Development of Plasma Fluid Model for a Microwave Rocket Supported by a Magnetic Field

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Abstract. A fluid model of plasma transport is developed to reproduce a plasma pattern induced by microwave irradiation when an external magnetic field is applied to the breakdown volume. Transport coefficients in the fluid model are evaluated using a fully kinetic simulation under a magnetic field to maintain consistency of electron transport between the particle and fluid models. The electron-density profile and propagation speed of the ionization front obtained by the fluid model agree with those of the particle model. Multidimensional or longer time-scale simulations can be conducted using the fluid model in the case of the application of an external magnetic field, with the simulation reducing computational cost compared to the fully kinetic model.

1. Introduction

Microwave rockets have been proposed to conduct frequent and flexible operations for small satellites to accelerate several areas of space utilization, particularly in the fields of business, scientific research, and national defense. It can achieve this by reducing the launch cost of a payload mass [1–5]. During the launch of a microwave rocket, intense microwaves are transmitted from a ground-based oscillator to the vehicle, which has a parabolic mirror to focus the incident beam. A dense plasma is generated inside the microwave-rocket nozzle as the intense field induced by mirror focusing breaks down the ambient air. A strong shock wave is generated when the plasma absorbs the energy of the incident microwave, providing an impulsive thrust to the vehicle due to interactions between the shock wave and the thruster wall. The launch cost of small satellites is decreased as the fuel needed on the vehicle can be reduced or removed by transmitting energy for propulsion from the ground. There is a large cost required to develop the ground-based microwave oscillator; however, this initial burden will become cost effective over time with repetitive launches of inexpensive rockets [5].

Flight demonstrations of microwave rockets has been conducted by irradiating a microwave beam using a high-power gyrotron [1–5]. A parabolic thruster was placed in a chamber to evaluate the ambient pressure dependence of thrust performance as the microwave rocket travels in rarefied gas conditions at higher altitudes. The higher thrust performance was obtained at atmospheric pressure, while forming a filamentary plasma. The filamentary plasma was caused by a standing wave, which was induced by wave reflection of the overcritical plasma. The thrust performance degraded with a decrease in the ambient pressure, and the plasma pattern transitioned from the discrete plasma to a diffusive breakdown pattern due to infrequent elastic collision. One-dimensional (1D) particle simulations for the charged-particle transport were conducted to identify the mechanism behind the decrease in the thrust performance at lower pressures. An increase in propagation speed of the ionization front and a decrease of the energy-absorption rate by the plasma lowered the thrust performance at rarefied gas conditions [6]. To improve the thrust performance, the propagation speed of the ionization front must be suppressed by applying an external magnetic field to the breakdown region of the microwave rocket. Electron cyclotron resonance (ECR) heating was induced by satisfying the resonant condition to increase the energy-absorption rate of the plasma. This improved thrust performance due to an enhanced shock wave inside the rocket nozzle [7,8]. Previous simulations did not address multidimensionality of the plasma structure when an external magnetic field was applied, as the computational requirements of such a particle simulation were huge. In addition, the largely increased time-scale physics of a microwave plasma cannot be completely captured using the fully kinetic model when a magnetic field is applied. A fluid model of the plasma transports, which has a smaller computational load, was coupled with the electromagnetic wave propagation in past studies [9–14]; however, the breakdown physics under the magnetic field was not examined using the fluid model. It is necessary to develop the fluid model under the magnetic field to examine the multidimensional plasma structure and the breakdown physics with a longer time scale.

The objective of this study is to construct a fluid model, which has simulations that require a smaller computational load than the particle model, so as to reproduce the breakdown physics induced by microwave irradiation when a magnetic field is applied. The transport coefficients under the magnetic field are evaluated using the fully kinetic model, and these coefficients are introduced in the fluid model to maintain consistency between the particle and fluid models. The 1D plasma structure simulated by the fluid model is compared with the 1D breakdown structure obtained by the particle model to validate the developed fluid model.

2. Numerical methods

2.1. Electromagnetic field calculation

An electromagnetic wave simulation is coupled with the fluid model of the plasma transport via the current density to examine the breakdown physics induced by microwave irradiation as interactions between the incident microwave and plasma play an important role for formation of the plasma pattern. Maxwell's equations are numerically integrated using the finite-difference time-domain (FDTD) method with Mur's boundary conditions [15] to reproduce the propagation of the incident microwave;

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \tag{1}$$

$$\nabla \times \mathbf{H} = \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mathbf{J}, \tag{2}$$

where $\mathbf{E}$ is the electric field, $\mathbf{B}$ is the magnetic flux density, $\mathbf{H}$ is the magnetic field, t is the time, ϵ_0 is the electric permittivity in vacuum, and $\mathbf{J}$ is the current density. Here, $\mathbf{B} = \mu\mathbf{H}$ with the magnetic permeability μ. The current density $\mathbf{J}$ in Eq. (2) is evaluated using the fluid description of plasma transport to model interactions between the dense plasma and propagation of the incident microwave. The time step of the FDTD simulation is determined by the Courant-Friedrichs-Lewy condition for light speed.

