Modelling ground conductivity for computing the electric field associated with geomagnetically induced currents using the Finite Element Method. (A mid latitude case study)

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Abstract: The study of geomagnetically induced currents (GIC) in technological systems connected to the Earth such as power lines and pipelines during adverse space weather conditions requires the computation of the electric field induced in the Earth. These computations can be achieved through solving Maxwell's equations with appropriate boundary conditions. COMSOL Multiphysics, a finite element method (FEM) simulation package is used to compute the electric field resultant from measured geomagnetic field variation under the assumption of a horizontally layered Earth model. The study is based on the calculation of the GIC in a transformer grounding point at an electrical substation in South Africa for which a previous study determined the conductivity profile. The investigation aims at studying the effects of varying the number, thickness and conductivity of layers when computing the electric field associated with GIC, thus, enhancing the understanding of the distribution of the induced horizontal electric field within the various layers of the Earth during a geomagnetic storm. The measured GIC and the modelled GIC are compared to determine the best representation of the Earth. This kind of study is important in understanding the layers which matter most in the effective modelling of GIC for this particular substation. The results based on this case study indicates that for a layered Earth model where the top layers have a low conductivity compared to the underlying layers the deeper high conductivity layers have a significant influence on the accuracy of the modelled GIC.

1. Introduction

Geomagnetically induced currents (GIC) are enhanced during geomagnetic storms. A geomagnetic storm arises when the magnetosphere is highly disturbed. This occurs when the interplanetary magnetic field turns southward for a significantly long period [1]. The variations in the magnetosphere-ionosphere currents have a great influence on the geomagnetic perturbations on the surface of the Earth. As such, the magnetosphere-ionosphere currents are considered to be the primary source of surface electric field [2]. The variations of the surface magnetic field determine the characteristics of the surface electric field. The extent to which space weather affects grounded technological systems is an expanding research arena. Such knowledge aids in understanding the vulnerability of several technological systems to GIC during adverse space weather.

An investigation of the variations of the induced electric field in the various layers of the Earth is presented. The conductivity profile used in the model was derived by Ngwira et al., [3] for the Earth below a South African 400 kV substation at Grassridge (33.7°S, 25.6°E).

For the computation of the induced field, finite element method (FEM) software package, COMSOL Multiphysics is used. This is a rarely explored technique in the computation of electric fields connected with GIC computation. A comparison is made of GIC derived from an electric field modelled using the FEM with measured GIC at Grassridge during the geomagnetic storm of October 29, 2003. The case study uses geomagnetic field data measured at Hermanus (34.4°S, 19.2°E) as model input.

2. GIC modelling in mid- and low- latitude

The computation of GIC in any technological system requires firstly the determination of the surface electric field where the system is grounded. The second step requires determination of the system parameters which are unique and dependent on system configuration [4]. The electric field associated with GIC can be computed or modelled in several ways. In reality, the computations are complex due to the inhomogeneity of the ground conductivity and the non-uniformity of the electric field. Hence some reasonable assumptions are made in order to simplify the computations.

For middle- and low- latitude regions, the plane wave model is often deemed adequate as the regions are not subjected to auroral electrojets. The model assumes that the magnetic field propagates as a plane wave directed vertically down to the Earth resulting in components of the field being parallel with the surface of the Earth. The GIC computations require magnetic field time series data which is typically only measured at magnetic observatories. The coupling between the magnetic field due to the telluric currents and those due to the space currents are neglected to further simplify the computations [5].

In cases where the surface electric field is computed over a layered Earth (consisting of horizontal layers of different conductivities as shown in Figure 1, the apparent conductivity of each layer is considered [2]. The horizontal components of the electric field are given by equation 1 and 2

$$E_y = -\mu_0^{-1} Z B_x \tag{1}$$

$$E_x = \mu_0^{-1} Z B_y \tag{2}$$

where E_y is the East-directed component of the electric field, E_x is the North-directed component of the electric field and B_x and B_y are the corresponding orthogonal horizontal magnetic field components, Z is the surface impedance and μ_0 is the permeability of free space.

