radius in the crystals and (b) input pump power are shown in Figure 5. The maximum laser output power is achieved for mode sizes of 300 μm radius and below. However, below 300 μm the risk for optical damage increases considerably with less than 5% gain in output power. As such, the actual laser system was developed with a 300 μm pump and laser mode radius in both crystals. The laser was pulsed at 50 kHz and achieved a maximum of 45.1W average power with a slope efficiency of more than 66% (vs. incident pump power). The experimentally measured data of the laser in pulsed mode is also shown in Figure 5(b). The simulation results were in very good agreement with measured data, especially at high pump powers. The simulated continuous wave (CW) slope results provide an upper bound for pulsed performance. Simulated transmitted pump power through both crystals at full pump power equaled the experimentally measured transmitted power of 7.1 W.
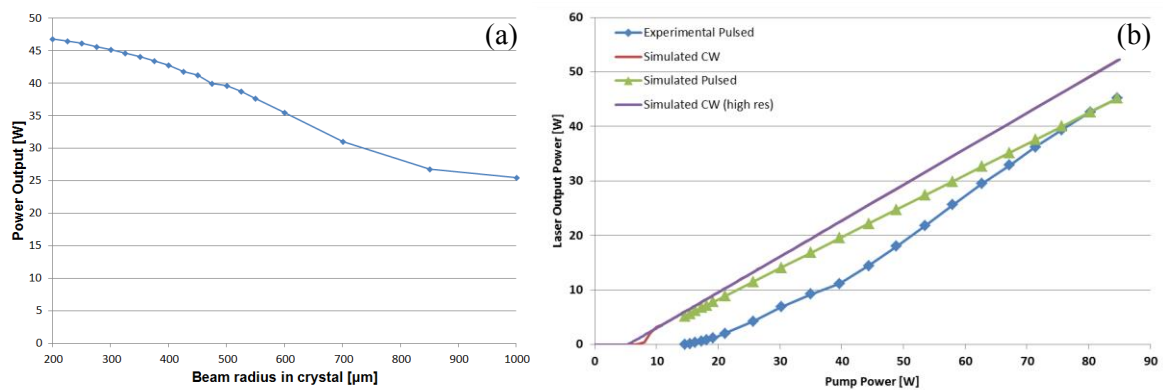


**Figure 5.** (a) Simulation results for laser output power as a function of laser mode radius in the crystal shows a steady increase in laser efficiency for smaller beam sizes. (b) Simulation results for laser output power in continuous wave and pulsed mode as compared to experimentally measured values for the actual laser in pulsed mode.

The output power discrepancies at lower pump powers are due to a number of factors including that model parameters (mode size, pump wavelength etc.) are best known at full power and that the model does not account for thermal lensing (the model assumes a perfect mode overlap between pump and laser mode that does not vary with pump power). Furthermore the model does not account for smaller pump sizes at low pump powers intrinsic to the operation of the Tm-fibre laser, the shift in pump wavelength with increasing pump power and the known water absorption loss around 40 - 50W of pump power (as the pump wavelength shifts through a water absorption line).

## 5. Conclusion

A novel, highly flexible and easy to use laser dynamic simulation library has been demonstrated by exploiting the graphical abstraction and vector capabilities of the Matlab Simulink numerical integration platform. The model's ability to simulate multi-mode/multi-gain media resonators was validated against measured results for a previously published efficient dual-crystal Ho:YLF oscillator [8]. The model was used to determine optimal laser cavity parameters such as the ideal laser mode size in the laser crystal. Measured and simulation results were in good agreement, especially at the high pump powers where simulation input parameters closely matched actual parameters.

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