2.2. Fluid model of plasma transport

A precise plasma structure is obtained if the particle model for the charged-particle transport is coupled with electromagnetic wave propagation; however, the particle model uses substantial computational resources. In this study, the fluid description of plasma transport is made based on a quasi-neutral assumption to reduce the computational cost. A simple diffusion equation for electrons is numerically integrated based on past studies [11–14];

$$\frac{\partial n_e}{\partial t} - \nabla \cdot (D_{eff} \nabla n_e) = \nu_i n_e, \tag{3}$$

where n_e is the electron density, D_{eff} is the effective diffusion coefficient, and ν_i is the ionization frequency. The effective diffusion coefficient is written as $D_{eff} = (\alpha D_e + D_a)/(\alpha + 1)$ with the ambipolar diffusion coefficient $D_a = (\mu_i D_e + \mu_e D_i)/(\mu_i + \mu_e)$ and $\alpha = \tau_M \nu_i$. Here, $\tau_M = \epsilon_0/[en(\mu_i + \mu_e)]$ is Maxwell's relaxation time, where μ_e is the electron mobility, μ_i is the ion mobility, D_e is the electron diffusion coefficient, and D_i is the ion diffusion coefficient. The diffusion coefficient for free electrons is evaluated by the fully kinetic simulation coupled with collisional reactions in order to maintain consistency between the fluid and particle models. The diffusion coefficient of ion D_i is assumed as 0 because the time scale of plasma propagation is much shorter than that of the ion movement. The time step for the diffusion equation is set as one period of the incident microwave because the time scale for plasma evolution is much longer than the wave propagation [11–14], and time integration is implicitly conducted using the Crank-Nicolson method. The electron current density is estimated as $\mathbf{J} = -en_e\mathbf{v_e}$ in the fluid model, where $\mathbf{v_e}$ is the electron velocity. The electron velocity is calculated by integrating an equation of motion with the electric force, Lorentz force, elastic collision, and electron-pressure gradient;

$$\frac{\partial \mathbf{v_e}}{\partial t} = -\frac{e(\mathbf{E} + \mathbf{v}_e \times \mathbf{B})}{m_e} - \nu_m \mathbf{v_e} - \frac{k_B}{m_e n_e} \nabla (n_e T_e), \tag{4}$$

where m_e is the electron mass, μ_m is the elastic collision frequency, k_B is the Boltzmann constant, and T_e is the electron temperature. The time step for the equation of motion is the same as for the FDTD simulation, and the Euler explicit method is employed to integrate the equation of motion for electrons. The current density is fed back to the FDTD simulation to reproduce interactions between the electromagnetic wave propagation and the plasma.

2.3. Fully kinetic simulation to evaluate transport coefficients under a magnetic field

It is necessary to assess the transport coefficients (ionization frequency, elastic collision frequency, diffusion coefficient, and electron temperature) to integrate the diffusion equation and equation of motion of electrons in the fluid description of a plasma [16]. Data sets of transport coefficients are evaluated against the root-mean-square (RMS) field value using the 1D particle-in-cell (PIC) with the Monte Carlo collision (MCC) model. The local RMS field is calculated by solving Maxwell's equation in the plasma fluid and FDTD coupling simulation. Using the local RMS field, the transport coefficients in the fluid model are interpolated from data sets evaluated by the PIC-MCC simulation. Charged particles have cyclotron motion under the external magnetic field, and these must be considered by coupling the 1D PIC-MCC model with the FDTD simulation when transport coefficients are estimated.

