THE ELECTROMAGNETIC FIELD

Theorem: A vector field is known completely if (i) the divergence $\vec{\nabla}\cdot\vec{V}(\vec{x}) = s(\vec{x})$ is known, (ii) the curl $\vec{\nabla}\times\vec{V}(\vec{x}) = \vec{c}(\vec{x})$ is known, and (iii) the asymptotic behaviour, e.g. $\lim_{|\vec{x}|\to\infty} \vec{V}(\vec{x}) = 0$, is known. In this case, the vector is : $\underline{\vec{V}(\vec{x}) = -\vec{\nabla}\phi(\vec{x}) + \vec{\nabla}\times\vec{A}(\vec{x})}$ (III.22)

with

$$\left\{\begin{array}{ll} \phi(\vec{x}) = \dfrac{1}{4\pi}\displaystyle\int \frac{s(\vec{x}')}{|\vec{x}-\vec{x}'|}\, d^3x' & \text{(III.23)} \\[2ex] \vec{A}(\vec{x}) = \dfrac{1}{4\pi}\displaystyle\int \frac{\vec{c}(\vec{x}')}{|\vec{x}-\vec{x}'|}\, d^3x' & \text{(III.24)} \end{array}\right.$$

——— × ———

TUTORIAL: show (III.22) by taking the gradient of (III.23), and the curl of (III.24). Hint: Use

$$\nabla^2_{\vec{x}}\left(\frac{1}{|\vec{x}-\vec{x}'|}\right) = \nabla^2_{\vec{x}'}\left(\frac{1}{|\vec{x}-\vec{x}'|}\right) = -4\pi\delta(\vec{x}-\vec{x}')$$

and use spherical coordinates $d^3x = r^2 dr\, d\Omega$, $d\Omega = \sin\theta\, d\theta\, d\varphi$, $\int d\Omega = 4\pi$, and

$$\nabla^2 = \frac{1}{r}\frac{\partial^2}{\partial r^2} + \ldots$$

——— × ———

Eqs. (I.17) & (I.20) shows that $\vec{\nabla}\times\vec{A} = \vec{B}$, but $\vec{\nabla}\cdot\vec{A}$ does not enter anywhere. Hence, $\vec{\nabla}\cdot\vec{A}$ can be specified to be whatever one wishes.

TUTORIAL

(i) Show that the "Coulomb gauge": $\vec{\nabla}\cdot\vec{A} = 0$ corresponds to an instantaneous Coulomb potential, i.e. $\Phi(\vec{x}, t) = \int \frac{\rho(\vec{x}', t)}{|\vec{x}-\vec{x}'|} d^3x'$

where the time "t" is the same in ρ as in Φ

(ii) Show that if $\vec{A} \to \vec{A}' = \vec{A} + \vec{\nabla}\Lambda$ and

$\Phi \to \Phi' = \Phi - \frac{1}{c}\frac{\partial\Lambda}{\partial t}$ then $\vec{E}' = \vec{E}$ & $\vec{B}' = \vec{B}$

(iii) Show that if $\vec{\nabla}\cdot\vec{A} = -\frac{\partial\Phi}{\partial t}$ or

$\partial^\mu A_\mu = 0$ then $\Box^2 \Phi(\vec{x}, t) = 4\pi\rho(\vec{x}, t)$

and $\Box^2 \vec{A} = \frac{4\pi}{c}\vec{J}$. (This is the Lorentz gauge: $\partial^\mu A_\mu = 0$)

— x —

FREE EM FIELD (a real field)

$$A^\mu(x) = \sum_{\lambda=1}^{2} \int d^3k \frac{1}{(2\pi)^{3/2}} \frac{1}{\sqrt{2\omega_k}} \left[\epsilon^\mu(\vec{k},\lambda)\, a(\vec{k},\lambda)\, e^{-ik\cdot x} + \epsilon^{\mu *}(\vec{k},\lambda)\, a^*(\vec{k},\lambda)\, e^{ik\cdot x}\right] \quad (\text{III}.27)$$

The EM field (photon) is charge neutral, and is transverse (2 degrees of freedom). From originally 4 dof (A^μ is a 4-vector) reduce to 3 ($\partial^\mu A_\mu = 0$) and then to 2 by fixing $\Lambda(x)$.

Canonical quantization of EM field:

For KG we had $[\hat{\phi}(\vec{x},t), \hat{\pi}(\vec{y},t)] = i\delta^{(3)}(\vec{x}-\vec{y})$

For the EM field: $\hat{\pi}_i = \dfrac{\partial \hat{\mathcal{L}}}{\partial(\partial_0 A_i)}$, $\hat{\pi}^\nu = \dfrac{\partial \mathcal{L}}{\partial(\partial_0 \hat{A}_\nu)}$

From (I.21) $\hat{\pi}^\nu = \hat{F}^{0\nu}$, hence

$$\hat{\pi}^0 = \hat{F}^{00} = 0 \quad \nabla\!\!! \qquad (\text{III}.26)$$

The naive canonical quantization fails.

Solution $\begin{cases} \text{a) use the Coulomb gauge [EASY]} \\ \text{b) modify the canonical quantization [HARD]} \end{cases}$

a) Coulomb gauge, $A^0(x) = 0$, $\vec{\nabla}\cdot\vec{A}(x) = 0$
(in free space)

$$\vec{A}(x) = \int d^3k \sum_{\lambda=1}^{2} \frac{1}{\sqrt{(2\pi)^3 2\omega}} \left[\vec{e}(k,\lambda)\, a(k,\lambda)\, e^{-ik\cdot x} + \vec{e}^{\,*}(k,\lambda)\, \hat{a}^\dagger(k,\lambda)\, e^{ik\cdot x}\right] \quad (\text{III}.27)$$

TUTORIAL

Show that

$$[A^i(\vec{x},t), \hat{A}^j(\vec{y},t)] = i\int \frac{d^3k}{(2\pi)^3}\left(\delta_{ij} - \frac{k_i k_j}{\vec{k}^2}\right) \times e^{i\vec{k}\cdot(\vec{x}-\vec{y})} \quad (\text{III}.28)$$

HINT:

Use $\displaystyle\sum_{\lambda=1}^{2} e^{i*}(k,\lambda)\, e^{j}(k,\lambda) = \delta_{ij} - \frac{k_i k_j}{\vec{k}^2}$.

——— x ———

Since the theory is gauge invariant $A_\mu \to A'_\mu = A_\mu + \partial_\mu \Lambda$ gives $\mathcal{L}' = \mathcal{L}$, we can use any gauge we wish.

DIRAC FIELD

The "classical" Dirac Lagrangian, leading to Dirac eqn. is

$$\mathcal{L}_D = \bar{\psi}(x)\,(i\gamma^\mu \partial_\mu - m_0)\,\psi(x) \quad (\text{III}.29)$$

The Euler-Lagrange's eqns. for ψ & $\bar{\psi}$

$$\partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \psi)} - \frac{\partial \mathcal{L}}{\partial \psi} = 0 \quad \& \quad \partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \bar{\psi})} - \frac{\partial \mathcal{L}}{\partial \bar{\psi}} = 0 \quad (\text{III}.30)$$

give

$$\begin{cases} \bar{\psi}(x)\,(i\gamma^\mu \overleftarrow{\partial_\mu} + m_0) = 0 \\ (i\gamma^\mu \overrightarrow{\partial_\mu} - m_0)\,\psi(x) = 0 \end{cases} \quad (\text{III}.31)$$

This Dirac field has no classical limit, but it is treated as such, and then "quantized" (2nd quantization)

$$\hat{\psi}(x) = \sum_s \int d^3p \left[\psi^{(+)}_{\vec{p},s}(x)\, \hat{b}(\vec{p},s) + \psi^{(-)}_{-\vec{p},-s}(x)\, \hat{d}^{\dagger}(\vec{p},s) \right] \quad (\text{III}.32)$$

$\hat{b}$ & $\hat{d}$: annihilation ops.

$\hat{b}^{\dagger}$ & $\hat{d}^{\dagger}$: creation ops.

<u>Pauli Exclusion Principle</u>:

$$|\vec{p}, \vec{p}\,'\rangle = \pm\, |\vec{p}\,', \vec{p}\rangle \quad (\text{III}.33)$$

(+) sign → bosons, satisfy commutation relations

(−) sign → fermions, " anti " "

For Fermions use $\{\,,\}$ instead of $[\,,]$.

e.g. $\{\hat{b}^{\dagger}_{\vec{p}\,'}, \hat{b}^{\dagger}_{\vec{p}}\} = 0$, which leads to:

$$|\vec{p}, \vec{p}'\rangle = C^{-1}_{\vec{p}'}\, C^{-1}_{\vec{p}}\, \underbrace{\hat{b}^{\dagger}_{\vec{p}}\, \hat{b}^{\dagger}_{\vec{p}'}}_{-\hat{b}^{\dagger}_{\vec{p}'}\, \hat{b}^{\dagger}_{\vec{p}}}\, |0\rangle$$

$$= -|\vec{p}'\vec{p}\rangle \qquad (\text{III}.34)$$

Pauli Principle – antisymmetric state
i.e. if $\vec{p} = \vec{p}'$ then $|\vec{p}, \vec{p}\rangle = -|\vec{p}\,\vec{p}\rangle = 0$
No two electrons in the same state

Anticommutation relations:

$$\hat{\pi}_a = \frac{\partial \mathcal{L}_D}{\partial(\partial_0 \psi_a)} \qquad \text{with } \mathcal{L}_D = i\psi^{\dagger}_a \partial_0 \psi_a + \dots$$

$$\pi_a = i\psi^{\dagger}_a \qquad (\text{III}.35)$$

Postulate Canonical anticommutation relation

$$(\text{III}.36) \qquad \{\hat{\psi}_a(\vec{x}, t), \hat{\pi}_b(\vec{y}, t)\} = i\delta_{ab}\, \delta^{(3)}(\vec{x} - \vec{y})$$

This implies (show it)

$$\{\hat{b}(\vec{p}', s'), \hat{b}^{\dagger}(\vec{p}, s)\} = \{\hat{d}(\vec{p}', s'), d^{\dagger}(\vec{p}, s)\}$$

$$= \delta_{ss'}\, \delta^{(3)}(\vec{p} - \vec{p}') \qquad (\text{III}.37)$$

and all others are zero.