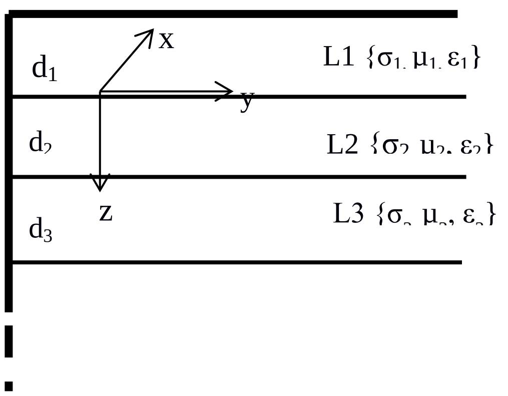

Figure 1. A 2D representation of the Earth's structure consisting of 3 layers, L1, L2 and L3 of depth d_1, d_2 and d_3. Each layer has a different conductivity (σ), permeability (μ) and permittivity (ε). In reality the number of layers can vary. The layers are all assumed to have infinite extent in the x-direction and the bottom layer is assumed to have infinite extent also in the z-direction. The dimension of the layers in the y-axis is finite to allow localisation of the computations

3. Computing the electric field by means of the finite element method

Techniques such as the complex image method (CIM) [6], method of auxiliary sources (MAS) [7] have been applied in the computation of magnetic field and electric field associated with GIC modelling. Recent studies are considering the use of Finite element method [8] to compute the electric field associated with GIC. Only the horizontal components (E_x and E_y) of the electric field are computed and equation 3 is used for GIC computation. E_z is always zero if there is no horizontal variation in the Earth’s conductivity, thus the Earth’s electric field is always horizontal [5]. The coefficients (*a* and *b*) are computed separately using appropriate methods [9].

$$\mathrm{GIC(t)} = a\mathrm{E_x}(t) + b\mathrm{E_y}(t) \tag{3}$$

where *a* and *b* are the network coefficients which can be derived from the network topology and impedances.

FEM is a technique that divides the problem domain into non-overlapping simple elements to reduce the complexity of the structural geometry. The partial differential equation (PDE) governing the field variable under investigation is solved for each element. This reduces the complexities by finding a solution of the PDE for each. The element solutions are integrated to give the overall solution for the whole structure.

Within the COMSOL Multiphysics package, the *ACDC module* was used as it is designed to solve Maxwell's equation [10]. In FEM procedures the general steps that are followed are, defining the geometry, allocating material properties, imposing appropriate boundary conditions and selecting the type and size of the elements (mesh). Computations are handled by the FEM software and the results are analysed by means of graphic displays.

3.1 The model
The construction of the layered Earth model is done in a 2D domain in the y-z plane. The interest is therefore in the variation of the induced electric field $E_{(x,y)}$ along the z- axis. The most important thing at this stage is to make sure that all layers are properly joined to each other for electrical continuity. This is achieved through the built-in "*union*" function of the COMSOL software.

3.2 Magnetic field data
The measured magnetic field is introduced in the model as a piecewise cubic interpolated function applied to the surface of the Earth. One magnetic field component is loaded at a time, that is either the B_x or B_y component. In the current case, magnetic field data measured at one-minute intervals are used.

3.3 Boundary Conditions
To keep the magnetic field uniform within the model, a "perfect magnetic conductor" boundary condition was set on the left and right edges of the flat Earth model. This boundary condition, implying **n** x **H** = 0, prevents tangential components of the magnetic field at the edges of the model. **n** is the unit normal to the surface. The "magnetic insulation" boundary condition was set at the bottom of the model to ensure that the induced currents can only flow in the y- direction.

3.4 *The system equation*
The layers of the Earth are considered to be sub-domains. For each sub-domain, Ampere's law, (equation 4) is solved.

$$\sigma\frac{\mathrm{d}\mathbf{A}}{\mathrm{dt}} + \nabla\times\left(\mathbf{B}(\mu_0\mu_r)^{-1}\right) - \sigma\mathbf{v}\times\mathbf{B} = \mathbf{J} \tag{4}$$

where **A** is the magnetic potential vector, **B** is the applied magnetic flux density, μ_r is the relative permeability **J** is the current density, **v** is the velocity of the conductor which is 0 in the current case as there is no moving media in the model, μ_0 is the permeability of free space and σ is the conductivity of the Earth . The electric field (**E**) is then inferred from the expression in equation 5:

$$\mathbf{E} = \mathbf{J}\sigma^{-1}. \tag{5}$$

3.5 The Mesh
An *extremely fine* mesh size of triangular elements was used to provide a reasonable compromise between spatial resolution and computation time. This yielded a total number of 11137 elements and 23096 degrees of freedom. The size of the mesh has a significant bearing on the computation time.

While fewer elements may take a shorter time to converge, the accuracy of the solution may be compromised.

3.6 Computation

The final step of the process is to allow the computation of the desired variables. A *time dependent* study was used for the computation of the electric field within the several layers of the domain. The electric field at the interface between Earth and air is then used to compute the GIC that is likely to be driven into any grounded technological system in the vicinity.