In the 1D PIC module with three velocity components, the equations of motion for electrons and ions are numerically integrated to trace the motion of the charged particles under electromagnetic fields calculated from the FDTD simulation. The Buneman-Boris method [17] is utilized to integrate the equation of motion, which guarantees that no work is induced by the magnetic field. Current density is not fed back to the FDTD module for the assessment of transport coefficients using the particle model of the plasma. To compare plasma patterns between the fluid and particle models, the current density is fed back to the FDTD simulation when the 1D breakdown structure is reproduced using the fully kinetic model. The current density at the computational grids is evaluated by a linear weighting method in the fully kinetic model. The MCC module is coupled with the PIC module to combine collisional reactions with the motion of the charged particles. The Nanbu method [18] is utilized to determine the reaction type and whether a collision exits for pure nitrogen gas. Phelps's cross section is used to introduce eighteen reactions (e.g., elastic collision, vibrational excitation, rotational excitation, electron-impact excitation, and ionization) for nitrogen gas [19]. The MCC module is coupled with the PIC module using the Birdsall method [20].

To evaluate the transport coefficients [16], the ionization frequency ν_i is deduced in the fully kinetic model by:

$$\nu_i = \frac{\ln[N_e(t)] - \ln[N_e(t_0)]}{t - t_0}, \tag{5}$$

where N_e is the total particle numbers of electrons and t_0 is the initial time. The electron-diffusion coefficient is assessed by:

$$D_e = \frac{\langle [x_e(t) - \langle x_e(t_0)\rangle]^2\rangle - \langle [x_e(t) - \langle x_e(t_0)\rangle]^2\rangle}{2(t - t_0)}, \tag{6}$$

where x_e is the electron position, and $\langle \cdots \rangle$ indicates the average value of all particles [16]. The electron temperature T_e is calculated as:

$$T_e = \frac{2}{3k_B}\langle \frac{1}{2} m_e(\mathbf{v}_e - \bar{\mathbf{v}}_e)^2\rangle, \tag{7}$$

where $\langle \cdots \rangle$ indicates the average over all electrons in the simulation domain. The elastic collision frequency is estimated by counting the number of collisions in the particle code.

3. Simulation condition

The 1D breakdown simulation is conducted in nitrogen gas to compare the fluid model with the particle model when an external magnetic field is applied to the breakdown volume, which is induced by microwave irradiation. The maximum intensity of the incident microwave E_y is 5 MV/m with a frequency of 110 GHz in an ambient pressure of 0.1 atm. The external magnetic field B_z is selected as 3.93 T to obtain electron cyclotron resonance (ECR) heating via coupling of electron cyclotron motion with field oscillations from the incident microwaves. The electrons and ions diffuse in the x-direction, the electric field oscillates in the y-direction, and the magnetic field is applied in the z-direction. The 1D simulation domain has 2.25λ ($\lambda = 0.27$ cm, which is the microwave wavelength), and the number of grid points is 1,800 for the fluid model. As a comparison, 40,000 grid points are used in the fully kinetic simulation to resolve the Debye length of the dense plasma. The initial plasma spot with a Gaussian distribution is set on the 1D simulation domain. The electrons initially have velocities corresponding to a temperature of 1 eV in the particle simulation. Neutral particles of 300 K are assumed as a background gas. A microwave is irradiated from the left to right boundaries to reproduce a breakdown pattern in nitrogen, and the plasma propagates toward the left boundary from the initial plasma spot.

4. Results

4.1. Evaluation of transport coefficients using 1D PIC-MCC model

To introduce the ionization frequency, elastic collision frequency, electron-diffusion coefficient, and the electron temperature into the fluid model, they are evaluated using the 1D PIC-MCC simulation for nitrogen gas at ambient pressure of 0.1 atm. Microwave beams of 0 to 15 MV/m are irradiated to assess the transport coefficients under an external magnetic field of 3.93 T. The current density is not fed back to the FDTD simulation for evaluation of the transport coefficients. The electron-diffusion coefficient of 3.93 T is smaller than 0 T because the external magnetic field disturbs the electron transport (Fig. 1), suppressing propagation of the ionization front inside the microwave-rocket nozzle. Confinement effects from the external magnetic field are successfully considered in the estimation of transport coefficients by utilizing the fully kinetic model. The transport coefficients evaluated by the PIC-MCC model are locally interpolated in the FDTD and plasma fluid coupling simulation.

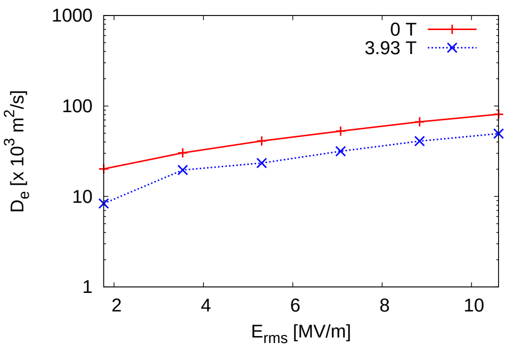


Figure 1. Electron-diffusion coefficients evaluated by the PIC-MCC simulation at an ambient pressure of 0.1 atm. The 110-GHz microwaves are irradiated to a Gaussian plasma spot under magnetic fields of 0 T and 3.93 T.