<u>TUTORIAL</u>

Show that $\mathcal{L}_D$, Eq. (III.29) is invariant under the "global" gauge transformation

$$\psi(x) \to \psi'(x) = \psi(x)\, e^{i\alpha} \qquad \alpha = \text{const.}$$

using (I.31) & (I.32) together with $\psi' \simeq (1 + i\alpha + \dots)\psi$ show that the conserved current is $J_\mu = i\bar{\psi}\gamma_\mu\psi$.

Local Gauge Invariance:

$$\psi(x) \longrightarrow \psi'(x) = e^{i\theta(x)}\,\psi(x) \qquad (\text{III}.38)$$

TUTORIAL

i) Show that the gauge transformation (III.38) does not leave $\mathcal{L}_D$ invariant. Hence there is no conserved Noether current.

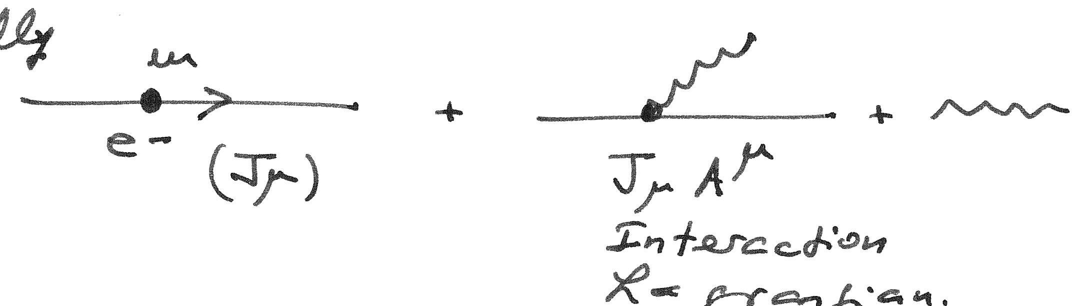
ii) Show that in order to re-establish local gauge invariance a new term must be added to $\mathcal{L}_D$, of the form $(\bar{\psi}\gamma_\mu\psi)A^\mu$, with
$$A_\mu \rightarrow A'_\mu = A_\mu + \partial_\mu\theta(x).$$

iii) Show that the conserved current of the new $\mathcal{L}_D$ is the electromagnetic current $J_\mu(x) = \bar{\psi}(x)\gamma_\mu\psi(x)$ with $|e| = 1$.

———— × ————

$$\boxed{\mathcal{L}_D = \bar{\psi}(x)(i\gamma^\mu\partial_\mu - m_0)\psi(x) + J_\mu(x)A^\mu(x) - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}} \qquad (\text{III}.39)$$

Graphically

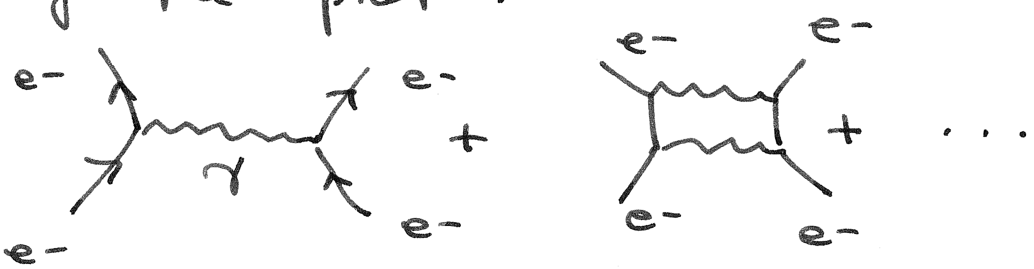


The local gauge symmetry dictates the form of the interaction !

<u>Matter Particles</u> : $e^{\pm}$ Quantum excitations of Dirac field.

<u>Gauge Particle</u> : γ Quantum excitation of the electromagnetic field

e.g. the $e^- e^-$ interaction is mediated by the photon:

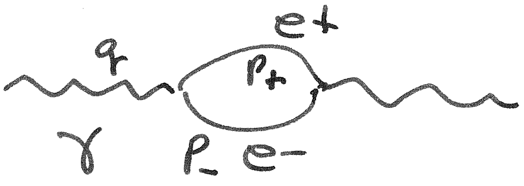


<u>THE QUANTUM VACUUM</u>

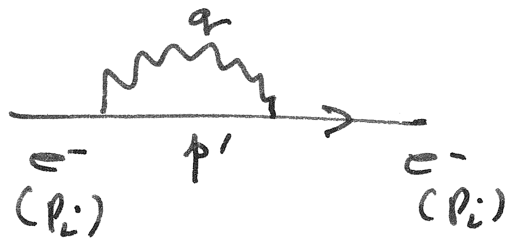
CLASSICAL PHYSICS	$\Delta E = 0$	First Law of Thermodynamics
QUANTUM PHYSICS	$\Delta E \neq 0$	Heisenberg's Uncertainty Relation

$$\boxed{\Delta E \Delta t \simeq \hbar} \quad (\text{III}.40)$$

Examples of energy-momentum violation processes:



$q^2 = (p_+ + p_-)^2 = 2m_e^2 + 2E^2 - 2|\vec{p}|^2 \neq 0$



$p_i^2 = (q + p')^2$

$m_e^2 \qquad m_e^2 + 2q\cdot p' \neq m_e^2$

Feynman diagrams : INVENT "virtual particles"

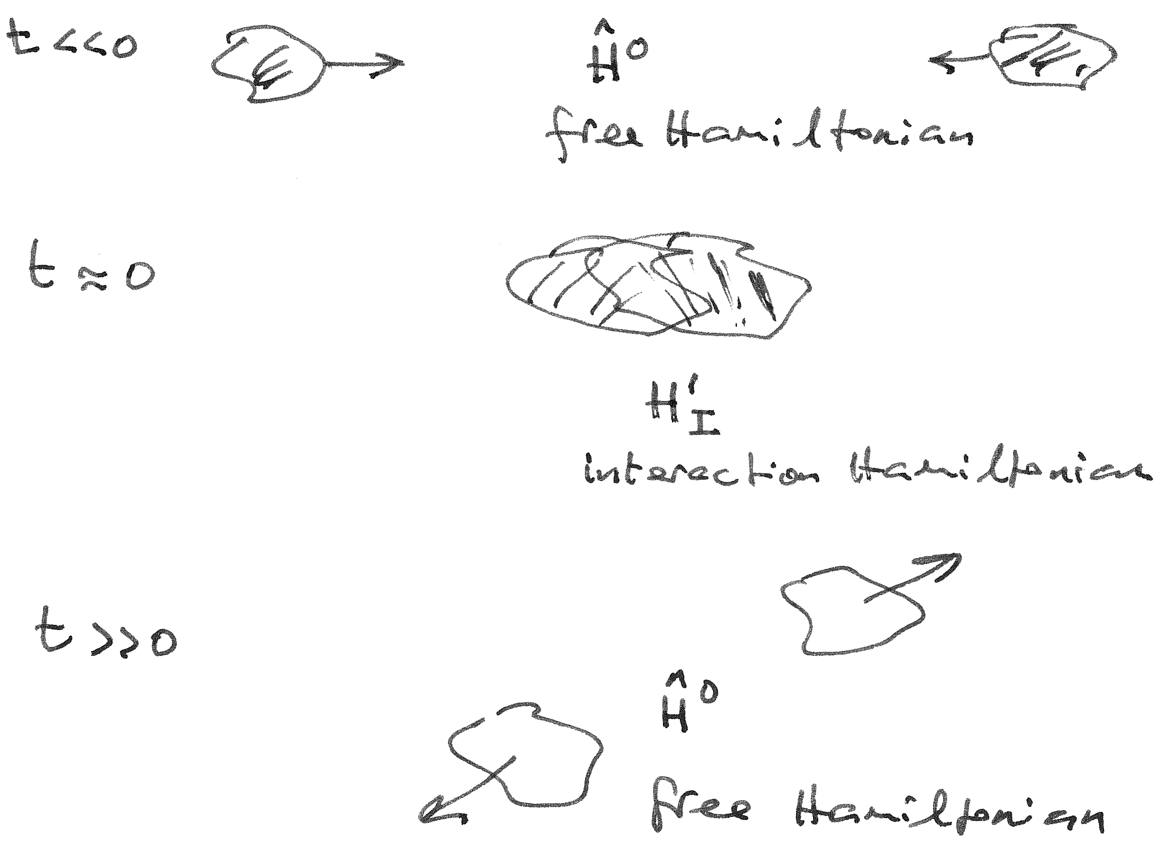
LECTURE 4. SIGMA MODEL

The standard sigma model involves pion, sigma, and nucleon fields. We discuss here an abridged version without nucleons and with no spontaneous symmetry breaking. The goal is to introduce Feynman diagrams.