4. Modelled Cases

The study considers 3 cases. These layer combinations were selected so that characteristics of the electric field could be deduced from these three situations which are close representation of the ground conductivity structure. For case 1, the purpose was to evaluate if the poorly are sufficient for calculating the electric field associated with the GIC. Case 2 evaluate the effects of a less resistive layer underlying a high resistivity layer on the calculated electric field. Case 3 evaluate the effects of the terminating half-space on the surface electric field.

Case 1: 7 layers (high resistivity layers only). Resistivity in Ωm = [440,420,400,380,330,300,260] and the corresponding depth in km [1.1, 3.9, 7, 14, 24, 50, 100]

Case 2: 2 layers, (layer 1 has a resistivity which is the average of the high resistive layers on top while layer 2 has a low resistivity). Resistivity in Ωm = [361, 10] and depth in km = [200, 190]

Case 3: 3 layers (Same layers as for Case 2, plus a deepest layer of infinite extent and medium resistivity). Resistivity in Ωm = [361, 10, 16] and depth in km = [200, 190,∞]

In all cases, the depth of the layers as originally derived are preserved. The model inputs are the resistivity and magnetic field.

5. Results discussion and conclusion

Dong et al, [8] evaluated the accuracy of 2D FEM electric field modelling by comparing FEM simulated electric field with analytical values. This current study has demonstrated the applicability of FEM in electric field modelling through the good correlation between measured GIC and GIC derived from the electric field obtained by FEM. The use of FEM comes with an advantage of handling complex geometry related to the geological structure. The aim of this study is identify the suitable Earth conductivity profiles to be used for GIC modelling for the Grassridge substation.

The results presented focuses on the period with high magnetic perturbations on October 29, 2003 i.e. during 06:00-08:00 hours UT. The network coefficients used for the GIC modelling are a = -80 AkmV^{-1} and b = 15 A kmV^{-1}, which were derived by Koen [11] for the Grassridge substation. Figure 2, 3 and 4 show the electric field variation and GIC modelled for the selected period. For evaluation of the modelled GIC, the maximum modelled GIC, correlation coefficient and the root mean square of the difference between the modelled and measured results for the three cases and are calculated. The maximum measured GIC for this particular storm was 12.56 A.

Figure 2 shows that for Case 1 the magnitude of the induced electric field reduces with depth. For these 7 consecutive layers, the conductivity is actually increasing with depth. The GIC modelled with the surface electric field has 91% correlation with the measured GIC. The maximum value of the modelled GIC during this storm was 12.0 A and the RMSE of 0.98 A.

For the second case which considers an average of the 7 layers (low conducting) above a highly conducting layer, the modelled GIC has a correlation coefficient of 89 %, RMSE of 1.00 A and maximum modelled GIC of 12.35 A . Figure 3 shows how the variation of the electric field on the surface compares with the maximum field induced in the two layers. What is interesting to note is that the electric field induced on the surface is the same as that induced on the first layer at some point. For example, during the period 06:30-07:00, the surface electric field (E_y) is the same as that in the first layer, but this is not true of E_x for the same period. However, the same can be said for E_x

during the period 07:00-08:00. The reasons for this behavior in the electric field are yet to be investigated.

The third case which considers the deepest layer to have infinite extent results in a GIC with a correlation coefficient of 88 %, RMSE of 1.15 A and a maximum modelled GIC of 10.78 A.

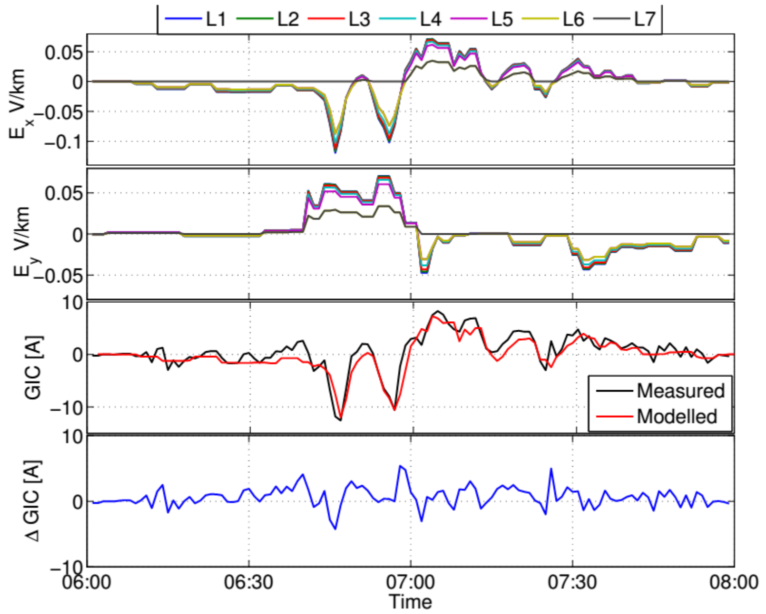

Figure 2. The estimated horizontal field components for each layer, the modelled GIC and the difference between the measured and modelled GIC for Case 1.