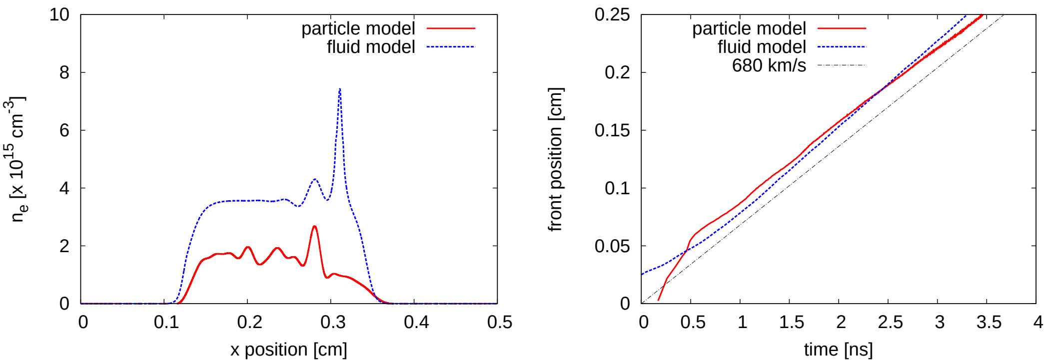
4.2. Comparison of 1D plasma structures between particle and fluid models under magnetic fields

The 1D fluid and particle simulations are conducted in nitrogen gas to compare plasma structures and propagation speeds obtained using both models at an ambient pressure of 0.1 atm. Microwaves of 5-MV intensity and a frequency of 110 GHz are injected from the left to the right boundaries under a magnetic field of 3.93 T. The current density is fed back to the FDTD simulation to reproduce an interaction between the dense plasma and the electromagnetic wave propagation in the fluid and particle models.

Overcritical plasma is formed due to the quick ionization at the initial plasma spot (Fig. 2(a)), which reflects the incident microwave. The ionization front propagates toward the left boundary because seed electrons diffuse from the bulk plasma. Sequential ionization occurs in front of the bulk plasma when the seed electrons are heated due to the strong electric field generated by an overlap of the incident and reflected microwaves. The maximum electron density of the fluid model is larger than that of the particle model. The density profile obtained by the fluid model agrees with that of the particle model because the transport coefficients in the fluid model are evaluated using the particle simulation. The time evolution of the ionization-front position from the initial plasma spot is assessed by tracking the electron densities exceeding 1×10^{13} cm^{-3} (Fig. 2(b)). The propagation speed of the ionization front is estimated as 680 km/s at 0.1 atm under the magnetic field of 3.93 T, and the propagation speeds of both fluid and particle models are the same. Under the external magnetic field, the multidimensional plasma pattern or the longer time-scale physics can be examined using the fluid model because consistency of the density profile and the propagation speed is maintained between the particle and fluid models.

5. Conclusion

Under an external magnetic field, the breakdown pattern of a microwave plasma was reproduced by coupling the fluid or particle descriptions of the plasma transports with an electromagnetic wave propagation. Confinement effects from the magnetic field were introduced in the fluid model from the transport coefficients utilized in the fluid model. These coefficients were evaluated by



(a) Electron number densities (b) Front position of plasma

Figure 2. Electron number densities and front position of plasma obtained by the fluid and particle models. At 0.1 atm, 110-GHz microwave is irradiated to breakdown volume under external magnetic field of 3.93 T.

the particle code, which reproduces cyclotron motions of charged particles under a magnetic field. The particle and fluid simulations were conducted with current feedback to the FDTD simulation to compare the microwave-plasma patterns obtained by each model. The electron-density profile and the propagation speed of the ionization front obtained by the fluid model agree with those found in the particle model because consistency of electron transports was maintained between the particle and fluid models. It is possible to conduce the multidimensional or longer time-scale simulations can be performed using the fluid model when a magnetic field is applied to the breakdown volume induced by microwave irradiation.

Acknowledgements
The development of numerical models was supported by Dr. Jean-Pierre Boeuf at the University of Toulouse. The computations in this work were performed on a FUJITSU PRIMEHPC FX100 at the Japan Aerospace Exploration Agency. This work was supported by JSPS KAKENHI Grant Number JP16J09910.

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