QUANTUM MECHANICS: Three representations (or "pictures"): (i) Schrödinger: operators are time independent, (ii) Heisenberg wave functions (state vectors) are time independent. (iii) Interaction both wave functions & operators are time-dependent.

Assumption: Interaction takes place at and around a given time, e.g. $t=0$.

Scattering process



The transition is effected by the "time-evolution" operator:

$$\Psi_I(t) = U(t,t_0)\,\Psi_I(t_0) \qquad (\text{IV}.1)$$

where $U(t,t_0)$ is determined from $H_I'(t)$. Schrödinger's eqn. then becomes ($\hbar = 1$)

$$i\frac{\partial \Psi_I(t)}{\partial t} = \hat{H}_I'(t)\,\Psi_I(t) = \hat{H}_I'\left[U(t,t_0)\,\Psi_I(t_0)\right]$$

$$= i\frac{\partial}{\partial t}\left[U(t,t_0)\,\Psi_I(t_0)\right] \qquad (\text{IV}.2)$$

where Eq. (IV.1) was used in the last step above. This leads to the evolution equation

$$\boxed{i\frac{d}{dt}U(t,t_0) = \hat{H}_I'(t)\,U(t,t_0)} \qquad (\text{IV}.3)$$

Solving this equation SOLVES THE SCATTERING PROBLEM

Knowledge of $H_I'(t)$ is required in (IV.3). In some cases only partial knowledge is available, or the problem is too complicated. In this case there is an approximate method: PERTURBATION THEORY:

From (IV.3) one obtains after integration and using $U(t,t)=1$

$$\boxed{U(t,t_0) = 1 - i\int_{t_0}^{t} dt_1\, \hat{H}_I'(t_1)\,U(t_1,t_0)} \qquad (\text{IV}.4)$$

This is NOT a solution, as the unknown ($U(t,t_0)$) appears also under the integral in the right-hand-side.

Performing a first iteration one writes for $U(t_1, t_0)$ (inside the integral) a similar integral equation, i.e.

$$U(t_1, t_0) = 1 - i \int_{t_0}^{t_1} dt_2 \, \hat{H}_I'(t_2) \, U(t_2, t_0) \quad (\text{IV}.5)$$

Substituting in (IV.4) gives

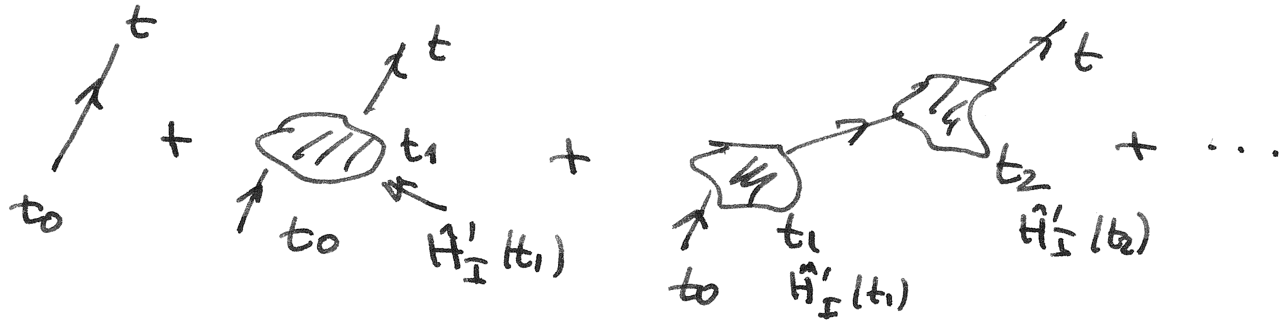
$$U(t, t_0) = 1 - i \int_{t_0}^{t} dt_1 \, \hat{H}_I'(t_1) \left[1 - i \int_{t_0}^{t_1} dt_2 \, \hat{H}_I'(t_2) \, U(t_2, t_0) \right] \quad (\text{IV}.6)$$

Repeating the procedure, now for $U(t_2, t_0)$, etc. leads to the infinite series

$$U(t, t_0) = 1 - i \int_{t_0}^{t} dt_1 \, \hat{H}_I'(t_1) + (-i)^2 \int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2 \, \hat{H}_I'(t_1) \hat{H}_I'(t_2)$$

$$+ \cdots (-i)^n \int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2 \cdots \int_{t_0}^{t_{n-1}} dt_n \, \hat{H}_I'(t_1) \hat{H}_I'(t_2) \cdots \hat{H}_I'(t_n) \, U(t_n, t_0) \quad (\text{IV}.7)$$

The idea is that if the interaction is not too "strong" the series converges rapidly and only a few terms are needed. Graphically this expansion can be pictured as



If there is a coupling involved in the interaction, e.g. g, then the diagrams are of order $\mathcal{O}(g^0 = 1)$, $\mathcal{O}(g)$, $\mathcal{O}(g^2)$, etc.

It is possible to rearrange Eq. (IV.7) so that the upper limit of integration is always $\underline{\underline{t}}$.
[see the book Quang Ho-Kim & X.Y. Pham, Elementary Particles & their Interactions, Springer, (1998)]. The result is

$$U(t,t_0) = 1 - i\int_{t_0}^{t} dt_1\, \hat{H}'_I(t_1) + \frac{(-i)^2}{2!}\int_{t_0}^{t} dt_1 \int_{t_0}^{t} dt_2 \tag{IV.8}$$

$$* \; T\left[\hat{H}'_I(t_1)\, \hat{H}'_I(t_2)\right] + \dots + \frac{(-i)^n}{n!}\int_0^t dt_1 \int_0^t dt_2 \cdots \int_0^t dt_n$$

$$* \; T\left[H'_I(t_1) \cdots \hat{H}'_I(t_n)\right],$$

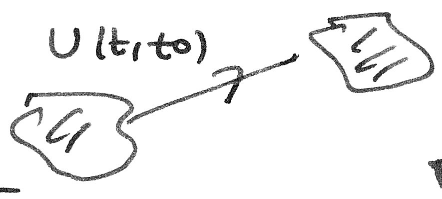
where the "time ordered" product is

$$T\left[\hat{A}(t_1)\,\hat{A}(t_2)\right] = \hat{A}(t_1)\,\hat{A}(t_2)\,\theta(t_1-t_2) + \hat{A}(t_2)\,\hat{A}(t_1)\,\theta(t_2-t_1), \tag{IV.9}$$

and $\theta(z) = \begin{cases} 0 & z<0 \\ 1 & z \geq 0 \end{cases}$ is the Heavyside step funct.

S-Matrix

The scattering process, as described above using the time evolution operator $U(t,t_0)$ can be pictured as



$$\Phi_I(i,t) = U(t,t_0)\,\Phi_i$$

$$\Phi_I(t_0) \equiv \Phi_i$$

The probability of finding $\Phi_I(i,t)$ in the final state is

$$\langle \Phi_f | \Phi_I(i,t)\rangle = \langle \Phi_f | U(t,t_0) | \Phi_i\rangle \equiv U_{fi}(t,t_0) \tag{IV.10}$$

Taking the limit $t_0 \to -\infty$ and $t \to +\infty$ gives the so called S-matrix

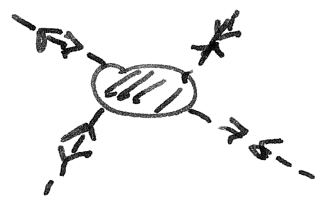
$$U(\infty, -\infty) \equiv S_{fi} \qquad (\text{IV}.11)$$

$$S_{fi} = T\left\{ \exp\left[-i \int_{-\infty}^{\infty} dt \int d^3x \, \hat{\mathcal{H}}_I'(t, \vec{x}) \right] \right\} \qquad (\text{IV}.12)$$

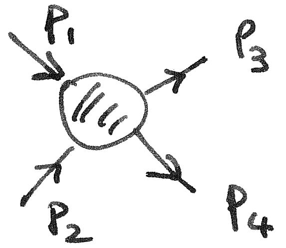
where $\hat{\mathcal{H}}_I'$ is the Hamiltonian density. The Taylor series expansion of (IV.12) gives (IV.9).

DECAY RATES

SCATTERING



Mandelstam (kinematical) variables s, t, u:



$$\left\{ \begin{aligned} s &= (p_1 + p_2)^2 = (p_3 + p_4)^2 \\ t &= (p_1 - p_3)^2 = (p_4 - p_2)^2 \\ u &= (p_1 - p_4)^2 = (p_2 - p_3)^2 \end{aligned} \right. \qquad (\text{IV}.13)$$

TUTORIAL: Show that $s + t + u = m_1^2 + m_2^2 + m_3^2 + m_4^2$.

s-channel $\equiv$

$$\left\{ \begin{aligned} s &= (E_1 + E_2)^2 = (E_3 + E_4)^2 = E_{CM}^2 \\ t &= m_1^2 + m_3^2 - 2E_1 E_3 + 2|\vec{p}|^2 \cos\theta_{CM} \end{aligned} \right.$$

Center of mass (CM) system
$\vec{p}_1 + \vec{p}_2 = 0 = \vec{p}_3 + \vec{p}_4$

s: square of CM energy

t: proportional to the scattering angle (cosine).