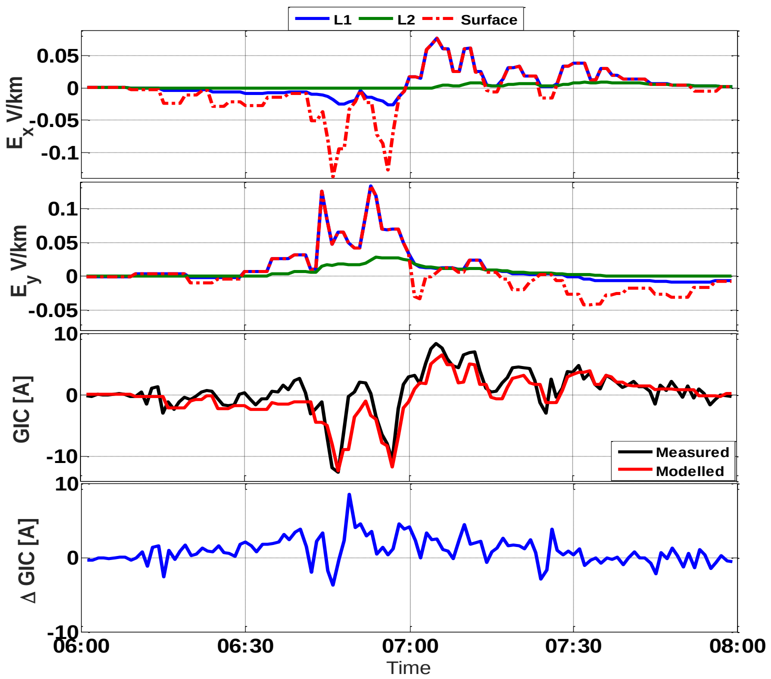

Figure 3. The estimated horizontal electric field components for each layer, the modelled GIC and the difference between the measured and modelled GIC for Case 2.

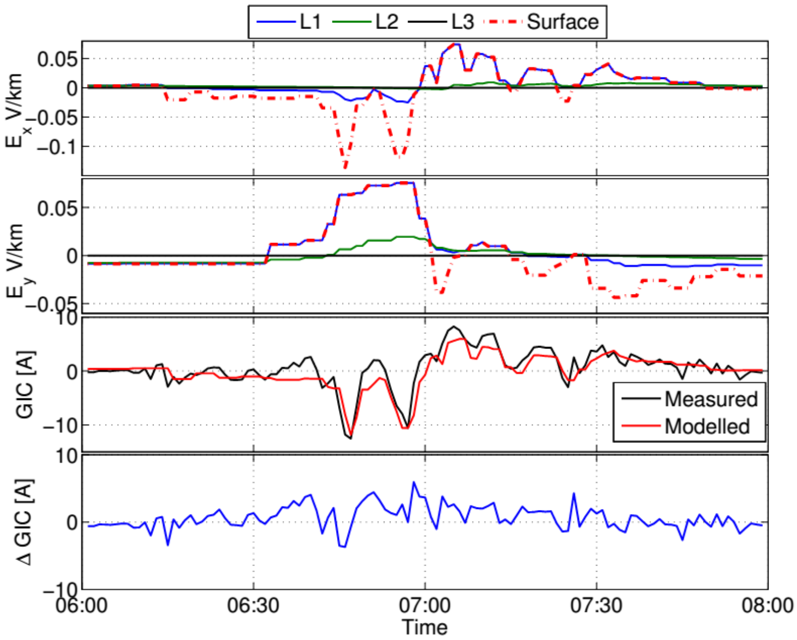

Figure 4. The estimated horizontal field components for each layer, the modelled GIC and the difference between the measured and modelled GIC for Case 3.

It was noted that the FEM software introduces an arbitrarily phase shift observed in the modelled and measured GIC, (time lag in figure 2 and lead Figures 3 and 4). The lag/lead of the modelled GIC was adjusted for maximum correlation with the measured GIC.

From the current results it is indicated that the configuration of layers considered for a GIC model can have a significant impact on the maximum modelled GIC, however we note that the correlation coefficient and the root mean square error obtained from the for the three cases are very close to each other. These results therefore indicate that quality GIC estimation could be achieved with simplified Earth representation instead of a 7-layer structure such as Case1. Another significant conclusion that the deeper layers have a significant contribution to the surface electric field induced during a geomagnetic storm.

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