T & M Scattering amplitude

$$S_{fi} = \delta_{fi} + i\, T_{fi} \;;\quad T_{fi} = (2\pi)^4 \delta^{(4)}(P_f - P_i)\, M_{fi} \quad (\text{IV}.14)$$

no-interaction Dynamics $\quad P_{i,f} = \sum_j (P_{i,f})_j$

Of interest is $M_{fi} \equiv \langle f | M | i \rangle$, as it gives the decay rate or the differential cross section

$$\underbrace{d\Gamma}_{\text{decay rate}} \propto |\langle f|M|i\rangle|^2 \, dLips|_f \;;\quad \frac{d\sigma}{d\Omega} \propto |\langle f|M|i\rangle|^2$$

$dLips$: Lorentz-invariant phase space of the final state

$\Gamma \sim \frac{1}{\tau}$ ← lifetime

See Quang Ho-Kim & X.Y. Pham (op. cit.)

LINEAR SIGMA-PION MODEL

$\sigma^0 \qquad \pi^- \qquad \pi^0 \qquad \pi^+$

$\mu \qquad m_{\pi^\pm} \simeq m_{\pi^0}$

Pions: (i) $\hat\phi_a(x)$ $(a = 1,2,3) \Rightarrow (\pi^+, \pi^-, \pi^0)$ real fields

or (ii) $\underbrace{\hat\varphi(x) = \frac{1}{\sqrt{2}}(\phi_1(x) \pm i\phi_2(x)}_{\substack{\pi^\pm \\ \text{complex field}}}$ & $\underbrace{\phi_3(x)}_{\substack{\pi^0 \\ \text{real field.}}}$

Free Lagrangian densities

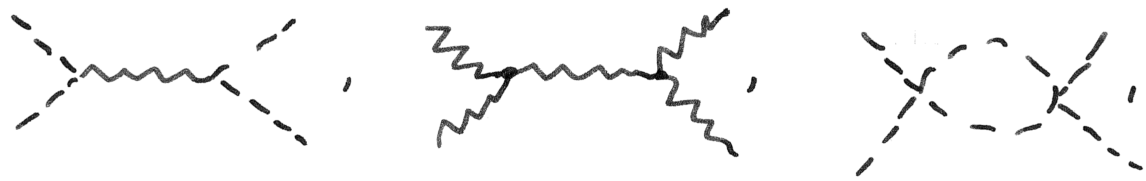
$$\mathcal{L}_\pi = \frac{1}{2}\partial^\mu \hat\phi_3(x)\partial_\mu \hat\phi_3(x) - \frac{1}{2} m_0^2 \hat\phi_3^2(x) + \partial^\mu \hat\varphi^\dagger(x)\partial_\mu \hat\varphi(x) - m_0^2 \hat\varphi^\dagger(x)\hat\varphi(x) \qquad (m_0 \equiv m_\pi)$$

$$\mathcal{L}_\sigma = \frac{1}{2}\partial^\mu \hat\sigma(x)\partial_\mu \hat\sigma(x) - \frac{1}{2}\mu^2 \sigma^2(x) \;;\quad \underbrace{\hat\sigma = \hat\sigma^\dagger}_{\text{neutral particle.}}$$

Interaction Lagrangians:

e.g. $-g\,\hat{\phi}_3^4(x)$ $\quad -g''\,\hat{\sigma}^4(x)$ $\quad -g'\,\hat{\sigma}^2(x)\hat{\phi}_3^2(x)$ $\quad -\lambda\,\hat{\sigma}(x)\,\hat{\phi}^2(x)$ $\quad -\lambda\hat{\sigma}^3(x)$

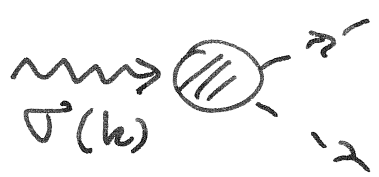
No derivatives, just fields. All couplin constants are dimensionless. Pion is a pseudoscalar, Sigma is a scalar. Hence no $3\pi, 5\pi, 7\pi$ in a point. Iteration of the above processes lead to higher order terms, e.g.

 , etc.

The interaction Lagrangians don't contain derivatives, hence

$$\hat{\mathcal{H}}_I \equiv \underbrace{\frac{\partial \mathcal{L}_I}{\partial(\partial_0 \hat{\phi}_i)}}_{=0} \partial_0 \phi_i - \hat{\mathcal{L}}_I = -\hat{\mathcal{L}}_I \quad (\text{IV}.15)$$

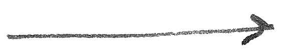
EXAMPLE: $\sigma \to \pi^0 \pi^0$ decay



$\sigma(k)$ $\quad \pi^0(p_1)$ $\quad \pi^0(p_2)$

$$S_{fi} = (\delta_{fi}) - i\langle f|\hat{\mathcal{H}}_I|i\rangle + \cdots$$

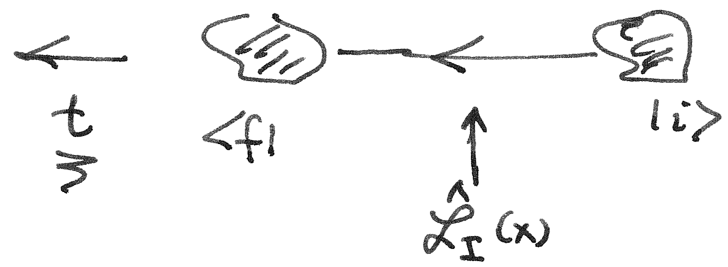
$$S_{fi}^{(1)} = +i\langle f|\hat{\mathcal{L}}_I|i\rangle$$



t

$$(\text{IV}.16) \quad \boxed{S_{fi}^{(1)} = -i\lambda\langle f|\int d^4x\, \hat{\phi}_3^2(x)\hat{\sigma}(x)|i\rangle}$$

$$\langle f | \hat{\mathcal{L}}_I(x) | i \rangle$$



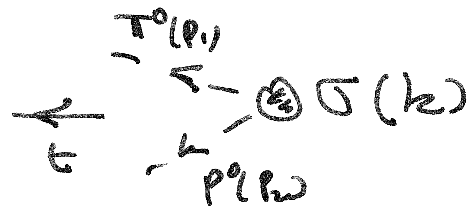
It all happens at the space-time point x. It has been drawn as separate only for clarity.

<u>Procedure</u>

1) write down $|i\rangle$

2) write down $\langle f|$

3) Insert $\hat{\mathcal{L}}_I(x)$



4) Identify the terms that lead to $\# \langle 0|0\rangle$ where $\#$ will be S_{fi}

1) 2): $\langle f| = \langle \pi^0(p_1)\, \pi^0(p_2)| = \langle 0| \hat{a}_{\vec{p}_1} \hat{a}_{\vec{p}_2} \quad C^{-1}_{\vec{p}_1} \; C^{-1}_{\vec{p}_2}$

$|i\rangle = \hat{c}^\dagger_k |0\rangle \quad C^{-1}_{\vec{k}}$

3) $S^{(1)}_{fi} = -i\lambda \, C^{-1}_{\vec{p}_1} C^{-1}_{\vec{p}_2} C^{-1}_{\vec{k}} \langle 0| \hat{a}_{\vec{p}_1} \hat{a}_{\vec{p}_2} \mathcal{L}_I \hat{c}^\dagger_k |0\rangle$

$= -i\lambda \,(C'_1)^{-1} \langle 0| \hat{a}_{\vec{p}_1} \hat{a}_{\vec{p}_2} \int d^4x \; \hat{\phi}_3(x)\, \hat{\phi}_3(x)\, \hat{\sigma}(x)\, \hat{c}^\dagger_{\vec{k}} |0\rangle$

Need to substitute $\hat{\phi}_3\, \hat{\phi}_3\, \hat{\sigma}$, but the final result should be proportional to $\langle 0|0\rangle = 1$.

$$\hat{\sigma}(x) = \int d^3k' \left[\underbrace{\hat{c}_{\vec{k}'}\, \phi^{(+)}_{\vec{k}'}(x)}_{\text{surviving term}} + \underbrace{\hat{c}^\dagger_{\vec{k}'}\, \phi^{(-)}_{-\vec{k}'}(x)}_{\text{cannot contribute}} \right]$$

Notice: $\hat{\sigma}(x)\, \hat{c}^{\dagger}_{\vec{k}} |0\rangle \Rightarrow \begin{cases} \hat{c}_{\vec{k}'}\, \hat{c}^{\dagger}_{\vec{k}} |0\rangle & (a) \\ \hat{c}^{\dagger}_{\vec{k}'}\, \hat{c}^{\dagger}_{\vec{k}} |0\rangle & (b) \end{cases}$

(a) is correct: $\hat{c}^{\dagger}_{\vec{k}} |0\rangle \propto |\sigma(k)\rangle$

& $\hat{c}_{\vec{k}'}\, \hat{c}^{\dagger}_{k} |0\rangle = \hat{c}_{k'} |\sigma(k)\rangle \Rightarrow |0\rangle$

(b) is not contributing as it would produce $2\,\sigma$ mesons at x. – (there is only one in the initial state).

Similarly

$$\langle 0|\, \hat{a}_{\vec{p}_1}\, \hat{a}_{\vec{p}_2}\, \hat{\phi}_3(x)\, \hat{\phi}_3(x) \dots$$

involves $\hat{a}^{\dagger}_{\vec{p}_1'}$ & $\hat{a}_{\vec{p}_1'}$ $\qquad \hat{a}^{\dagger}_{\vec{p}_2'}\, \hat{a}_{\vec{p}_2'}$

only the combination $\langle 0|\, \underbrace{\hat{a}_{\vec{p}_1}\, \hat{a}_{\vec{p}_2}}_{\text{create } \pi^0\pi^0}\, \underbrace{\hat{a}^{\dagger}_{\vec{p}_1'}\, \hat{a}^{\dagger}_{\vec{p}_2'}}_{\text{annihilate } \pi^0\pi^0} \dots$ contributes for the same reasons as in a).

$$S_{fi} = -i\lambda (C's)^{-1} \langle 0|\, \hat{a}_{\vec{p}_1}\, \hat{a}_{\vec{p}_2} \int d^4x \int d^3p_1' \int d^3p_2' \int d^3k'\; \hat{a}^{\dagger}_{\vec{p}_1'}\, \phi^{(-)}_{-\vec{p}_1'}(x)\, \hat{a}^{\dagger}_{\vec{p}_2'}\, \phi^{(-)}_{-\vec{p}_2'}(x)\, \hat{c}_{\vec{k}'}\, \phi^{(+)}_{\vec{k}'}(x)\, \hat{c}^{\dagger}_{k} |0\rangle$$

$$= -i\lambda (C's)^{-1} \langle 0|\, \hat{a}_{\vec{p}_1}\, \hat{a}_{\vec{p}_2}\, \hat{a}^{\dagger}_{\vec{p}_1'}\, \hat{a}^{\dagger}_{\vec{p}_2'}\, \hat{c}_{\vec{k}'}\, \hat{c}^{\dagger}_{k} \int d^4x \int d^3p_1' \int d^3p_2' \int d^3k'\; \phi^{(-)}_{-\vec{p}_1'}(x)\, \phi^{(-)}_{-\vec{p}_2}(x)\, \phi^{(+)}_{\vec{k}'}(x) |0\rangle$$

Let us concentrate on the algebra of the operators:

$$VV \equiv \langle 0| \hat{a}_{\vec{p}_2} \underbrace{\hat{a}_{\vec{p}_1} \hat{a}^\dagger_{\vec{p}_1'}} \hat{a}^\dagger_{\vec{p}_2'} \underbrace{\hat{c}_{\vec{k}'} \hat{c}^\dagger_{\vec{k}}} |0\rangle =$$

$$\Rightarrow \quad = \delta(\vec{p}_1 - \vec{p}_1') + \hat{a}^\dagger_{\vec{p}_1'} \hat{a}_{\vec{p}_1}$$

replace by $[\hat{c}_{\vec{k}'}, \hat{c}^\dagger_{\vec{k}}]$ as $\hat{c}_{\vec{k}'}|0\rangle = 0$ & $[\hat{c}_{\vec{k}'}, \hat{c}^\dagger_{k}] = \delta^{(3)}(\vec{k}-\vec{k}')$

(& used $\hat{a}_{\vec{p}_2} \hat{a}_{\vec{p}_1} = \hat{a}_{\vec{p}_1} \hat{a}_{\vec{p}_2}$)

$$VV = \langle 0| \hat{a}_{\vec{p}_2} \left[\delta(\vec{p}_1 - \vec{p}_1') + \hat{a}^\dagger_{\vec{p}_1'} \hat{a}_{\vec{p}_1} \right] \hat{a}^\dagger_{\vec{p}_2'} \delta^{(3)}(\vec{k}-\vec{k}') |0\rangle$$

$$= \langle 0| \left\{ \delta^{(3)}(\vec{p}_1 - \vec{p}_1') \underbrace{\hat{a}_{\vec{p}_2} \hat{a}^\dagger_{\vec{p}_2'}}_{[\hat{a}_{\vec{p}_2}, \hat{a}^\dagger_{\vec{p}_2'}]} + \underbrace{\hat{a}_{\vec{p}_2} \hat{a}^\dagger_{\vec{p}_1'}}_{[\hat{a}_{\vec{p}_2}, \hat{a}^\dagger_{\vec{p}_1'}]} \underbrace{\hat{a}_{\vec{p}_1} \hat{a}^\dagger_{\vec{p}_2'}}_{[\hat{a}_{\vec{p}_1}, \hat{a}^\dagger_{\vec{p}_2'}]} \right\} |0\rangle$$

$$\times \delta^{(3)}(\vec{k} - \vec{k}')$$

as $\langle 0|\hat{a}^\dagger_{\vec{p}_2'} = 0$

as $\hat{a}_{\vec{p}_1}|0\rangle = 0$

as $\hat{a}_{\vec{p}_2'}|0\rangle = 0$

$$VV = \delta^{(3)}(\vec{k}-\vec{k}') \left\{ \delta^{(3)}(\vec{p}_1 - \vec{p}_1') \delta^{(3)}(\vec{p}_2 - \vec{p}_2') + \right.$$

$$\left. + \delta^{(3)}(\vec{p}_2 - \vec{p}_1') \delta^{(3)}(\vec{p}_1 - \vec{p}_2') \right\}$$

$$\times \underbrace{\langle 0|0\rangle}_{1}$$

substituting in S_{fi} the $(C's)^{-1}$ cancel out with those in the plane waves ϕ's.

$$S_{fi} = -i\lambda \int d^4x \int d^3p_1' \int d^3p_2' \int d^3k' \underbrace{e^{-ik'\cdot x}}_{\text{from } \phi^{(+)}_{\vec{k}'}(x)} \underbrace{e^{ip_1'\cdot x}}_{\phi^{(-)}_{\vec{p}_1'}} \underbrace{e^{ip_2'\cdot x}}_{\phi^{(-)}_{\vec{p}_2'}}$$

$$\times \delta^{(3)}(\vec{k}-\vec{k}') \left[\delta^{(3)}(\vec{p}_2 - \vec{p}_2') \delta^{(3)}(\vec{p}_1 - \vec{p}_1') + \right.$$

$$\left. + \delta^{(3)}(\vec{p}_2 - \vec{p}_1') \delta^{(3)}(\vec{p}_1 - \vec{p}_2') \right]$$

If $\begin{cases} \vec{p}_2 = \vec{p}_2' & \text{then } E_2 = E_2' \\ \vec{p}_1 = \vec{p}_1' & \text{the } E_1 = E_1' \end{cases}$

$$\int d^3k' \, e^{-ik'^0 \cdot x^0} \, e^{+i\vec{k}' \cdot \vec{x}} \, \delta^{(3)}(\vec{k}-\vec{k}') =$$

$$= \underbrace{e^{-ik'^0 x^0}}_{= e^{ik^0 x^0}} \, e^{i\vec{k}\cdot\vec{x}} = e^{-ik\cdot x}$$

because if $|\vec{k}'| = |\vec{k}| \Rightarrow k'^0 = k^0$, $k^0 = \sqrt{|\vec{k}|^2 + \mu^2}$

and similarly for the other two momentum integrals.

$$S_{fi} = -i\lambda \int d^4x \, e^{-ik\cdot x} \left[e^{i(P_1+P_2)\cdot x} + e^{i(P_2+P_1)\cdot x} \right]$$

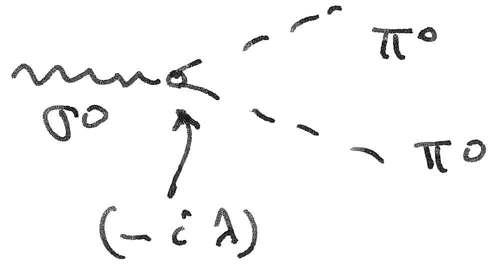
$$= -2i\lambda \, (2\pi)^4 \, \delta^{(4)}(P_1+P_2-k)$$

$$= (2\pi)^4 \, \delta^{(4)}(P_1+P_2-k) \, \underbrace{[-2i\lambda]}_{iM_{fi}}$$

[From (IV.14) $S^{(1)}_{fi} = i(2\pi)^4 \delta^{(4)}(P_f - P_i) M_{fi}$]

$$\boxed{iM_{fi} = -2i\lambda}$$

Feynman rule:



But a factor 2 must be introduced because the final state is $\pi^0(P_1)\,\pi^0(P_2) = \pi^0(P_2)\,\pi^0(P_1)$ so $M_{fi} = 2(-i\lambda)$.– this symmetrization does not enter in $\sigma \to \pi^+\pi^-$ as the pions are different.

<u>TUTORIAL</u>: Compute $S_{fi}^{(1)}$ for the decay

$$\sigma(k) \to \pi^+(P_1)\, \pi^-(P_2)$$

LECTURE 5 - QCD I

Global symmetries of the strong interactions among hadrons

SU(2) [Isospin] Symmetry:

$$\begin{pmatrix} p^+ \\ n^0 \end{pmatrix} \quad \begin{pmatrix} \pi^+ \\ \pi^0 \\ \pi^- \end{pmatrix} \quad \begin{pmatrix} \rho^+ \\ \rho^0 \\ \rho^- \end{pmatrix} \quad \text{etc.}$$

SU(2) generated by the Pauli matrices σ_i.

$\begin{pmatrix} p^+ \\ n^0 \end{pmatrix}$ Isospin $I = \frac{1}{2}$ N° of states $2I+1 = 2$.

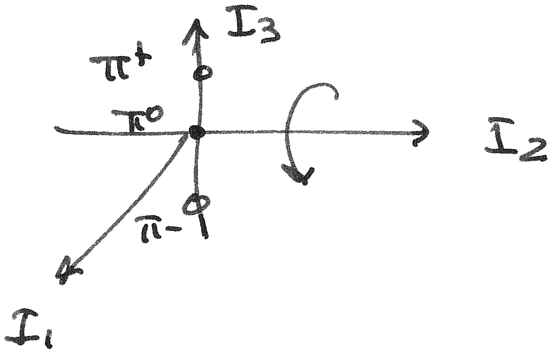
$$I_3 = \begin{cases} +1/2 & p^+ \\ -1/2 & n^0 \end{cases}$$

The electromagnetic interaction breaks SU(2) $m_{p^+} \neq m_{n^0}$, but it is a small effect.

Strong interaction: Invariant under I-spin rotations as well as charge conjugation C. However I & C do not commute

$$C\,|\pi^+\rangle = |\pi^-\rangle \neq |\pi^+\rangle .$$

Introduce G-parity $G = C\, e^{i\pi I_2}$



$$e^{i\pi I_2}\,|\pi^+\rangle = |\pi^-\rangle$$

$$C\,|\pi^-\rangle = |\pi^+\rangle$$

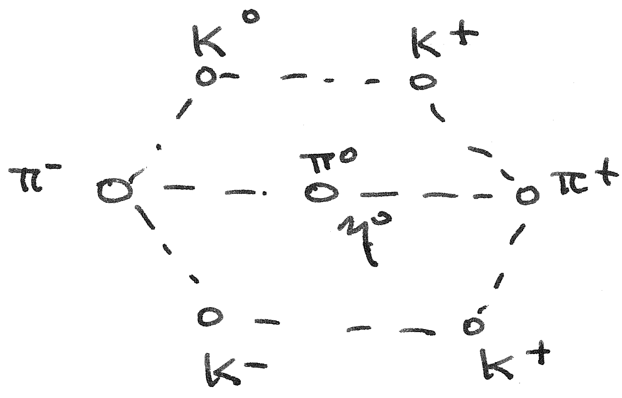
$$\therefore \quad G\,|\pi^+\rangle = |\pi^+\rangle$$

$$G\,|\pi^-\rangle = |\pi^-\rangle$$

G-parity of π is 1.

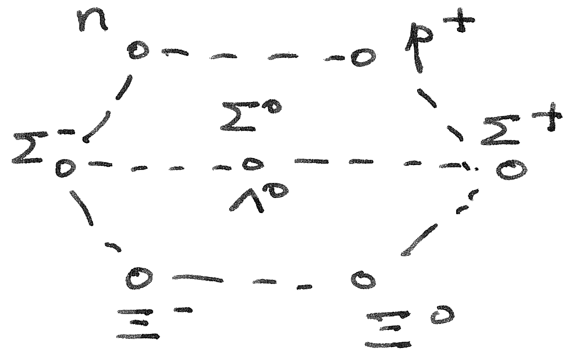
From SU(2) $\Longrightarrow$ SU(3)

Meson octet:
(Spin = 0)



SU(3)

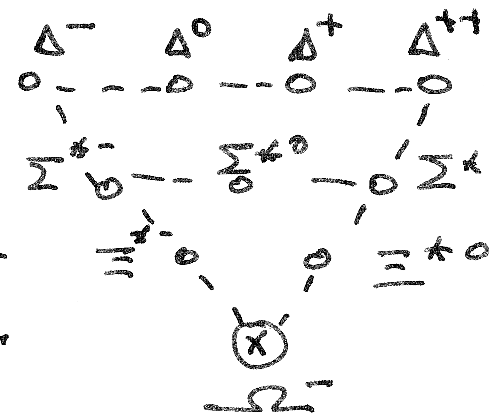
Baryon octet
(Spin = 1/2)



SU(3)

Baryon decuplet
(Spin = 3/2)

The Ω^- had not yet been discovered when SU(3) was proposed.



SU(3)

Hadrons transform under the irreducible representations of the unitary groups, SU(2), SU(3). The fundamental representation is $\underline{\underline{3}}$ for SU(3). Particles: QUARKS: up, down, strange. Not seen in experiments. Remained hypothetical until their discovery in the late 1960's, but they do not exist as free particles. QUARK CONFINEMENT.

Quarks:

$$\begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix} \quad \begin{matrix} Q/|e| \\ 2/3 \\ -1/3 \end{matrix}$$

$|p^+\rangle = |uud\rangle$; $|n^0\rangle = |udd\rangle$; $|\pi^+\rangle = |\bar{d}u\rangle$

$|\pi^-\rangle = |\bar{u}d\rangle$; $|\pi^0\rangle = \frac{\bar{u}u - \bar{d}d}{\sqrt{2}}$; $|\Delta^{++}\rangle = |uuu\rangle$

Problem $|\Delta^{++}\rangle$ should obey Pauli's principle but the ground state is made of three identical quarks $|\underset{u}{\Uparrow}\underset{u}{\Uparrow}\underset{u}{\Uparrow}\rangle$ $S = 3/2$.

(i) give up Pauli, or (ii) invoke a new quantum number.

Still, case is not closed $\Rightarrow$ Additional evidence?

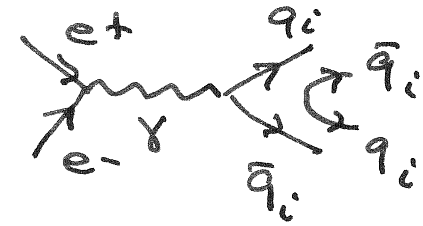
$\pi^0 \to \gamma\gamma$

The decay width is $\Gamma(\pi^0 \to \gamma\gamma)\big|_{THY} = \frac{7.73}{9}$ eV

from theory, but $\Gamma(\pi^0 \to \gamma\gamma)\big|_{EXP} = 7.8 \pm 0.5$ eV

from experiment. A factor 3 in the S-matrix ($9 = 3^2$ in the rate) is needed.

$e^+e^- \to$ hadrons



$$R \equiv \frac{\sigma(e^+e^- \to had)}{\sigma(e^+e^- \to leptons)} = \sum_{i=1}^{n_F} e_i^2$$

e.g. uds $R\big|_{THY} = \left(\frac{2}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 = \frac{2}{3}$

$R\big|_{EXP} \approx 2$ (again a factor 3)

New mesons at $E \approx 3$ GeV uncover the charm quark

$R_{EXP} \approx 3.3$; $R_{THY} = \frac{10}{9}$ still a factor 3 missing

Further up at $E \approx 5$ GeV : bottom quark

$R_{EXP} \approx 3.6$; $R_{THY} = \frac{11}{9}$ still a 3 missing.

Fritzsch & Gell-Mann : 1972

"COLOUR" degree of freedom

$N_C = 3$

Each quark comes in three varieties 1,2,3 or red, green, blue, etc. But hadrons are colour singlets, i.e. they have no net colour. The gluons, which transmit the strong force also carry colour. Hence $\mathcal{L}_{QCD}$ is NONLINEAR !

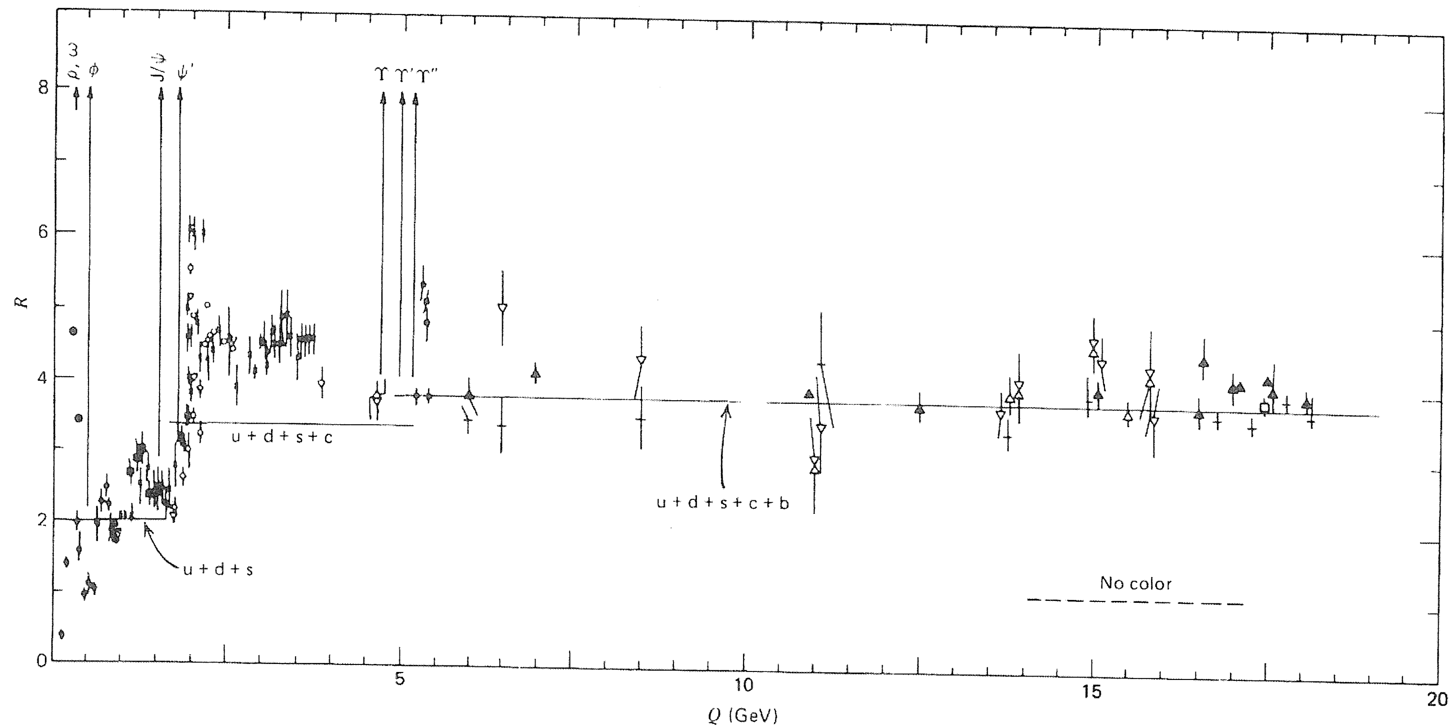
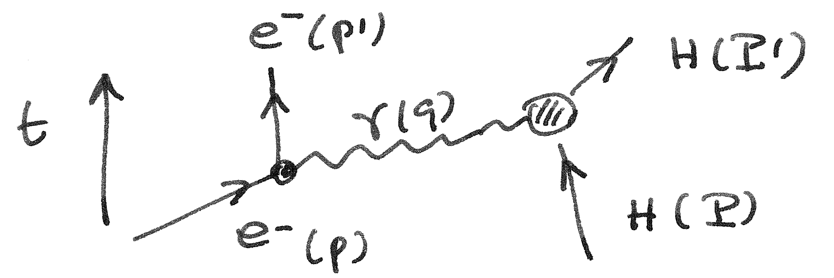


FIGURE 15.9
Plot of $R = \sigma(e^+ + e^- \rightarrow hadrons)/\sigma(e^+ + e^- \rightarrow \mu^+ + \mu^-)$ vs. energy, indicating the predictions of the standard model with three colors.

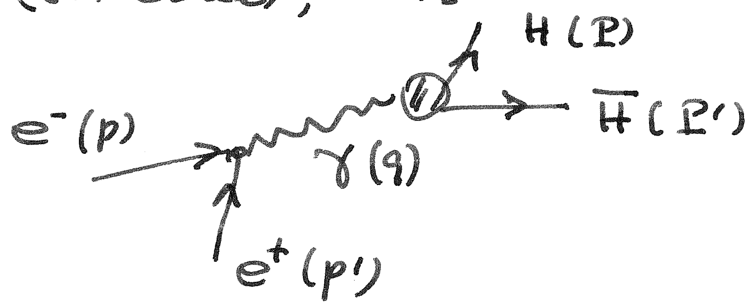
ELECTROMAGNETIC STRUCTURE OF HADRONS (FORM FACTORS)

Consider e^- H(adron) scattering where $H = \pi^\pm$ or $p^+_i \ldots$



elastic scattering (final state is also $e^- H$)

In CM system
$$\underline{q^2 = -2|\vec{p}|\,|\vec{p}'|\,(1-\cos\theta)}$$
neglecting the electron mass. Hence $q^2 < 0$, i.e. <u>space-like</u>. Instead, in $e^+e^- \to H\bar{H}$, $q^2 > 0$ (timelike), viz.



$$\underline{q^2 = (p+p')^2 > 0}$$

<u>The electron</u>: Dirac eqn. predicts $\mu_{e^-} = \mu_B = \frac{e\hbar}{2m_e}$

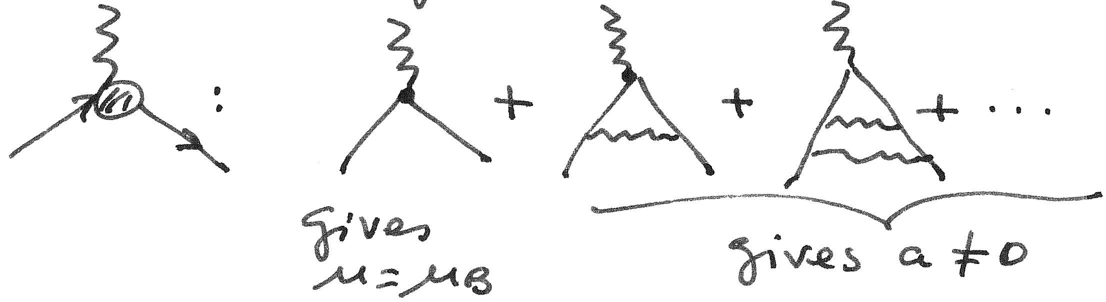
or $g_{e^-} = 2$ and $a = \frac{g-2}{2} = 0$, but $a_{Exp} = 0.0011\ldots$

i) e^- is not pointlike (is made of constituents)

ii) e^- is pointlike, but anomaly has a different origin. (The correct assumption)

<u>Vacuum fluctuations</u>: $\underline{\underline{\Delta E \, \Delta t \sim \hbar}}$

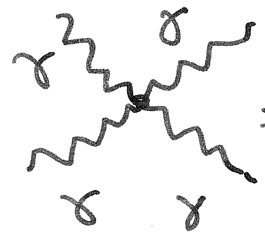
A photon probing an electron:



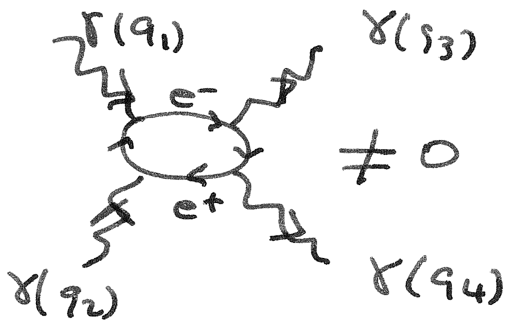
Quantum Electrodynamics (QED).

$$\mathcal{L}_{QED} = \bar{\psi}(x)(i\gamma^\mu \partial_\mu - m_0)\psi(x) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + J_\mu(x)A^\mu(x)$$

At this level $\mathcal{L}_{QED}$ is quadratic in the fields, hence linear in the equations of motion. Since γ has no electric charge there is no "direct" interaction among photons (which would create nonlinearity). However, in a quantum field theory, the vacuum is part of the theory. For instance

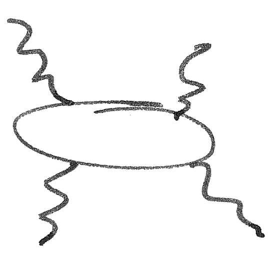


$= 0$ but

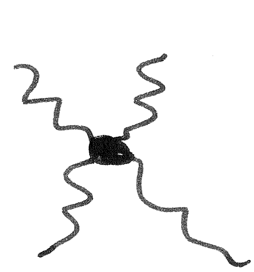


$\neq 0$

If the mass of the e^- is much larger than the momenta of the photons then



$\approx$



which is effectively a non-linear effect.

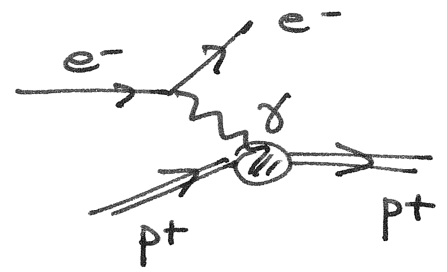
Computing all diagrams up to a certain order gives today:

$$\begin{cases} a_{EXP} = 0.001\ 159\ 652\ 180.73\ (0.28) \\ a_{THY} = 0.001\ 159\ 652\ 181.82\ (0.78) \end{cases}$$

precision better than one in a billion !!

BEST MEASURED & CALCULATED NUMBER

<u>The proton</u>:



If p^+ were pointlike $\mu_{p^+} \approx \mu_B$, as the e^-, with small corrections due to vacuum fluctuations. The electron would be probing a Coulomb electric field with the potential:

$$V(r) = V_{COUL}(r) = \frac{|e|}{4\pi r}$$

However, $\mu_{p^+} \simeq 1.79\, \mu_B$! In addition, the partner of p^+, i.e. the neutron has $\mu_{n^0} \simeq -1.91\, \mu_B$! Neither p^+ nor n^0 could be pointlike, and the potential should become

$$V(r) = \frac{1}{4\pi} \int d^3 y \, \frac{\rho(\vec{y})}{|\vec{x} - \vec{y}|}$$

where $\rho(\vec{y})$ is the charge distribution. In the limit of a pointlike object

$$\lim \rho(y) = |e|\, \delta^3(y)$$

and

$$\lim V(r) = \frac{|e|}{4\pi r} = V_{COUL}.$$

It will become useful to compute the Fourier transform of $V_{COUL}(r)$, i.e. V_{COUL} in "momentum space", i.e.

$$V_{COUL}(\vec{q}) = \frac{1}{4\pi}\int d^3x \; e^{-i\vec{q}\cdot\vec{x}} \; V_{COUL}(r)$$

The integral can be calculated easily as follows

$$\int d^3x \, e^{-i\vec{q}\cdot\vec{x}}\left(\frac{1}{r}\right) = \lim_{\mu\to 0}\int d^3x \; e^{-i\vec{q}\cdot\vec{x}} \; \frac{e^{-\mu r}}{r} =$$

$$= \lim_{\mu\to 0} 2\pi \int_0^\infty\int_{-1}^{1} \frac{e^{-\mu r}}{r} r^2 dr \; e^{-i|\vec{q}|r\cos\theta} \, d(\cos\theta)$$

$$= \lim_{\mu\to 0} 2\pi \int_0^\infty \frac{e^{-\mu r}}{r} r^2 \, \frac{2\sin|\vec{q}|r}{|\vec{q}|r} \, dr$$

$$= \lim_{\mu\to 0} \frac{4\pi}{|\vec{q}|} \underbrace{\int_0^\infty e^{-\mu r}\sin|\vec{q}|r}_{\frac{|\vec{q}|}{\mu^2+|\vec{q}|^2}} = \lim_{\mu\to 0}\frac{4\pi}{\mu^2+|\vec{q}|^2} = \frac{4\pi}{|\vec{q}|^2}$$

including the factor $\frac{1}{4\pi}$ & $|e|$

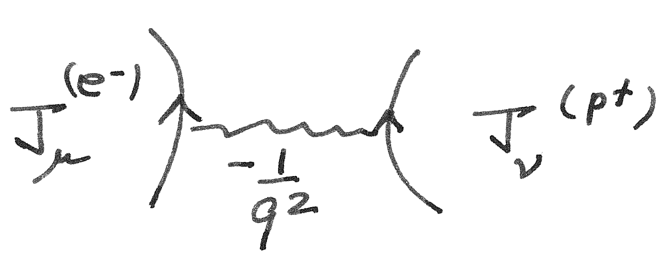
$$\boxed{V_{COUL}(\vec{q}) = \frac{|e|}{|\vec{q}|^2}}$$

which is the Fourier transform of the Coulomb potential

The relativistic extension is $-|\vec{q}|^2 \Longrightarrow q^2 = q_0^2 - |\vec{q}|^2$

$$\boxed{V_{COUL}(q) = -\frac{|e|}{q^2}}$$

Interpreted as



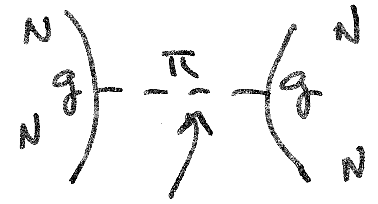
$(J_\mu = |e|\,\bar{\psi}\gamma_\mu\psi)$

A one-photon exchange between EM currents corresponds to a Coulomb interaction.

If μ is kept finite in this calculation, one has instead of the Coulomb potential, the Yukawa (short range) potential

$$V_Y(r) = g\frac{1}{4\pi}\frac{e^{-\mu r}}{r} = g\int \frac{d^3q}{(2\pi)^3}\frac{e^{i\vec{q}\cdot\vec{x}}}{|\vec{q}|^2+\mu^2}$$

which tried to explain the short range nucleon-nucleon interaction in terms of pion exchange, where $\mu = m_\pi$.



$$\frac{1}{|\vec{q}|^2+\mu^2}$$

The prediction was confirmed with the discovery of the π. But the physics is not correct.

Returning to the nucleon, and postulating an extended charge distribution $\rho(\vec{y})$,

$$V_N(\vec{q}) = \frac{1}{4\pi}\int d^3x\, e^{i\vec{q}\cdot\vec{x}}\int d^3y\,\frac{\rho(\vec{y})}{|\vec{x}-\vec{y}|}$$

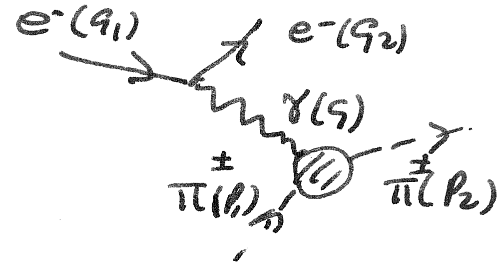
with $\vec{z} = \vec{x}-\vec{y}$ and $\int d^3x\, e^{i\vec{q}\cdot\vec{x}}\frac{1}{r} = \frac{4\pi}{|\vec{q}|^2}$

$$V(\vec{q}) = \frac{1}{4\pi}\int d^3z\,\frac{e^{-i\vec{q}\cdot\vec{z}}}{|\vec{z}|}\int d^3y\, e^{-i\vec{q}\cdot\vec{y}}\rho(\vec{y})$$

$$V(\vec{q}) = \frac{1}{|\vec{q}|^2}\int d^3y\, e^{i\vec{q}\cdot\vec{y}}\rho(\vec{y}) \equiv \frac{F_N(q^2)}{|\vec{q}|^2}$$

where $F_N(q^2)$ is the Fourier transform of the charge distribution, called the EM form factor. For a point nucleon $F_N(q^2) = |e|$, otherwise only $F_N(0) = |e|$.

Pion Form Factor



$$\langle \pi(p_2) | J_\mu^{EM}(0) | \pi(p_1) \rangle$$
$$= (p_1+p_2)_\mu F_\pi(q^2) + (p_2-p_1) G_\pi(q^2)$$

from general Lorentz invariance considerations.

Since $\partial_\mu J^\mu_{EM} = 0 = q^\mu J_\mu^{EM} = (p_2-p_1)^\mu J_\mu^{EM}$

$$0 = \underbrace{(p_2^2 - p_1^2)}_{m_\pi^2 - m_\pi^2 = 0} F_\pi(q^2) + \underbrace{(p_2-p_1)^2}_{\neq 0} \underbrace{G_\pi(q^2)}_{\text{MUST VANISH}}$$

Hence for $\pi^\pm$ there is only one form factor, $F_\pi(q^2)$. It does not correspond to a point particle, hence $\pi^\pm$ has electromagnetic structure.

Nucleon Form Factors

$$\langle N(p_2) | J_\mu^{EM}(0) | N(p_1) \rangle = \bar{u}(p_2) \left[F_1(q^2)\gamma_\mu + \frac{i\sigma_{\mu\nu}q^\nu}{2M} F_2(q^2) \right] \times u(p_1)$$

$F_1^{p^+}(0) = 1 \; (|e|)$; $F_1^{n^0}(0) = 0$

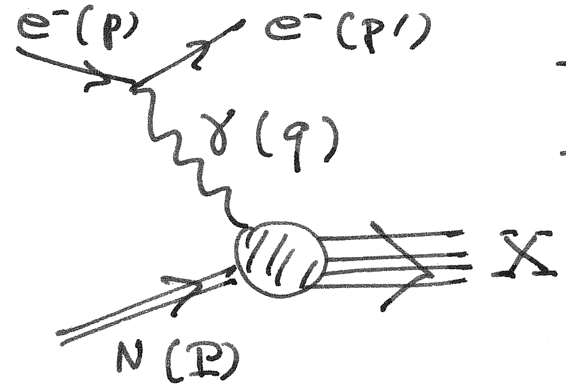
$F_2^{p^+}(0) = 1.79$; $F_2^{n^0}(0) = -1.91$ in μ_B unit

The nucleon being a Dirac particle would have already a $\mu_N = \mu_B$. The deviation is due to substructure.

Elastic experiments cannot reveal more than this. To discover the substructure one needs inelastic scattering data.

Deep Inelastic e⁻- Nucleon scattering

SLAC 1968 – 1972.



$$-q^2 = -(p-p')^2 \equiv Q^2 > 0$$

Two "structure functions"

$$W_{1,2}(Q^2, \nu)$$

$$\nu = \frac{P \cdot q}{M} = E - E' \quad \text{(LAB frame)}$$

$P_{LAB} = (M, \vec{0})$; $p = (E, \vec{p})$; $p' = (E', \vec{p}')$ Bjorken $x \equiv \frac{-q^2}{2M\nu} = \frac{Q^2}{2M\nu}$

Bjorken (1966) predicted that

$$\lim_{\substack{Q^2 \to \infty \\ \nu \to \infty}} \begin{pmatrix} W_1(Q^2, \nu) \\ \nu \frac{W_2}{M}(Q^2, \nu) \end{pmatrix} = \begin{pmatrix} F_1(x) \\ F_2(x) \end{pmatrix}$$

i.e. while $W_{1,2}$ depend on Q^2 & ν, in the Bj. limit they become functions of only one variable, x !!

In addition, the framework in which this prediction was made (infinite P) indicated that if the nucleon had constituents, then they behaved as quasi-free particles.

SLAC experiments found cross sections much larger than expected, with events (scattered electrons) at very large angles: Rutherford revived